
Tutorial 6

Drag Forces and Aerodynamics

A long copper wire, 12 mm in diameter, is stretched taut and exposed to 10°C wind at a velocity of 27.5 m/s. What is the drag force per metre length of wire?

We can consider the wire as an “infinite cylinder”, to use the language of the drag coefficient diagram.

We need to work out the Reynolds number of the wire in this flow:

$$Re = \frac{\rho DV}{\mu}$$

The density of air in normal conditions is close enough to 1.2 kg/m³. For a more accurate estimate, use $P = \rho RT$: $\rho = P/RT = 101300/(287(273+10)) = 1.247 \text{ kg/m}^3$. Let's go with that value.

The viscosity of air at 10°C is $1.8 \times 10^{-5} \text{ Pa.s}$ (p20 Study Guide)

So with a diameter of 0.012 m and velocity of 27.5 m/s we have

$$Re = \frac{\rho DV}{\mu} = 1.247(0.012)27.5/1.8 \times 10^{-5} = 22860$$

The drag coefficient diagram gives a C_D of 1.3 for an “infinite cylinder” at this Re .

The profile area of the wire per metre is $A = D(1) = 0.012 \text{ m}^2$

The drag force per metre can then be calculated as

$$Drag = C_D \rho \frac{U^2}{2} A = 1.3(1.247)27.5^2(0.012)/2 = \underline{7.356 \text{ N}}$$

A subject weighing 76 kg tests a 8.4 m diameter parachute. At what terminal velocity will the subject hit the ground? Assume $\rho = 1.2 \text{ kg/m}^3$ and $\mu = 2 \times 10^{-5} \text{ Pa.s}$.

We need a drag force equal to 76 kg of mass based on a fixed velocity. The actual force is

$$D = mg = 76(9.8) = 745 \text{ N}$$

The area of the chute is $\pi D^2/4 = \pi(8.4^2)/4 = 55.42 \text{ m}^2$

Then the drag force is:

$$Drag = C_D \rho \frac{U^2}{2} A = C_D(1.2)V^2(55.42/2) = 33.25C_D V^2 = 745 \text{ N}$$

For circular disks $C_D = 1.12$ (study guide)

Then $V = (745/(33.25(1.12)))^{0.5} = 4.473$ m/s (similar speed to jumping off a fence)

A steel ball 3 mm in diameter falls through an oil bath of density 908 kg/m^3 and kinematic viscosity $1.46 \times 10^{-4} \text{ m}^2/\text{s}$. What is the terminal velocity of the ball? Density (steel) = 7700 kg/m^3

The ball's mass is $m = \rho V = \rho(4/3 \pi D^3/8) = 7700(4/3\pi(0.003^3)/8) = 108.9 \times 10^{-6} \text{ kg}$ (about 0.1 g)

The ball's profile area is $A = \pi D^2/4 = \pi(0.003^2)/4 = 7.069 \times 10^{-6} \text{ m}^2$

The terminal velocity occurs when the drag force D = the weight force $W = mg = 1.067 \times 10^{-3} \text{ N}$

$$\text{Drag} = C_D \rho \frac{U^2}{2} A = C_D(908)V^2(7.069 \times 10^{-6}/2) = 3.209 \times 10^{-3} C_D V^2 = 1.067 \times 10^{-3} \text{ N}$$

So we need $C_D V^2 = 1.067 \times 10^{-3} / 3.209 \times 10^{-3} = 0.332$

We can proceed iteratively by guessing a value of C_D , calculating V and Re to locate a value of C_D , and repeating to convergence:

Say $C_D = 1$

Then $V = (0.332)^{0.5} = 0.576$ m/s

$Re = 0.576(0.003)/1.46 \times 10^{-4} = 11.84$ (laminar)

For spheres in this range we have $C_D = 4.5$

Then $V = (0.332/4.5)^{0.5} = 0.272$ m/s

$Re = 0.272(0.003)/1.46 \times 10^{-4} = 5.59$ (laminar)

For spheres in this range we have $C_D = 7$

Then $V = (0.332/7)^{0.5} = 0.218$ m/s

$Re = 0.218(0.003)/1.46 \times 10^{-4} = 4.09$ (laminar)

For spheres in this range we have $C_D = 7.5$

Then $V = (0.332/7.5)^{0.5} = 0.210$ m/s

The solution converges on a velocity of 0.2 m/s

An airplane weighs 1.837 tonnes and has a wing area of 28 m^2 . What angle of attack must the wings make with the horizontal for a 160 km/h cruising speed? Use $\rho = 1.2 \text{ kg/m}^3$ and the "typical" lift and drag coefficients in the lectures.

Let's convert that airspeed to SI units first of all: $V = 160/3.6 = 44.44$ m/s

Now let's look at the aerodynamic lift equation:

$$Lift = C_L \rho \frac{U^2}{2} A_w$$

The lift is an equilibrium force, equal to the aircraft's weight in Newtons. Convert the mass to weight:

$$Lift = weight = mg = 1837(9.8) = 18 \times 10^3 \text{ N}$$

All that remains is the lift coefficient C_L :

$$C_L = \frac{Lift}{\rho \frac{U^2}{2} A_w} = 18 \times 10^3 / (1.2(44.44^2)28/2) = 0.5425$$

The diagram gives a lift coefficient of approximately 0.55 for an angle of attack of 5.0° . This is a suitable angle of attack within the accuracy of this model.

What wing area is required to support a 2.2 tonne aircraft cruising at 27m/s when the wing has a 5° angle of attack?

We can go straight to the lift equation:

$$Lift = C_L \rho \frac{U^2}{2} A_w$$

Where C_L is a lift coefficient (assume the diagram on p 149 Study Guide is accurate).

At cruising speed the lift must equal the weight: $Lift = mg = 2.2 \times 10^3(9.8) = 21.56 \times 10^3 \text{ N}$

At 5° the lift coefficient is 0.65 (Study Guide)

$$\text{So } C_L \rho U^2 A_w / 2 = 0.65(1.2)27^2 A_w / 2 = 21.56 \times 10^3 \text{ N}$$

$$\text{Thus } A_w = 21.56 \times 10^3(2) / (0.65(1.2)27^2) = 75.83 \text{ m}^2$$

A kite weighs 1.12 kg and has an area of 0.75 m^2 . On a day when the wind speed is 9 m/s the kite rises at an angle of attack of 8° to the horizontal, reaching a maximum height when the string makes an angle of 45° to the horizontal, at which point the tension in the kite is measured as 30 N. What are the approximate drag and lift coefficients for this simple wing?

At the cruising altitude the lift force = the weight force + the vertical component of the tension, and the drag force = the horizontal component of the tension:

$$Lift = 1.12(9.8) + 30\sin(45) = 32.19 \text{ N}$$

$$Drag = 30\cos(45) = 21.21 \text{ N}$$

We need to resolve the kite area into horizontal (drag) and vertical (lift) components*:

$$A_L = 0.75 \cos(8) = 0.742 \text{ m}^2$$

$$A_D = 0.75 \sin(8) = 0.104 \text{ m}^2$$

Then the lift and drag equations are

$$\text{Lift} = C_L \rho \frac{U^2}{2} A_L = C_L (1.2) 9^2 0.742 / 2 = 32.19 \text{ N}$$

So $C_L = 32.19(2) / (0.742(9^2)1.2) = 0.893$

And $\text{Drag} = C_D \rho \frac{U^2}{2} A_D = C_D (1.2) 9^2 0.104 / 2 = 21.21 \text{ N}$

So $C_D = 21.21(2) / (0.104(9^2)1.2) = 4.196$

*This is a more accurate way of representing the area of a wing in its two principle profile areas. Published values of C_L and C_D may or may not use this formulation.

The basic specifications for an F14 jet are as follows:

- Wingspan 64 m spread, 38 m swept
 - Total Wing area $A_w = 54.5 \text{ m}^2$
 - Drag profile area $A_p = 11.5 \text{ m}^2$
 - Mass 27,700 kg fully loaded
 - 2 x GE F110 Turbofan engines
 - Thrust 61.4 kN from each engine, 124.7 kN each with afterburners lit
 - Maximum airspeed Ma2.34 or 2485 km/h at high altitude
 - Peak Thrust/Weight ratio 0.91
- a) Using the data, derive a drag coefficient for the F14 at maximum thrust (with afterburners) and airspeed. Assume air temperature -56.5°C and pressure 26 kPa absolute at 10 km alt.
 - b) The lift coefficient at top airspeed is rated at 0.054. What lift does that generate? Is the fully loaded aircraft climbing or descending at 10 km altitude and top speed?
 - c) A 1:32 scale model F14 weighs 2.5 kg. What are the ratios of wing span, area and weight to the full scale? Assuming the drag and lift coefficients are in similar ratios to the peak performance coefficients in a) and b), what thrust does the model require at cruising speed?
 - d) Given c) and assuming a model drag coefficient 100x that of the prototype peak performance drag coefficient, what is the model's cruising speed near sea level at 15°C?

A diagram adds nothing to this, but you should make a quick sketch so you don't mix A_w with A_p etc.

a) Thrust = 2 x 124.7 = 249.4 kN = Drag at top speed (Thrust = Drag at any constant V)

$$\text{Drag} = C_D \rho V^2 A_p / 2 = 249.4 \times 10^3 \text{ N}$$

$$V = 2485 \text{ [km/h]} \times 1000 \text{ [m/km]} / 3600 \text{ [s/h]} = 690.3 \text{ m/s}$$

$$A_p = 11.5 \text{ m}^2 \text{ (given)}$$

$$\rho = P/RT = 26 \times 10^3 / (287(273.3 - 56.5)) = 0.42 \text{ kg/m}^3$$

Thus $C_d = \frac{249.4 \times 10^3}{(0.42(690.3^2)11.5/2)} = 0.217$ (dimensionless)

b) $\text{Lift} = C_L \rho V^2 A_w / 2 = 0.054(0.42)690.3^2(54.5)/2 = 294500 = 294.5 \text{ kN}$

The loaded aircraft weighs 27700 kg or $27700(9.8) = 271.5 \text{ kN}$

The lift exceeds the load, so the aircraft is still climbing at this speed and altitude.

c) The wing span ratio is just the linear scale, $L_r = 1/32 = 0.03125$

The wing area is much less than this: $A_r = 1/32^2 = 0.000976 = L_r/32$

The weight ratio is smaller still: $W_r = 2.5/27700 = 0.00009025 = L_r/346.3$

At cruising speed, Lift = Weight and Drag = Thrust

We don't know the lift or drag coefficients, but we are guessing (as advised) that they are in a similar ratio to before:

$$C_D/C_L = 0.217/0.054 = 4.02$$

We can form similar ratios of weight/thrust:

$$\text{Thrust} = C_d \rho V^2 A_p / 2 = \text{Drag}$$

$$\text{Weight} = C_L \rho V^2 A_w / 2 = \text{Lift}$$

$$\text{So Thrust/Weight} = C_d A_p / C_L A_w = C_d / C_L \times A_p / A_w = 4.02 \times 11.5 / 54.5 = 0.85$$

$$\text{And Thrust} = 0.85 \times \text{Weight} = 0.85 \times 9.8 \times 2.5 = \underline{\underline{20.8 \text{ N}}}$$

This is about 2.1 kg of thrust, or a thrust to weight ratio of 0.84. The data show a peak thrust/weight ratio of 0.91 so this seems reasonable, if a bit high for minimum cruising speed. Drag and thrust coefficients don't necessarily have similar ratios at different Reynolds numbers.

Note that model and prototype aircraft DO NOT fly at the conditions suggested by similitude, these are laboratory settings only. Actual flight settings for a model are worked out separately from the prototype.

d) Assuming $C_d = 100(0.217) = 21.7$ (reasonable given the rapid increase in C_d for small Re)

$$\text{Drag} = C_d \rho V^2 A_p / 2 = \text{Thrust} = 20.8 \text{ N}$$

$$A_p = 11.5/32^2 = 0.0112 \text{ m}^2$$

$$\rho = P/RT = 101325 / (287(288)) = 1.23 \text{ kg/m}^3 \text{ (normal air density)}$$

$$\text{So } V = (2(\text{Thrust}) / (C_d \rho A_p))^{1/2} = (2(20.8) / (21.7(1.23)0.0112))^{1/2} = 11.8 \text{ m/s} = \underline{\underline{42.5 \text{ km/h}}}$$

Velocities much larger or smaller than this would not be applicable to a model at normal cruising flight speeds but to laboratory simulations with a fixed support for the model. Check your answer to decide if say 2 m/s or 200 m/s are reasonable cruise speeds for a 2-m wingspan model plan.