

Normal Depth

A rough formed concrete prismatic channel draining an industrial area is 1.7 m wide at the base, with side slopes of 1:4 and bedslope of 0.5 mm per metre. The channel is 400 m long and 1 m deep to the top of the concrete edge. Assuming steady uniform flow, no downstream constrictions and at least 150 mm freeboard to the water surface, what is the maximum flowrate the channel can carry? What is the flow velocity at that stage?

The key phrase is “steady uniform flow”, which points to the Manning equation. In discharge form, we have:

$$Q = VA = AR^{2/3} \frac{\sqrt{S_o}}{n} = \frac{A^{5/3}}{P^{2/3}} \frac{\sqrt{S_o}}{n}$$

Define the terms:

A = flow area = Wy + ky² where W is the base width (1.7 m) and k the side slope (1:4 = 1:k ; k = 4)

P = wetted perimeter = W + 2y√(1+k²)

A and P depend on the flow depth, y, which we are told is 0.15 m freeboard less than the channel depth of 1.0 m: ie y = 1 – 0.15 = 0.85 m

So = bed slope = 0.0005 m/m

n = channel roughness. Looking up “rough formed concrete” we find (study guide) “concrete (formed, no finish) 0.013 – 0.018” Let’s use a high value in that range, say n = 0.017, since we don’t want to overestimate (economic reasons), and definitely don’t want to underestimate (safety and serviceability reasons).

So we can calculate the terms and put it back together:

$$A = Wy + ky^2 = 1.7(0.85) + 4(0.85^2) = 4.335 \text{ m}^2$$

$$P = W + 2y\sqrt{1+k^2} = 1.7 + 2(0.85)(1 + 4^2)^{0.5} = 8.709 \text{ m}$$

$$\text{And } Q = 4.335^{5/3} 0.0005^{0.5} / (8.709^{2/3} 0.017) = 3.581 \text{ m}^3/\text{s}$$

The average velocity is

$$V = Q/A = 3.581/4.335 = 0.826 \text{ m/s}$$

A square concrete channel in a water treatment plant design is to be 200 m long and 2 m wide, but the design engineer has not yet estimated a suitable depth. Growth forecasts suggest a maximum flowrate of 12.7 m³/s in peak periods. The channel is to be cut into a shallow hillside at a slope of 5 mm/m and is to be lined in concrete box culvert sections. Assuming steady uniform flow, how deep does the channel need to be if a freeboard of 100 mm is required?

Assuming steady uniform flow means “use Manning’s equation”:

$$Q = VA = AR^{2/3} \frac{\sqrt{S_o}}{n} = \frac{A^{5/3}}{P^{2/3}} \frac{\sqrt{S_o}}{n}$$

However in this case we don’t know the flow depth in advance, so we can’t calculate the area or wetted perimeter.

Let’s use the approximate depth for wide square channels as a first guess, and see how close it is:

The roughness of manufactured concrete pipes and box sections is tabulated as 0.012:

$$y \approx \left(\frac{Qn}{W\sqrt{S_o}} \right)^{3/5} = (12.7(0.012)/(2(0.005)^{0.5}))^{3/5} = 1.046 \text{ m}$$

Let’s just check the flowrate that this depth gives:

$$A = Wy = 2(1.046) = 2.092 \text{ m}^2$$

$$P = W + 2y = 2 + 2(1.046) = 4.092 \text{ m}$$

$$\text{And } Q = 2.092^{5/3} 0.005^{0.5} / (4.092^{2/3} 0.012) = 7.88 \text{ m}^3/\text{s}$$

This is not even nearly right, because the channel cannot be called wide or shallow. It will do as a first estimate though. To increase the flowrate we need to increase the depth: Say $y = 1.5 \text{ m}$

$$A = Wy = 2(1.5) = 3.0 \text{ m}^2$$

$$P = W + 2y = 2 + 2(1.5) = 5 \text{ m}$$

$$Q = 3.0^{5/3} 0.005^{0.5} / (5^{2/3} 0.012) = 12.57 \text{ m}^3/\text{s}$$

That’s a pretty good second guess! A few more iterations give $y = 1.513 \text{ m}$. With a freeboard of 100 mm the channel needs to be $1.513 + 0.1 = 1.613 \text{ m}$ deep.

A smooth prismatic concrete channel is 2.1 m wide at the base, with side slopes of 1:4 and bedslope of 0.7 mm per metre. The channel is 300 m long and 1.2 m deep. Assuming steady uniform flow and at least 200 mm freeboard to the water surface, what is the maximum flowrate the channel can carry? What is the flow velocity at that stage?

The key phrase is “steady uniform flow”, which points to the Manning equation. In discharge form, we have:

$$Q = VA = AR^{2/3} \frac{\sqrt{S_o}}{n} = \frac{A^{5/3}}{P^{2/3}} \frac{\sqrt{S_o}}{n}$$

Define the terms:

A = flow area = $Wy + ky^2$ where W is the base width (2.1 m) and k the side slope (1:4 = 1:k ; k = 4)

P = wetted perimeter = $W + 2y\sqrt{1+k^2}$

A and P depend on the flow depth, y, which we are told is 0.2 m freeboard less than the channel depth of 1.0 m: ie $y = 1.2 - 0.2 = 1.0$ m

S_o = bed slope = 0.0007 m/m

n = channel roughness. Looking up “smooth concrete” we find (study guide) “concrete (trowel finish) 0.012 – 0.015”. Let’s use a high value in that range, say $n = 0.014$, since we don’t want to overestimate (economic reasons), and definitely don’t want to underestimate (safety and serviceability reasons).

So we can calculate the terms and put it back together:

$$A = Wy + ky^2 = 2.1(1.0) + 4(1.0^2) = 6.1 \text{ m}^2$$

$$P = W + 2y\sqrt{1+k^2} = 2.1 + 2(1.0)(1 + 4^2)^{0.5} = 10.35 \text{ m}$$

$$\text{And } Q = 6.1^{5/3} 0.0007^{0.5} / (10.35^{2/3} 0.014) = 8.104 \text{ m}^3/\text{s}$$

The average velocity is

$$V = Q/A = 8.104/6.1 = 1.328 \text{ m/s}$$

A square manufactured concrete channel is to be 150 m long and 1.5 m wide. A maximum flowrate of 15.1 m³/s is expected. The channel slope is 4 mm/m. Assuming steady uniform flow, how deep does the channel need to be constructed if a freeboard of 300 mm is required?

Assuming steady uniform flow means “use Manning’s equation”:

$$Q = VA = AR^{2/3} \frac{\sqrt{S_o}}{n} = \frac{A^{5/3}}{P^{2/3}} \frac{\sqrt{S_o}}{n}$$

However in this case we don’t know the flow depth in advance, so we can’t calculate the area or wetted perimeter.

Let's use the approximate depth for wide square channels as a first guess, and see how close it is:

The roughness of manufactured concrete pipes and box sections is tabulated as 0.012:

$$y \approx \left(\frac{Qn}{W\sqrt{S_o}} \right)^{3/5} = (15.1(0.012)/(1.5(0.004)^{0.5}))^{3/5} = 1.474 \text{ m}$$

Let's just check the flowrate that this depth gives:

$$A = Wy = 1.5(1.474) = 2.212 \text{ m}^2$$

$$P = W + 2y = 1.5 + 2(1.474) = 4.448 \text{ m}$$

$$\text{And } Q = 2.212^{5/3} 0.004^{0.5} / (4.448^{2/3} 0.012) = 7.311 \text{ m}^3/\text{s}$$

This is not even nearly right, because the channel cannot be called wide or shallow. It will do as a first estimate though. To increase the flowrate we need to increase the depth: Say $y = 2.5 \text{ m}$

$$A = Wy = 1.5(2.5) = 3.75 \text{ m}^2$$

$$P = W + 2y = 1.5 + 2(2.5) = 6.5 \text{ m}$$

$$Q = 3.75^{5/3} 0.004^{0.5} / (6.5^{2/3} 0.012) = 13.7 \text{ m}^3/\text{s}$$

Say $y = 2.7 \text{ m}$

$$A = Wy = 1.5(2.7) = 4.05 \text{ m}^2$$

$$P = W + 2y = 1.5 + 2(2.7) = 6.9 \text{ m}$$

$$Q = 4.05^{5/3} 0.004^{0.5} / (6.9^{2/3} 0.012) = 15.0 \text{ m}^3/\text{s}$$

That's a pretty good third guess! A few more iterations give $y = 2.722 \text{ m}$. With a freeboard of 300 mm the channel needs to be $2.722 + 0.3 = 3.022 \text{ m}$ deep.

Critical Depth

At what critical depth will a 4 m wide spillway with vertical sides flow when the flowrate is $32.3 \text{ m}^3/\text{s}$? What depth above the spillway is the still water behind the weir? Check your result with the sharp-crested weir equation.

Spillways and many other types of depth controls flow at the critical depth for a given discharge:

$$Q_c = \sqrt{\frac{gA^3}{B}}$$

Where A is the flow area and B is the surface flow width at critical depth. For channels with vertical sides, this reduces to

$$Q_c = \sqrt{\frac{g y_c^3}{B}} = \sqrt{g B^2 y_c^3}$$

The critical discharge equals the flowrate when the flow depth $y = y_c$:

$$y_c = \sqrt[3]{\frac{Q^2}{g B^2}} = (32.3^2 / (9.8(4^2)))^{1/3} = 1.881 \text{ m}$$

The still water behind the spillway has the same total energy as the spillway itself, but it is all expressed as potential energy. The kinetic energy of the spillway is expressed as a velocity head:

$$H = y_c + V^2/2g$$

Where $V = Q/A = Q/(B y_c) = 32.3/(4(1.881)) = 4.293 \text{ m/s}$

Thus $H = y_c + V^2/2g = 1.881 + 4.293^2/19.62 = 2.820 \text{ m}$

As a check, try the sharp crested-weir equation with a correction for spillway length:

$$Q = 1.83 L' H^{3/2} \quad \text{where } L' = L - 0.2H = 4 - 0.2(2.82) = 3.44 \text{ m}$$

$$Q = 1.83(3.44)2.82^{1.5} = 29.81 \quad \text{OK}$$

An engineer needs to design a channel to safely convey runoff from a residential area. Assuming a grassy swale with side slopes of 1:4, the designer considers a v-shaped channel 0.5 m deep. Assuming a maximum Froude number of 0.8, what is the channel's maximum bedslope for subcritical flow? Does this provide a safe footing for children crossing the channel? (use a velocity-depth limit of $V.d < 0.4$)

The Froude number governs the transition from subcritical to supercritical flow. Grassy channels must be designed for subcritical flow or they will fail due to erosion. A Froude number of 0.8 is a safe maximum:

$$Fr = \frac{V}{c} = \frac{V}{\sqrt{gy}} \quad \text{CORRECTION OVER PAGE.}$$

Where V is the normal flow velocity:

$$V = \left(\frac{A}{P} \right)^{2/3} \frac{\sqrt{S_o}}{n}$$

$$\text{So } \frac{V}{\sqrt{gy}} = \left(\frac{A}{P} \right)^{2/3} \frac{\sqrt{S_o}}{n\sqrt{gy}} < 0.8$$

For a 1:4 v-shaped drain 0.5 m deep,

Correction: Use $Fr = \frac{V}{\sqrt{gA/B}} = \left(\frac{A}{P}\right)^{2/3} \frac{\sqrt{S}}{n\sqrt{gA/B}} < 0.8$

MANNING Froude

$$A = ky^2 = 4(0.5^2) = 1 \text{ m}^2$$

$$P = 2y\sqrt{1+k^2} = 2(0.5)\sqrt{1+4^2} = 4.123 \text{ m}$$

Also $B = 2 \times 4 \times 0.5 = 4 \text{ m}$

The roughness of a short grassy swale is about 0.03:

So $S_o < \left(0.8n\sqrt{gA/B} \left(\frac{P}{A}\right)^{2/3}\right)^2 = (0.8(0.03)(9.8(0.5))^{0.5} 4.123^{2/3})^2 = 0.0187 \text{ m/m}$

$= 0.0093 \text{ m/m}$

The velocity of the channel is then

$$V = \left(\frac{A}{P}\right)^{2/3} \frac{\sqrt{S_o}}{n} = (1/4.123)^{2/3} \frac{0.0187^{0.5}}{0.03} = 1.773 \text{ m/s}$$

~~0.0187~~ ~~0.03~~ ~~1.25~~

ABOUT $\frac{1}{2}$ THE
SLOPE (DUE TO
AVERAGE DEPTH
IN FROUDE NO.)

and the safe footing limit is $V_{xd} = 1.773(0.5) = 0.886 >> 0.4$

~~1.773~~ ~~0.5~~

This is not a safe design. A piped design may be needed to divert the runoff away from public spaces.

A well-maintained grassy v-shaped channel with side slopes of 1:4 is to be 0.6 m deep. Assuming a maximum Froude number of 0.8, what is the channel's maximum bedslope for subcritical flow? What is the maximum flow velocity within the channel? Is this safe for children?

The Froude number governs the transition from subcritical to supercritical flow. Grassy channels must be designed for subcritical flow or they will fail due to erosion. A Froude number of 0.8 is a safe maximum:

$$Fr = \frac{V}{c} = \frac{V}{\sqrt{gy}}$$

Where V is the normal flow velocity:

$$V = \left(\frac{A}{P}\right)^{2/3} \frac{\sqrt{S_o}}{n}$$

So $\frac{V}{\sqrt{gy}} = \left(\frac{A}{P}\right)^{2/3} \frac{\sqrt{S_o}}{n\sqrt{gy}} < 0.8$

For a 1:4 v-shaped drain 0.6 m deep,

$$A = ky^2 = 4(0.6^2) = 1.44 \text{ m}^2$$

$$P = 2y\sqrt{1+k^2} = 2(0.6)\sqrt{1+4^2} = 4.948 \text{ m}$$

Also $B = 2 \times 4 \times 0.6 = 4.8 \text{ m}$

The roughness of a short grassy swale is about 0.03:

Correction: Use $Fr = \frac{V}{\sqrt{gA/B}} = \left(\frac{A}{P}\right)^{2/3} \frac{\sqrt{S}}{n\sqrt{gA/B}} < 0.8$

MANNING Froude

ie $S = \left(0.8n\sqrt{gA/B} \left(\frac{P}{A}\right)^{2/3}\right)^2$

$= (0.8(0.03)\sqrt{9.8 \cdot 1.44 / 4.8} (4.948)^{2/3})^2$

$= 0.0088 \text{ m/m}$

ABOUT $\frac{1}{2}$ THE
SLOPE (DUE TO
AVERAGE DEPTH
IN FROUDE NO.)

$$\text{So } S_o < \left(0.8n\sqrt{gy} \left(\frac{P}{A} \right)^{2/3} \right)^2 = (0.8(0.03)(9.8(0.6))^{0.5}(4.948/1.44)^{2/3})^2 = \cancel{0.0176} \text{ m/m} \\ \text{0.0088}$$

The velocity of the channel is then

$$V = \left(\frac{A}{P} \right)^{2/3} \frac{\sqrt{S_o}}{n} = (1.44/4.948)^{2/3} \cancel{0.0176}^{0.5/0.03} = \cancel{1.942} \text{ m/s} \\ \text{0.0088} \quad \text{1.37}$$

The depth-velocity relationship $V \cdot d < 0.4$ gives $0.6(\cancel{1.94}) > 0.4$ so the channel is unsafe for children.
 1.37

At what depth will a 15 m wide spillway with vertical sides flow when the flowrate is 208 m³/s? What depth above the spillway is the still water behind the weir?

Spillways and many other types of depth controls flow at the critical depth for a given discharge:

$$Q_c = \sqrt{\frac{gA^3}{B}}$$

Where A is the flow area and B is the surface flow width. For channels with vertical sides, this reduces to

$$Q_c = \sqrt{\frac{gBy_c^3}{B}} = \sqrt{gB^2y_c^3}$$

The critical discharge equals the flowrate when the flow depth $y = y_c$:

$$y_c = \sqrt[3]{\frac{Q^2}{gB^2}} = (208^2/(9.8(15^2)))^{1/3} = 2.697 \text{ m}$$

The still water behind the spillway has the same total energy as the spillway itself, but it is all expressed as potential energy. This equates to the spillway's potential energy (or depth head) plus kinetic energy (velocity head):

$$H = y_c + V^2/2g$$

Where $V = Q/A = Q/(By_c) = 208/(15(2.697)) = 5.142 \text{ m/s}$

Thus $H = y_c + V^2/2g = 2.697 + 5.142^2/19.62 = 4.045 \text{ m}$

As a check, try the broad-crested weir equation using that value of head:

$$Q = 1.67LH^{3/2} = 1.67(15)4.045^{1.5} = 204 \text{ m}^3/\text{s} \quad \text{OK}$$

Standard Step Method

A prismatic grass-lined channel 1.5 m wide and with 1:4 bank slopes has two reaches: a 30 m long reach at bedslope of 0.001 m/m upstream of a 50 m long reach at bedslope 0.0005 m. The channel outfalls over a concrete footpath which follows the shape of the channel, pouring freely into a deep drain covered with a grate. If the channel carries a flow of 0.7 m³/s, calculate the flow depth at the footpath and at the upstream point 80 m from the footpath. Is the footpath outfall safe for children ($V \cdot d < 0.4$)?

The standard step method allows us to calculate flow depths in non-uniform flow situations such as this: beginning with a critical depth calculation at the weir outfall we proceed upstream to determine the depths at 30 m and 80 m upstream:

Let's collate some data and formulae:

The roughness of a maintained grassy swale is about 0.03

The area, wetted perimeter and surface width of a flow y are given by:

$$A = Wy + ky^2$$

$$P = W + 2y\sqrt{1+k^2}$$

$$B = W + 2ky$$

At the outfall, critical depth conditions prevail:

$$Q_c = \sqrt{\frac{gA^3}{B}} = Q$$

We can trial values of y that produce this critical flowrate:

Try $y = 0.1$:

$$A = 1.5(0.1) + 4(0.1^2) = 0.19 \text{ m}^2$$

$$B = 1.5 + 2(4)0.1 = 2.3 \text{ m}$$

$$Q_c = (9.8(0.19^3)/2.3)^{0.5} = 0.171 \text{ m}^3/\text{s}$$

Try $y = 0.2$:

$$A = 1.5(0.2) + 4(0.2^2) = 0.46 \text{ m}^2$$

$$B = 1.5 + 2(4)0.2 = 3.1 \text{ m}$$

$$Q_c = (9.8(0.46^3)/3.1)^{0.5} = 0.555 \text{ m}^3/\text{s}$$

Try $y = 0.25$:

$$A = 1.5(0.25) + 4(0.25^2) = 0.625 \text{ m}^2$$

$$B = 1.5 + 2(4)0.25 = 3.5 \text{ m}$$

$$Q_c = (9.8(0.625^3)/3.5)^{0.5} = 0.826 \text{ m}^3/\text{s}$$

Try $y = 0.23$:

$$A = 1.5(0.23) + 4(0.23^2) = 0.557 \text{ m}^2$$

$$B = 1.5 + 2(4)0.23 = 3.34 \text{ m}$$

$$Q_c = (9.8(0.557^3)/3.34)^{0.5} = 0.712 \text{ m}^3/\text{s}$$

This is close enough given the other uncertainties in the calculation.

So we have a starting depth of 0.23 m/s. The flow velocity at the footpath is

$$V = Q/A = 0.7/0.557 = 1.257 \text{ m/s}$$

And the safe footing limit is

$$V \cdot d = 1.257(0.23) = 0.29 < 0.4 \text{ so this is a safe outfall.}$$

The wetted perimeter is then $P = W + 2y\sqrt{(1+k^2)} = 1.5 + 2(0.23)(1+4^2)^{0.5} = 3.397 \text{ m}$

And the friction slope at the outfall is

$$S_f = \left(\frac{nQ}{AR^{2/3}} \right)^2 = (0.03(0.7)/(0.557(0.557/3.397)^{2/3}))^2 = 0.01574 \text{ m/m}$$

We can now proceed upstream to the next step. The flow will be somewhat deeper than 0.23, tending towards the normal depth:

By trial and error, normal depth for 0.0005 m/m bedslope occurs at $y = 0.5378 \text{ m}$ (solve Manning's equation with this value to confirm)

We can locate where $y_2 = y_o = 0.5378$ by the step method:

$$A = 1.5(0.5378) + 4(0.5378^2) = 1.96 \text{ m}^2$$

$$P = 1.5 + 2(0.5378)(1+4^2)^{0.5} = 5.935 \text{ m}$$

$$V = Q/A = 0.7/1.96 = 0.356 \text{ m/s}$$

$$S_f = (0.03(0.7)/(1.96(1.24/5.935)^{2/3}))^2 = 0.0005 \text{ m/m (= bedslope for normal depth)}$$

The average friction slope over the section is

$$S_f = (0.01574 + 0.0005)/2 = 0.00812 \text{ m/m}$$

And the Step length is

$$\Delta L = \frac{\left(\frac{V_2^2}{2g} + y_2 \right) - \left(\frac{V_1^2}{2g} + y_1 \right)}{S_f - S_o}$$

$$= ((0.356^2/19.62 + 0.5378) - (1.257^2/19.62 + 0.23))/(0.00812 - 0.0005) = 30.67 \text{ m}$$

This is less than the reach length of 50 m, so we find the channel has established its normal depth at 30.67 m and will have this same depth at the channel section change at 50 m.

We can perform a similar calculation for the second reach:

By trial and error, normal depth for 0.001 m/m bedslope occurs at $y = 0.457$ m (solve Manning's equation with this value to confirm)

We can locate where $y_2 = y_o = 0.457$ by the step method:

$$A = 1.5(0.457) + 4(0.457^2) = 1.521 \text{ m}^2$$

$$P = 1.5 + 2(0.457)(1+4^2)^{0.5} = 5.269 \text{ m}$$

$$V = Q/A = 0.7/1.521 = 0.460 \text{ m/s}$$

$$S_f = (0.03(0.7)/(1.521(1.521/5.269)^{2/3}))^2 = 0.001 \text{ m/m (= bedslope for normal depth)}$$

The average friction slope over the section is

$$S_f = (0.001 + 0.0005)/2 = 0.00075 \text{ m/m}$$

And the Step length is

$$L = ((0.46^2/19.62 + 0.457) - (0.356^2/19.62 + 0.5378))/(0.00075 - 0.001) = 305 \text{ m}$$

That is it will take 305 m for the flow to establish at normal depth. We only have 30 m, so we should find the actual depth somewhere between 0.5378 and 0.457 m depth.

Try $y_2 = 0.5$:

$$A = 1.5(0.5) + 4(0.5^2) = 1.75 \text{ m}^2$$

$$P = 1.5 + 2(0.5)(1+4^2)^{0.5} = 5.623 \text{ m}$$

$$V = Q/A = 0.7/1.75 = 0.400 \text{ m/s}$$

$$S_f = (0.03(0.7)/(1.75(1.75/5.623)^{2/3}))^2 = 0.000683 \text{ m/m}$$

The average friction slope over the section is

$$S_f = (0.000683 + 0.0005)/2 = 0.000591 \text{ m/m}$$

And the Step length is

$$L = ((0.4^2/19.62 + 0.5) - (0.356^2/19.62 + 0.5378))/(0.000591 - 0.001) = 88 \text{ m}$$

Too far. Try $y_2 = 0.53$:

$$A = 1.5(0.53) + 4(0.53^2) = 1.919 \text{ m}^2$$

$$P = 1.5 + 2(0.53)(1+4^2)^{0.5} = 5.870 \text{ m}$$

$$V = Q/A = 0.7/1.919 = 0.364 \text{ m/s}$$

$$S_f = (0.03(0.7)/(1.919(1.919/5.870)^{2/3}))^2 = 0.000532 \text{ m/m}$$

The average friction slope over the section is

$$S_f = (0.000532 + 0.0005)/2 = 0.000516 \text{ m/m}$$

And the Step length is

$$L = ((0.364^2/19.62 + 0.53) - (0.356^2/19.62 + 0.5378))/(0.000516 - 0.001) = 15 \text{ m}$$

A little short. Try $y_2 = 0.525$

$$A = 1.5(0.525) + 4(0.525^2) = 1.89 \text{ m}^2$$

$$P = 1.5 + 2(0.525)(1+4^2)^{0.5} = 5.829 \text{ m}$$

$$V = Q/A = 0.7/1.89 = 0.370 \text{ m/s}$$

$$S_f = (0.03(0.7)/(1.89(1.89/5.829)^{2/3}))^2 = 0.000554 \text{ m/m}$$

The average friction slope over the section is

$$S_f = (0.000554 + 0.0005)/2 = 0.000527 \text{ m/m}$$

And the Step length is

$$L = ((0.370^2/19.62 + 0.525) - (0.356^2/19.62 + 0.5378))/(0.000527 - 0.001) = 26 \text{ m}$$

Close. Try $y_2 = 0.523$

$$A = 1.5(0.523) + 4(0.523^2) = 1.88 \text{ m}^2$$

$$P = 1.5 + 2(0.523)(1+4^2)^{0.5} = 5.813 \text{ m}$$

$$V = Q/A = 0.7/1.89 = 0.372 \text{ m/s}$$

$$S_f = (0.03(0.7)/(1.89(1.88/5.813)^{2/3}))^2 = 0.000563 \text{ m/m}$$

The average friction slope over the section is

$$S_f = (0.000563 + 0.0005)/2 = 0.000532 \text{ m/m}$$

And the Step length is

$$L = ((0.372^2/19.62 + 0.523) - (0.356^2/19.62 + 0.5378))/(0.000532 - 0.001) = 30 \text{ m}$$

The flow depth at the top of the 30 m reach is thus 0.523 m.

This laborious calculation can easily be transmitted to a spreadsheet for much easier trial and error.

A prismatic rock-lined channel 1.6 m wide and with 1:4 bank slopes has two reaches: a 40 m long reach at bedslope of 0.0015 m/m upstream of a 70 m long reach of bedslope 0.0004 m. The channel outfalls over a concrete weir of the same profile into a river several metres below. If the channel carries a flow of 1.3 m³/s, calculate the flow depth at the weir and at the upstream point 110 m from the weir.

The standard step method allows us to calculate flow depths in non-uniform flow situations such as this: beginning with a critical depth calculation at the weir outfall we proceed upstream to determine the depths at 70 m and 110 m upstream:

Let's collate some data and formulae:

The roughness of a rocky channel is about 0.045

The area, wetted perimeter and surface width of a flow y are given by:

$$A = Wy + ky^2$$

$$P = W + 2y\sqrt{1+k^2}$$

$$B = W + 2ky$$

At the outfall, critical depth conditions prevail:

$$Q_c = \sqrt{\frac{gA^3}{B}} = Q$$

We can trial values of y that produce this critical flowrate:

Try $y = 0.5$:

$$A = 1.6(0.5) + 4(0.5^2) = 1.8 \text{ m}^2$$

$$B = 1.6 + 2(4)0.5 = 5.6 \text{ m}$$

$$Q_c = (9.8(1.8^3)/5.6)^{0.5} = 3.2 \text{ m}^3/\text{s}$$

Try $y = 0.2$:

$$A = 1.6(0.2) + 4(0.2^2) = 0.48 \text{ m}^2$$

$$B = 1.6 + 2(4)0.2 = 3.2 \text{ m}$$

$$Q_c = (9.8(0.48^3)/3.2)^{0.5} = 0.582 \text{ m}^3/\text{s}$$

Try $y = 0.3$:

$$A = 1.6(0.3) + 4(0.3^2) = 0.84 \text{ m}^2$$

$$B = 1.6 + 2(4)0.3 = 4.0 \text{ m}$$

$$Q_c = (9.8(0.84^3)/4.0)^{0.5} = 1.2 \text{ m}^3/\text{s}$$

Try $y = 0.31$:

$$A = 1.6(0.31) + 4(0.31^2) = 0.88 \text{ m}^2$$

$$B = 1.6 + 2(4)0.31 = 4.08 \text{ m}$$

$$Q_c = (9.8(0.88^3)/4.08)^{0.5} = 1.279 \text{ m}^3/\text{s}$$

This is close enough given the other uncertainties in the calculation.

So we have a starting depth of 0.31 m. The flow velocity at the weir is

$$V = Q/A = 1.3/0.88 = 1.477 \text{ m/s}$$

The wetted perimeter is then $P = W + 2y\sqrt{(1+k^2)} = 1.6 + 2(0.31)(1+4^2)^{0.5} = 4.156 \text{ m}$

And the friction slope at the outfall is

$$S_f = \left(\frac{nQ}{AR^{2/3}} \right)^2 = (0.045(1.3)/(0.88(0.88/4.156)^{2/3}))^2 = 0.035 \text{ m/m}$$

We can now proceed upstream to the next step. The flow will be somewhat deeper than 0.31, tending towards the normal depth:

By trial and error, normal depth for 0.0004 m/m bedslope occurs at $y = 0.887 \text{ m}$ (solve Manning's equation with this value to confirm)

We can locate where $y_2 = 0.887$ by the step method:

$$A = 1.6(0.887) + 4(0.887^2) = 4.566 \text{ m}^2$$

$$P = 1.6 + 2(0.887)(1+4^2)^{0.5} = 8.914 \text{ m}$$

$$V = Q/A = 1.3/4.566 = 0.285 \text{ m/s}$$

$$S_f = (0.045(1.3)/(4.566(4.566/8.914)^{2/3}))^2 = 0.0004 \text{ m/m (= bedslope for normal depth)}$$

The average friction slope over the section is

$$S_f = (0.035 + 0.0004)/2 = 0.0177 \text{ m/m}$$

And the Step length is

$$\Delta L = \frac{\left(\frac{V_2^2}{2g} + y_2 \right) - \left(\frac{V_1^2}{2g} + y_1 \right)}{S_f - S_o}$$

$$= ((0.285^2/19.62 + 0.887) - (1.477^2/19.62 + 0.31))/(0.0177 - 0.0004) = 27.19 \text{ m}$$

This is less than the reach length of 70 m, so we find the channel has established its normal depth at 27.19 m and will have this same depth at the channel section change at 70 m.

We can perform a similar calculation for the second reach:

By trial and error, normal depth for 0.0015 m/m bedslope occurs at $y = 0.660$ m (solve Manning's equation with this value to confirm)

We can locate where $y_2 = y_o = 0.660$ by the step method:

$$A = 1.6(0.66) + 4(0.66^2) = 2.798 \text{ m}^2$$

$$P = 1.6 + 2(0.66)(1+4^2)^{0.5} = 7.042 \text{ m}$$

$$V = Q/A = 1.3/2.798 = 0.465 \text{ m/s}$$

$$S_f = (0.045(1.3)/(2.798(2.798/7.042)^{2/3}))^2 = 0.0015 \text{ m/m (= bedslope for normal depth)}$$

The average friction slope over the section is

$$S_f = (0.0015 + 0.0004)/2 = 0.0019 \text{ m/m}$$

And the Step length is

$$L = ((0.465^2/19.62 + 0.66) - (0.285^2/19.62 + 0.887))/(0.0019 - 0.0015) = 399 \text{ m}$$

That is it will take 399 m for the flow to establish at normal depth. We only have 40 m, so we should find the actual depth somewhere between 0.887 and 0.66 m depth.

Try $y_2 = 0.7$:

$$A = 1.6(0.7) + 4(0.7^2) = 3.08 \text{ m}^2$$

$$P = 1.6 + 2(0.7)(1+4^2)^{0.5} = 7.37 \text{ m}$$

$$V = Q/A = 0.7/1.521 = 0.422 \text{ m/s}$$

$$S_f = (0.045(1.3)/(3.08(3.08/7.37)^{2/3}))^2 = 0.001155 \text{ m/m}$$

The average friction slope over the section is

$$S_f = (0.001155 + 0.0004)/2 = 0.000778 \text{ m/m}$$

And the Step length is

$$L = ((0.422^2/19.62 + 0.7) - (0.285^2/19.62 + 0.887))/(0.000778 - 0.0015) = 252 \text{ m}$$

Try $y_2 = 0.844$:

$$A = 1.6(0.844) + 4(0.844^2) = 4.199 \text{ m}^2$$

$$P = 1.6 + 2(0.844)(1+4^2)^{0.5} = 8.56 \text{ m}$$

$$V = Q/A = 1.3/4.199 = 0.309 \text{ m/s}$$

$$S_f = (0.045(1.3)/(4.199(4.199/8.56)^{2/3}))^2 = 0.000501 \text{ m/m}$$

The average friction slope over the section is

$$S_f = (0.000501 + 0.0004)/2 = 0.000451 \text{ m/m}$$

And the Step length is

$$L = ((0.309^2/19.62 + 0.844) - (0.285^2/19.62 + 0.887))/(0.000451 - 0.0015) = 40.3 \text{ m}$$

The flow depth at the top of the 40 m reach is thus 0.844 m.

This laborious calculation can easily be transmitted to a spreadsheet for much easier trial and error:

Q	m ³ /s	1.3	1.3	
W	m	1.6	1.6	
k	m/m	4	4	Data
n		0.045	0.045	
So	m/m	0.0004	0.0015	
yo	mm	0.887225	0.659592	Trial for Qo = Q
yc	mm	0.31257	0.31257	Trial for Qc = Q
Ao	m ²	4.568234	2.795592	=W*yo+k*yo ²
Po	m	8.916246	7.039133	=W+2*yo*(1+k ²) ^{0.5}
Ac	m ²	0.890911	0.890911	=W*yc+k*yc ²
Bc	m	4.100559	4.100559	=W+2*k*yc
Pc	m	4.177517	4.177517	=W+2*yc*(1+k ²) ^{0.5}
Qo	m ³ /s	1.3	1.3	=Ao*(Ao/Po) ^(2/3) *So ^{0.5} /n
Qc	m ³ /s	1.3	1.3	=(9.8*Ac ³ /Bc) ^{0.5}
y1	m	0.31257	0.887225	=yc (control) or yo (uniform) or y (calculated by previous step)
A1	m ²	0.890911	4.568234	=W*y1+k*y1 ²
P1	m	4.177517	8.916246	=W+2*y1*(1+k ²) ^{0.5}
V1	m/s	1.45918	0.284574	=Q/A1
Sf1	m/m	0.033839	0.0004	=(n*Q/(A1*(A1/P1) ^(2/3))) ²
y2	m	0.887225	0.844	trial for new depth
A2	m ²	4.568234	4.199744	=W*y2+k*y2 ²
P2	m	8.916246	8.559802	=W+2*y2*(1+k ²) ^{0.5}
V2	m/s	0.284574	0.309543	=Q/A2
Sf2	m/m	0.0004	0.000501	=(n*Q/(A2*(A2/P2) ^(2/3))) ²
Sf	m/m	0.01712	0.000451	=(Sf1+Sf2)/2
ΔL	m	28.11989	40.47302	=((V2 ² /19.6+y2)-(V1 ² /19.6+y1))/(Sf-So)

Hydraulic Jump

A major dam has a low level sluice gate leading into a smooth concrete lined rectangular channel 55 m wide with shallow bedslope. The sluice gate releases 255 m³/s into the channel at a flow depth of 0.363 m. The normal flow depth in the channel is 3 m. Locate the hydraulic jump that must occur in the channel, the depth before the jump and the power dissipated by the apron and jump.

In this case we need to perform a supercritical standard step method and calculate the momentum function of the flow until it equals the momentum of the downstream normal flow depth, at which point a hydraulic jump occurs. This is a complex procedure that will benefit from application of a spreadsheet:

Create a table of the provided data and calculate the momentum function of the normal depth:

Q	254.7	
W	54.86	
y1	0.363	
yo	3.05	
Ao	167.323	=W*yo
Vo	1.522206	=Q/Ao
Mo	294.7294	=Vo^2*Ao/9.8+yo/2*Ao

Note that the momentum function uses the mean depth, which is half the total depth.

To start the standard step method, calculate the area, wetted perimeter, velocity and friction slope associated with y1 (the sluice gate depth):

A1	19.91418	=W*y1
P1	55.586	=W+2*y1
V1	12.78988	=Q/A1
Sf1	0.108648	=(0.013*Q/(A1*(A1/P1)^(2/3)))^2

Now add a guessed y2 (the depth of flow after frictional losses down the channel), and calculate A, P, V, Sf and the average Sf:

y2	0.4	
A2	21.944	=W*y2
P2	55.66	=W+2*y2
V2	11.60682	=Q/A2
Sf2	0.078756	=(0.013*Q/(A2*(A2/P2)^(2/3)))^2
Sf	0.093702	=(Sf1+Sf2)/2

Lastly include the step length calculation and the momentum function of the guessed y2:

L	15.4699	=-(V2^2/19.62+y2)-(V1^2/19.62+y1))/(Sf)
M1	306.0476	=V2^2*A2/9.8+y2/2*A2

Note that the step length function has a minus sign because in a supercritical step we move from upstream to downstream, the reverse of the direction (downstream to upstream) by which the formula was derived. Also note that there is no bed slope So in the denominator, as this is specified only as “shallow” and can be treated as zero.

Now we can gradually increase the value of y2 until the momentum function falls to that of the normal depth yo:

y2	0.416	
A2	22.82176	=W*y2
P2	55.692	=W+2*y2
V2	11.1604	=Q/A2
Sf2	0.069157	=(0.013*Q/(A2*(A2/P2)^(2/3)))^2
Sf	0.088903	=(Sf1+Sf2)/2

$$L = 21.77791 = -((V_2^2/19.62 + y_2) - (V_1^2/19.62 + y_1)) / (S_f)$$

$$M_1 = 294.8035 = V_2^2 * A_2 / 9.8 + y_2 / 2 * A_2$$

Here $M_1 = M_0$ at a step length L of 22 m, where the flow depth is 0.416 m

The power dissipated by the apron and jump is a function of the difference in total head between the sluice gate and the normal depth:

$$\text{Power} = \rho g Q \Delta H$$

$$H_1 = 8.700465 = y_1 + V_1^2 / 19.62$$

$$H_0 = 3.168099 = y_0 + V_0^2 / 19.62$$

$$\text{Power} = 13809117 = 9800 * Q * (H_1 - H_0)$$

The total power dissipated is 13.8 MW

A dam has a spillway leading into a smooth concrete lined rectangular channel 40 m wide with shallow bed slope. The sluice gate releases 200 m³/s into the channel at a flow depth of 0.41 m. The normal flow depth in the channel is 2.5 m. Locate the hydraulic jump that must occur in the channel, the depth before the jump and the power dissipated by the channel and jump.

In this case we need to perform a supercritical standard step method and calculate the momentum function of the flow until it equals the momentum of the downstream normal flow depth, at which point a hydraulic jump occurs. This is a complex procedure that will benefit from application of a spreadsheet:

Create a table of the provided data and calculate the momentum function of the normal depth:

Q	200	
W	40	
y ₁	0.41	
y ₀	2.5	
A ₀	100	=W*y ₀
V ₀	2.0	=Q/A ₀
M ₀	165.8	=V ₀ ² *A ₀ /9.8+y ₀ /2*A ₀

Note that the momentum function uses the mean depth, which is half the total depth.

To start the standard step method, calculate the area, wetted perimeter, velocity and friction slope associated with y₁ (the sluice gate depth):

A ₁	16.4	=W*y ₁
P ₁	40.82	=W+2*y ₁
V ₁	12.19512	=Q/A ₁
S _{f1}	0.084781	=(0.013*Q/(A ₁ *(A ₁ /P ₁) ^(2/3))) ²

Now add a guessed y₂ (the depth of flow after frictional losses down the channel), and calculate A, P, V, S_f and the average S_f:

y2	0.5	
A2	20	=W*y2
P2	41	=W+2*y2
V2	10	=Q/A2
Sf2	0.044011	=(0.013*Q/(A2*(A2/P2)^(2/3)))^2
Sf	0.064396	=(Sf1+Sf2)/2

Lastly include the step length calculation and the momentum function of the guessed y2:

L	37.1644	=-((V2^2/19.62+y2)-(V1^2/19.62+y1))/(Sf)
M1	209.0816	=V2^2*A2/9.8+y2/2*A2

Note that the step length function has a minus sign because in a supercritical step we move from upstream to downstream, the reverse of the direction (downstream to upstream) by which the formula was derived. Also note that there is no bedslope So in the denominator, as this is specified only as “shallow” and can be treated as zero.

Now we can gradually increase the value of y2 until the momentum function falls to that of the normal depth yo:

y2	0.65	
A2	26	=W*Y2
P2	41.3	=W+2*Y2
V2	7.692308	=Q/A2
Sf2	0.018534	=(0.013*Q/(A2*(A2/P2)^(2/3)))^2
Sf	0.051657	=(SF1+SF2)/2
L	83.70896	=-((V2^2/19.62+Y2)-(V1^2/19.62+Y1))/(Sf)
M1	165.4359	=V2^2*A2/9.8+Y2/2*A2

Here M1 = Mo at a step length L of 84 m, where the flow depth is 0.65 m

The power dissipated by the apron and jump is a function of the difference in total head between the sluice gate and the normal depth:

Power = $\rho g Q \Delta H$

H1	7.990071	=y1+V1^2/19.62
Ho	3.665882	=yo+Vo^2/19.62
Power	8475412	=9800*Q*(H1-Ho)

The total power dissipated is 8.475 MW

A concrete lined open channel ($n = 0.013$) is 2.5 m wide with vertical sides and a bedslope of 0.001 m/m. After an industrial accident the channel carries a flow of 5 m³/s.

- a) Assuming uniform steady flow, what is the flow depth at this discharge? (10 Marks)
- b) Is this deeper or shallower than the critical depth? What flow regime does this identify? What safety issues would you consider as a result? (15 Marks)
- c) Will a hydraulic jump be possible as debris from the accident begins to jam up in narrow sections downstream? (5 Marks)

- a) The normal depth is the depth at which Manning's equation equals the discharge:

$$Q = AR^{2/3}\sqrt{S}/n$$

$$A = Wy$$

$$P = W + 2y$$

$$R = A/P$$

There is no way to solve for y . Use trial and error:

$$\text{Say } y = 0.5 \text{ m}$$

$$A = 1.25 \text{ m}^2$$

$$P = 3.5 \text{ m}$$

$$Q = 1.25(1.25/3.5)^{2/3}\sqrt{0.001/0.013} = 1.5 \text{ m}^3/\text{s}$$

$$\text{Try } y = 1 \text{ m}$$

$$A = 2.5 \text{ m}^2$$

$$P = 4.5 \text{ m}$$

$$Q = 2.5(2.5/4.5)^{2/3}\sqrt{0.001/0.013} = 4.1 \text{ m}^3/\text{s}$$

$$\text{Try } y = 1.2 \text{ m} \dots Q = 5.3$$

$$\text{Try } y = 1.1 \text{ m} \dots Q = 4.7$$

$$\text{So } y \sim \underline{1.2 \text{ m}}, \text{ or a bit less.}$$

- b) Critical depth is given by a solution of

$$Q_c = \sqrt{gA^3/B} = Q$$

For rectangular channels (constant B), this gives $Q_c = \sqrt{g} y_c^3 B^2 = Q$

$$\text{So } y_c = \sqrt[3]{Q^2/gB^2} = \sqrt[3]{(5^2/9.8(2.5)^2)} = \underline{0.741 \text{ m.}}$$

Normal depth is deeper than critical, so the channel is flowing in the subcritical regime. The main safety issue is the depth of flow which will overwhelm a child. The velocity is

$$V = Q/A = 5/(2.5*1.2) = 1.67 \text{ m/s which is not fast but still dangerous for small children}$$

c) A hydraulic jump can only occur where the flow is supercritical. As this flow is subcritical, there is no risk of a jump occurring.