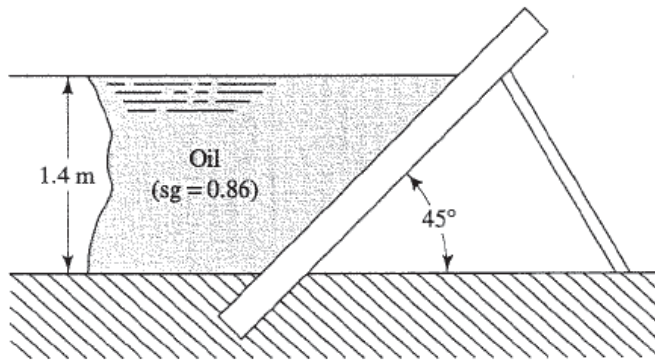


If the wall shown below is 4 m wide, calculate the total force on the wall due to the oil pressure. Also determine the location of the centre of pressure and show the resultant force on the wall.



Sloping Oil Bath Wall

Following the three-step method outlined in page 33 of the notes we have:

1. Calculate the net resultant force due to hydrostatic pressure

Resultant force = average pressure x area $F = \bar{P}A$

The pressure profile is linear, and the structure extends the full depth, so the average pressure occurs at half the depth:

$$\bar{P} = \rho g \frac{H}{2} = 860(9.8)0.7 = 5900 \text{ Pa}$$

The area is just the width x face length of the wall:

$$A = WH/\cos\theta = 4(1.4)/\cos(45) = 7.92 \text{ m}^2$$

Thus $F = 5900(7.92) = 46.72 \times 10^3 \text{ N}$

This force is directed at a normal to the sloping face of the wall.

2. Determine the *centre of pressure*, or position of the resultant force

We can calculate the centre of pressure as a vertical depth, then resolve that to a length on the face of the wall:

$$h_p = h_c + \frac{I_c \sin^2 \theta}{h_c A}$$

h_c is the depth to the centroid of the wall, which is $h_c = H/2 = 0.7 \text{ m}$

I_c is the moment of inertia of the wall, which is $I_c = W(H/\cos\theta)^3/12 = 4(1.4/\cos 45)^3/12 = 2.857 \text{ m}^4$

A is the face area of the wall as before:

$$h_p = 0.7 + 2.857 \sin^2 45 / (0.7 \times 7.92) = 0.958 \text{ m}$$

This is a vertical depth. Resolving it on the face of the wall, we have

$$L_p = h_p / \sin(45) = 0.958 / \sin(45) = 1.354 \text{ m}$$

This is a depth along the sloping face. We can convert this to a distance from the base of the wall, which we need in order to resolve the normal force resultant to a moment.

$$L_b = H/\cos 45 - L_p = 1.4/\cos(45) - 1.354 = 0.626 \text{ m}$$

3. Calculate moments and determine structural resisting forces and moments.

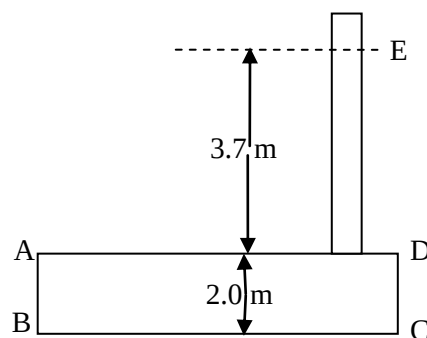
The static pressure overturning moment about the base of the wall is the resultant force normal to the wall multiplied by its distance to the base of the wall:

$$M_o = FL_b = 46.72 \times 10^3 (0.626) = 29.25 \times 10^3 \text{ Nm}$$

The supporting prop, assumed to be positioned opposite the water surface and normal to the back of the wall, must be capable of providing a resisting force equal to the overturning moment divided by its distance to the base of the wall:

$$F_r = M_o/L = 29.25 \times 10^3 / (1.4/\cos 45) = 14.77 \times 10^3 \text{ N}$$

Water rises to level E in the pipe attached to the tank ABCD. Determine and locate the resulting force acting on the wall AB which is 2.5 m wide.



The average pressure on the wall occurs at its centroid of area, halfway between A and B. This point is at a depth of $h_c = 3.7 + 2.0/2 = 4.7 \text{ m}$

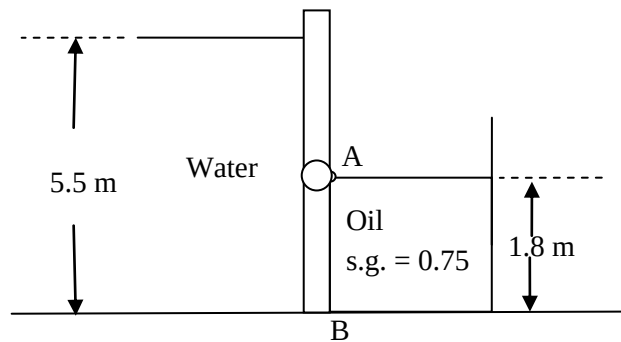
The pressure at this depth is $P = \rho g H = 9800(4.7) = 46.06 \times 10^3 \text{ Pa}$

The area of the wall is $A = 2(2.5) = 5 \text{ m}^2$

The net pressure force is thus $F = PA = 5(46.06 \times 10^3) = 230.3 \times 10^3 \text{ N}$

The line of this resultant is at $h_p = h_c + I_c/(h_c A) = 4.7 + bd^3/(12(h_c)5) = 4.7 + 2.5(2^3)/282 = 4.771$ m
 This is 0.929 m above the base of the tank.

Gate AB below is 1.2 m wide and is hinged at A. What horizontal force F must be applied at B to keep the gate AB closed?



Worked solution: The forces due to liquids must be determined and located relative to the hinge A, and any discrepancy balanced by a horizontal force F_h :

For the water, the gate centroid is at a depth

$$h_{cw} = 5.5 - 1.8/2 = 4.6 \text{ m}$$

The hydrostatic pressure at this depth is

$$P_w = \rho_w g h_{cw} = 1000(9.8)4.6 = 45.1 \text{ KPa}$$

The hydrostatic force is

$$F_w = P_w A = 45.1(1.2)1.8 = 97.4 \text{ KN}$$

This applies at a depth

$$h_{pw} = h_{cw} + I_c/h_{cw}A = 4.6 + (1.2(1.8)^3/12)/(4.6(1.2)1.8) = 4.66 \text{ m}$$

Relative to A, this depth is

$$h_{pwa} = 1.8 - (5.5 - 4.66) = 0.96 \text{ m}$$

Giving a moment force about A of

$$M_{pwa} = F_w \cdot h_{pwa} = 97.4(0.96) = 93.4 \text{ KN.m} \quad \text{anticlockwise}$$

Similarly for the oil we have

$$h_{co} = 1.8/2 = 0.9 \text{ m}$$

$$P_o = \rho_o g h_{co} = 750(9.8)0.9 = 6.6 \text{ KPa}$$

$$F_o = P_o A = 6.6(1.2)1.8 = 14.3 \text{ KN}$$

$$h_{po} = h_{poa} = h_{co} + I_c/h_{co}A = 0.9 + (1.2(1.8)^3/12)/(0.9(1.2)1.8) = 1.2 \text{ m}$$

$$M_{poa} = F_o \cdot h_{poa} = 14.3(1.2) = 17.2 \text{ KN.m} \quad \text{clockwise}$$

The difference in moments is

$$\Delta M = 93.4 - 17.2 = 76.2 \text{ KN.m}$$

This must be produced by a horizontal force acting at B, 1.8 m clockwise from A:

$$\Delta M = 1.8 F_h$$

$$F_h = \Delta M/1.8 = 76.2/1.8 = \underline{42.3 \text{ KN}}$$

There is no need to balance horizontal forces in this problem. Depending on the position of the balancing horizontal force (chosen arbitrarily to lie at B) there will be residual stresses in the gate that are taken up through elastic deformation.

An above-ground swimming pool is 4 m x 4 m in plan and 1.5 m deep. Determine the net hydrostatic force on each wall and the distance of this line of force from the ground. If the pool depth was doubled would the net force double or quadruple?

Following the three-step method outlined in page 33 of the notes we have:

1. Calculate the net resultant force due to hydrostatic pressure

$$\text{Resultant force} = \text{average pressure} \times \text{area} \quad F = \bar{P}A$$

The pressure profile is linear, and the structure extends the full depth, so the average pressure occurs at half the depth:

$$\bar{P} = \rho g \frac{H}{2} = 1000(9.8)1.5/2 = 7350 \text{ Pa}$$

The area is just the width x depth of the wall:

$$A = WH = 4(1.5) = 6.0 \text{ m}^2$$

Thus $F = 7350(6) = 44.1 \times 10^3 \text{ N}$

This force is directed at a normal to each pool wall.

2. Determine the *centre of pressure*, or position of the resultant force

We can calculate the centre of pressure as a vertical depth, then resolve that to a distance from the base of each wall:

$$h_p = h_c + \frac{I_c \sin^2 \theta}{h_c A}$$

The angle to the vertical is 90, hence $\sin^2 \theta = 1$

h_c is the depth to the centroid of the wall, which is $h_c = H/2 = 0.75 \text{ m}$

I_c is the moment of inertia of the wall, which is $I_c = W(H)^3/12 = 4(1.5)^3/12 = 1.125 \text{ m}^4$

A is the face area of the wall as before.

$$h_p = 0.75 + 1.125/(0.75 \times 6.0) = 1.0 \text{ m}$$

This is a vertical depth. We can convert this to a distance from the base of the wall, which we need in order to resolve the normal force resultant to a moment.

$$d_p = H - h_p = 1.5 - 1.0 = 0.5 \text{ m} = H/3$$

3. Calculate moments and determine structural resisting forces and moments.

The static pressure overturning moment about the base of the wall is the resultant force normal to the wall multiplied by its distance to the base of the wall:

$$M_o = Fd_p = 44.1 \times 10^3(0.5) = 22.05 \times 10^3 \text{ Nm}$$

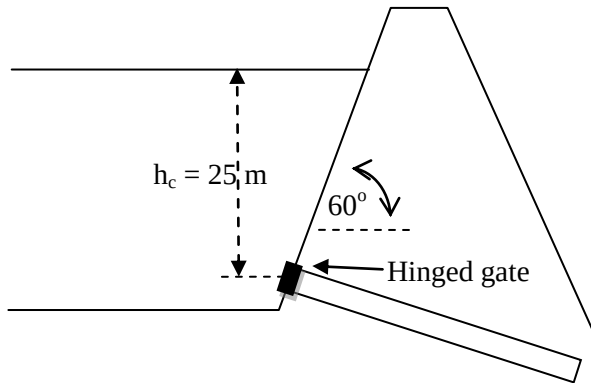
Adjacent walls of the swimming pool, connecting to each other by lined corners, must be capable of supplying a resisting moment equal to the overturning moment, in order to prevent a typical (and probably very funny) Australian backyard disaster from occurring.

If the depth of the pool was to double (making the disaster a virtual certainty) the calculation can be repeated as:

$$M_o = (F)d_p = (P[A])H/3 = (\rho g H/2[HW])H/3 = \rho g H^3 W/6 = 9800(3^3)4/6 = 176.4 \times 10^3 \text{ Nm}$$

Which is not 2 or 4 times worse, but 8x. Beware overfilling your pool!

The intakes to a hydroelectric power station are a group of 1 m diameter apertures on the water side of a dam wall sloping at 60° to the horizontal. The centre of each aperture is located at a depth of 25 m below the full water level. The apertures are held closed by circular gates hinged at the top. If the apertures are held closed by hydraulic hinge mechanisms, what maximum torque must each hinge be capable of supplying to open and safely close the gates?



Following the three-step method outlined in page 33 of the notes we have:

1. Calculate the net resultant force due to hydrostatic pressure

Resultant force = average pressure x area $F = \bar{P}A$

The average pressure occurs at the centroid of the aperture:

$$\bar{P} = \rho g h_c = 1000(9.8)25 = 245.0 \times 10^3 \text{ Pa}$$

The area is just the area of a circle:

$$A = \pi D^2/4 = \pi/4 = 0.7854 \text{ m}^2$$

Thus $F = 245.0 \times 10^3 (0.7854) = 192.4 \times 10^3 \text{ N}$

This force is directed at a normal to the sloping face of the aperture.

2. Determine the *centre of pressure*, or position of the resultant force

We can calculate the centre of pressure as a vertical depth, then resolve that to a length on the face of the wall:

$$h_p = h_c + \frac{I_c \sin^2 \theta}{h_c A}$$

h_c is the depth to the centroid of the aperture, which is 25 m

I_c is the moment of inertia of the aperture, which is $I_c = \pi D^4/64 = \pi/64 = 0.049 \text{ m}^4$

A is the area of the aperture as before:

$$h_p = 25 + 0.049 \sin^2 60 / (25 \times 0.7854) = 25.002 \text{ m}$$

To a very good approximation the normal force is at the centre of the aperture.

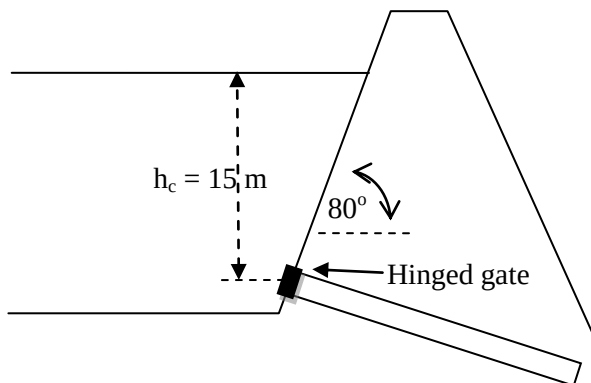
3. Calculate moments and determine structural resisting forces and moments.

The relevant opening moment is given by the pressure force x the distance between the hinge and the centre of pressure. Using the good approximation, we can easily see the distance is half the diameter of the aperture. The opening moment is thus

$$M_o = FD/2 = 192.4 \times 10^3 (0.5) = 96.2 \times 10^3 \text{ Nm}$$

The hinge mechanism must be capable of supplying at least this torque to the gate in opening and closing.

The low-level offtake on a deep reservoir is a 1 m square aperture on the water side of the dam wall, which slopes at 80° to the horizontal. The centre of the aperture is located at a depth of 15m below the full water level. The apertures are held closed by square gates hinged at the bottom. What maximum torque must each hinge be capable of supplying to open and safely close the gates?



Following the three-step method outlined in page 33 of the notes we have:

1. Calculate the net resultant force due to hydrostatic pressure

Resultant force = average pressure x area $F = \bar{P}A$

The average pressure occurs at the centroid of the aperture:

$$\bar{P} = \rho g h_c = 1000(9.8)15 = 147.0 \times 10^3 \text{ Pa}$$

The area is just the area of a square:

$$A = \text{side}^2 = 1.0 \text{ m}^2$$

Thus $F = 147.0 \times 10^3 (1.0) = 147.0 \times 10^3 \text{ N}$

This force is directed at a normal to the sloping face of the aperture.

2. Determine the *centre of pressure*, or position of the resultant force

We can calculate the centre of pressure as a vertical depth:

$$h_p = h_c + \frac{I_c \sin^2 \theta}{h_c A}$$

h_c is the depth to the centroid of the aperture, which is 15 m

I_c is the moment of inertia of the aperture, which is $I_c = \text{side}^4/12 = 1/12 = 0.0833 \text{ m}^4$

A is the area of the aperture as before:

$$h_p = 15 + 0.083 \sin^2 80 / (15 \times 1.0) = 15.01 \text{ m}$$

To a very good approximation the normal force is at the centre of the aperture.

3. Calculate moments and determine structural resisting forces and moments.

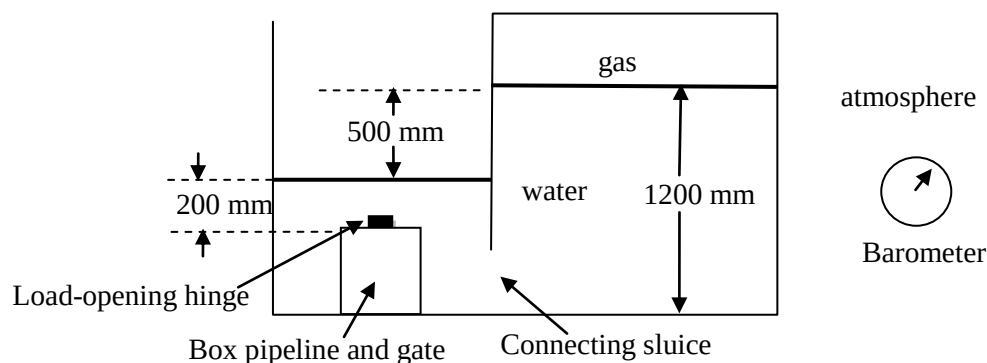
The relevant opening moment is given by the pressure force $\times$ the distance between the hinge and the centre of pressure. Using the good approximation, we can easily see the distance is half the side of the aperture. The opening moment is thus

$$M_o = FD/2 = 147.0 \times 10^3 (0.5) = 73.5 \times 10^3 \text{ Nm}$$

The hinge mechanism must be capable of supplying at least this torque to the gate in opening and closing.

The diagram shows a surge tank consisting of two chambers, one open to the atmosphere and another closed and filled with a low pressure gas. The two chambers are connected by a sluice opening as shown. The tank is designed to prevent pressure shock waves from rupturing a pipeline. At this moment the box pipeline is closed off so no dynamic pressures are in the system.

- What is the residual pressure in the gas, in gauge and absolute terms, if the barometric pressure reads 780 mm Hg? ($\rho = 13500 \text{ kg/m}^3$)
- The empty pipeline is kept closed by a square 500 x 500 mm gate which may open outwards into the tank. The hinge has a load-opening strength of 10 kN.m. What average pressure is required in the pipeline to open the gate in this configuration?



- The barometric pressure is $P = \rho g H = 13500(9.8)0.78 = 103.2 \text{ kPa}$

The pressure at the outside water surface is atmospheric $P_o = 103.2 \text{ kPa}$

The absolute pressure at the bottom of the tank is $P_a = 103200 + \rho_{\text{water}} g (1.200 - 0.500) = 110.1 \text{ kPa}$

The absolute pressure at the inside surface level is $110100 - \rho_{\text{water}}g(1.200) = 98.3 \text{ kPa}$

This is the absolute pressure in the gas.

The gauge pressure is absolute – atmospheric = $98.3 - 103.2 = -4.9 \text{ kPa}$

b) The average pressure on the gate is ρgh_c where h_c is the depth to the centroid of the gate:

$$h_c = 0.2 + 0.5/2 = 0.45 \text{ m}$$

$$P_{\text{average}} = 9800(0.45) = 4410 \text{ Pa}$$

The pressure force is the average pressure times the area of the gate:

$$F_p = 4410(0.5^2) = 1103 \text{ N}$$

The line of force is at a depth h_p :

$$h_p = h_c + I_c/(h_c.A) = h_c + b^4/(12h_c.A) = 0.45 + 0.5^4/(12(0.45)0.5^2) = 0.496 \text{ m depth}$$

This line is below the hinge at a distance $d = 0.496 - 0.2 = 0.296$

The closing moment of hydrostatic pressure is thus $F_p d = 1103(0.296) = 326.5 \text{ N.m}$

To this we can add the spring closing moment of 10 kN.m : $M_c = 10000 + 326.5 = 10326.5 \text{ N.m}$

The average pressure in the pipeline x the gate area x $\frac{1}{2}$ the gate depth is the opening moment:

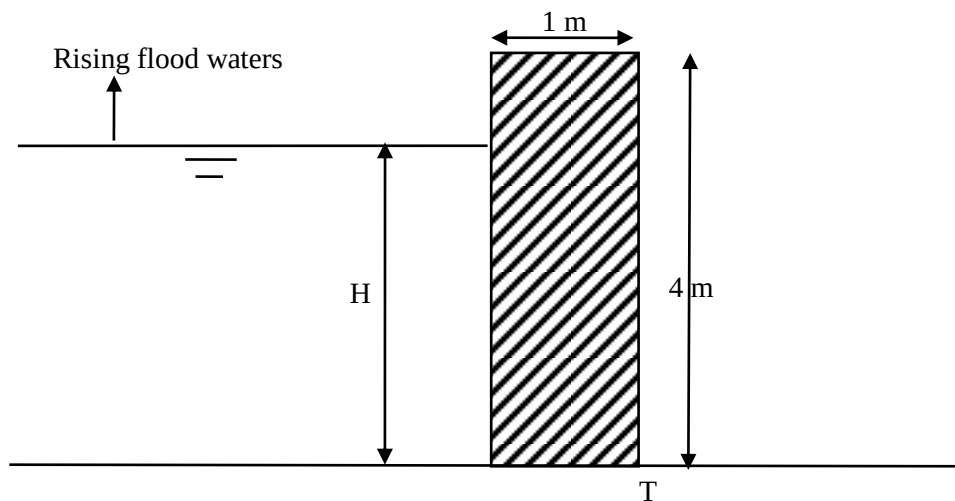
$$M_o = P_{\text{pipe}}(0.5^2)0.5/2$$

$$M_o = M_c = 10326.5 \text{ N.m when the gate is forced open}$$

$$P_{\text{pipe}} = 10326.5/(0.5^3/2) = 165.8 \text{ kPa is required to force open the gate}$$

A large concrete wall (as shown in cross-section below) of density 2500 kg/m^3 is used to temporarily plug a broken levee bank. Flood waters continue to inundate the area. Army sappers have laboured overnight to ensure the bedding and abutments of the block are firmly sandbagged to support the wall and retain water.

- Develop an expression for the overturning moment M_o per unit width of wall caused by floodwaters of height H . Use the outside toe T as a pivot.
- Calculate the resisting moment per unit width of wall due to the weight of the wall, also around the toe T.
- If the flood water keeps rising, is there a risk the concrete block will fail by falling over on its outside toe?



a) The average hydrostatic pressure is at half the depth:

$$P = 9800H/2$$

The overturning force per unit width of wall is the pressure times the height (=area/width):

$$F_o = PH = 9800H^2/2$$

The overturning force acts at a depth given by

$$h_p = h_c + I/Ah_c$$

$$h_c = H/2 \text{ (centroid of rectangular face)}$$

$$I = bh^3/12 = (1)H^3/12 = H^3/12 \text{ (second moment of rectangular face)}$$

$$A = bh = (1)H = H \text{ (area of rectangular face)}$$

$$h_p = H/2 + H^3/(12H.H/2) = H/2 + H/6 = 2H/3$$

This is offset from T by a distance $d = H - 2H/3 = H/3$

The overturning moment is thus given by $M_o = F_o.d = 9800H^2/2 \cdot H/3 = 9800 H^3/6$

b) The resisting moment is due to the weight of the wall. Using its density and cross-sectional area we have the weight per unit width of wall:

$$W = \rho g A = 2500(9.8)4(1) = 98 \text{ kN /m width of wall.}$$

This acts at a distance of 0.5 m from the toe. The resisting moment is then:

$$\underline{M_r = 0.5 \times 98 = 49 \text{ kN.m / m width}}$$

c) The wall overturns when the resisting and overturning moments are equal:

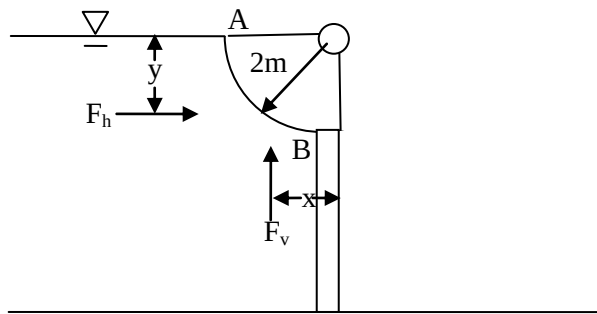
$$M_o = 9800H^3/6 = 49 \times 10^3$$

$$H = \sqrt[3]{(6(49 \times 10^3)/9800)} = 3.11 \text{ m}$$

The wall is at risk of overturning, sappers must sandbag the front wall to provide additional support.

Pressure Forces on Curved Surfaces

Determine and locate the components of force due to the water acting on the curved gate AB, per metre of length (curved sluice gates are common in dams, such as Copeton dam).



Worked solution: The curved surface profile in the horizontal direction is a rectangle:

$$h_c = h/2 = 1.0 \text{ m}$$

$$I_c = bh^3/12 = 1(2)^3/12 = 0.67 \text{ m}$$

Thus

$$P_h = \rho gh_c = 1000(9.8)1.0 = 9.8 \text{ KPa}$$

$$F_h = P_h A = 9.8(1.0)2.0 = \underline{19.6 \text{ KN}}$$

$$y = h_p = h_c + I_c/h_c A = 1.0 + 0.67/(1.0(1.0)2) = \underline{1.3 \text{ m}}$$

In the vertical direction, the force F_v is equal to the weight of water displaced by the semi-circle AB:

$$F_v = \rho g V = \rho g (\pi r^2/4) w = 1000(9.8)(\pi 2^2/4)1.0 = \underline{30.8 \text{ KN}}$$

This acts through the centroid of area of the semi-circle. From the study guide page 40 (centroids) we know that this is

$$x = 4/3 r/\pi = 4/3 2/\pi = \underline{0.85 \text{ m}}$$

The water side of the wall of a 100 m long dam is a quarter circle with a radius of 10 m. Determine the hydrostatic force (magnitude and line) when the dam is full.

The vertical weight force of one quarter of a 100x10 m cylinder of water is

$$W = L\rho g A/4 = 100\rho g \pi r^2/4 = 100(9800)\pi 10^2/4 = 76.97 \times 10^6 \text{ N}$$

This weight force acts through the centroid of the quarter circle:

$$\bar{x} = 4R/3\pi = 4.244 \text{ m from the centre of the circle (p 36 study guide)}$$

The vertical weight force thus produces a moment about the back toe of the wall of:

$$M_v = W \bar{x} = 76.97 \times 10^6 (4.244) = 326.7 \times 10^6 \text{ Nm}$$

The horizontal static pressure force acts on a projection of the curved wall, which can be analysed as a vertical 100x10 m rectangle:

The depth to the centroid of the projection is $h_c = R/2 = 5$

The average pressure is $\bar{P} = \rho g h_c = \rho g R/2 = 9800(5) = 49 \times 10^3 \text{ Pa}$

The area is $A = LR = 100(10) = 1000 \text{ m}^2$

The resultant horizontal force is $F_h = \bar{P}A = 49 \times 10^3(1000) = 49 \times 10^6 \text{ N}$

The depth of the centre of pressure is $h_p = h_c + I_c/(h_c A) = 5 + (100(10^3)/12)/(5 \times 1000) = 6.667 \text{ m}$

This can be resolved as a vertical distance to the back toe as $d_p = R - h_p = 10 - 6.667 = 3.333 \text{ m}$

The horizontal moment is thus:

$$M_h = F_h d_p = 49 \times 10^6 (3.333) = 163.3 \times 10^6 \text{ Nm}$$

The total moment about the back toe is the sum of horizontal and vertical moments:

$$M = M_h + M_v = 326.7 \times 10^6 + 163.3 \times 10^6 = 490 \times 10^6 \text{ Nm}$$

The total force is the sum of horizontal and vertical vectors:

$$(F_x, F_y) = (0, -76.97 \times 10^6) + (49 \times 10^6, 0) = (49 \times 10^6, -76.97 \times 10^6)$$

This acts through the coordinates (4.244, 3.333) relative to the back to of the wall, at an angle of

$$\theta = \tan^{-1}(-76.97/49) = -57.52^\circ$$

The water side of the wall of a 100 m long dam is a quarter circle with a radius of 10 m. Determine the hydrostatic force (magnitude and line) when the dam is full.

The vertical weight force of one quarter of a 100x10 m cylinder of water is

$$W = L\rho g A/4 = 100\rho g \pi r^2/4 = 100(9800)\pi 10^2/4 = 76.97 \times 10^6 \text{ N}$$

This weight force acts through the centroid of the quarter circle:

$$\bar{x} = 4R/3\pi = 4.244 \text{ m from the centre of the circle (p 36 study guide)}$$

The vertical weight force thus produces a moment about the back toe of the wall of:

$$M_v = W \bar{x} = 76.97 \times 10^6 (4.244) = 326.7 \times 10^6 \text{ Nm}$$

The horizontal static pressure force acts on a projection of the curved wall, which can be analysed as a vertical 100x10 m rectangle:

The depth to the centroid of the projection is $h_c = R/2 = 5$

The average pressure is $\bar{P} = \rho g h_c = \rho g R/2 = 9800(5) = 49 \times 10^3 \text{ Pa}$

The area is $A = LR = 100(10) = 1000 \text{ m}^2$

The resultant horizontal force is $F_h = \bar{P}A = 49 \times 10^3(1000) = 49 \times 10^6 \text{ N}$

The depth of the centre of pressure is $h_p = h_c + I_c/(h_c A) = 5 + (100(10^3)/12)/(5 \times 1000) = 6.667 \text{ m}$

This can be resolved as a vertical distance to the back toe as $d_p = R - h_p = 10 - 6.667 = 3.333 \text{ m}$

The horizontal moment is thus:

$$M_h = F_h d_p = 49 \times 10^6 (3.333) = 163.3 \times 10^6 \text{ Nm}$$

The total moment about the back toe is the sum of horizontal and vertical moments:

$$M = M_h + M_v = 326.7 \times 10^6 + 163.3 \times 10^6 = 490 \times 10^6 \text{ Nm}$$

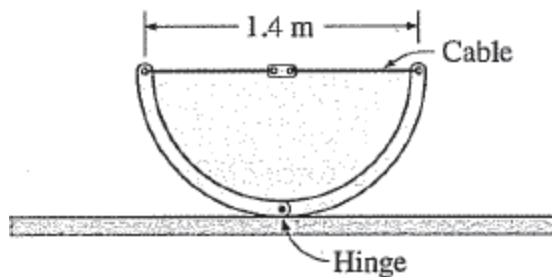
The total force is the sum of horizontal and vertical vectors:

$$(F_x, F_y) = (0, -76.97 \times 10^6) + (49 \times 10^6, 0) = (49 \times 10^6, -76.97 \times 10^6)$$

This acts through the coordinates (4.244, 3.333) relative to the back toe of the wall, at an angle of

$$\theta = \tan^{-1}(-76.97/49) = -57.52^\circ$$

A long, portable water trough of semicircular cross section, radius 0.7 m, is hinged at its base centreline and tied together by cables every 3 m along the length of the trough. Neglecting the weight of metal, calculate the tension in each cable when the trough is filled to the brim.



The objective is to determine a hydrostatic opening moment acting about the centreline hinge, and then calculate the resulting resisting moment supplied by the cables.

We can simplify the analysis by considering one half of a 3 m section of trough. The tension in the cable will be the same in either half, and the same in all other 3 m sections.

The vertical weight force of one quarter of a $3 \times 1.4\phi$ m cylinder of water is

$$W = L\rho g A/4 = 3\rho g \pi D^2/16 = 3(9800)\pi 1.4^2/16 = 11.31 \times 10^3 \text{ N}$$

This weight force acts through the centroid of the quarter circle:

$$\bar{x} = 4R/3\pi = 0.297 \text{ m from the centre of the circle (p 36 study guide)}$$

The vertical weight force thus produces an opening moment about the hinge of:

$$M_{ov} = W\bar{x} = 11.31 \times 10^3(0.297) = 3360 \text{ Nm}$$

The horizontal static pressure force acts on a projection of the curved wall, which can be analysed as a vertical 3×0.7 m rectangle:

The depth to the centroid of the projection is $h_c = R/2 = 0.35$

The average pressure is $\bar{P} = \rho g h_c = \rho g R/2 = 9800(0.35) = 3430 \text{ Pa}$

The area is $A = LR = 3(0.7) = 2.1 \text{ m}^2$

The resultant horizontal force is $F_h = \bar{P}A = 3430(2.1) = 7203 \text{ N}$

The depth of the centre of pressure is $h_p = h_c + I_c/(h_c A) = 0.35 + (3(0.7^3)/12)/(0.35 \times 2.1) = 0.4667 \text{ m}$

This can be resolved as a vertical distance to the hinge as $d_p = R - h_p = 0.7 - 0.4667 = 0.233 \text{ m}$

The horizontal overturning moment is thus:

$$M_{oh} = F_h d_p = 7203(0.233) = 1681 \text{ Nm}$$

The total overturning moment is the sum of horizontal and vertical moments:

$$M_o = M_{oh} + M_{ov} = 3360 + 1681 = 5041 \text{ Nm}$$

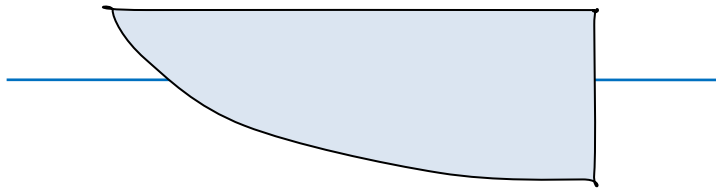
This can be resisted by a tensile force in the cable acting over the distance to the hinge:

$$M_r = M_o = TR$$

$$T = M_o/R = 5041/0.7 = 7201 \text{ N}$$

The tension in the cable is 7201 N, NOT twice this value. To see why not, consider removing the other half of the trough: what force must be applied to keep the remaining half in balance?

If the horizontal component of hydrostatic force on a curved surface was somehow *not* equal to the force on a projection of the surface onto a vertical plane, what could you conclude about the propulsion of a simple boat with a vertical stern (back end) as shown?



Assuming the diagram is complete and accurate, ie that the boat has no propulsion whatsoever, we would expect it to just sit there statically and not go anywhere. This means that all the forces on the boat must balance: weight equals buoyancy and all hydrostatic forces cancel out.

This provides us with an incredibly convenient analytical shortcut: In principle we would need to integrate elements of horizontal force on the curving face, varying as the angle of the hull to the waterline varies, just to calculate the net “backwards” force on the boat. In practice, as this thought experiment reveals, we only need to calculate the much simpler distribution of “forwards” forces on the plane vertical face, and then assert that they *must* be the equal and opposite to the curved distribution, so that they all cancel out.

We can use this idea to save a lot of time calculating curvilinear pressure distributions. Wherever we find pressure forces occurring in complex 3-dimensional shape, we can simplify the problem by *projecting* the shape onto a plane surface and calculating with the resulting simplified description. How useful is that?!

A sphere 250 mm in diameter, relative density 1.4, is submerged in a liquid having a density varying with depth y below the surface given by $\rho = 1000 + 0.03y \text{ kg/m}^3$. Find the depth at which the sphere will float in equilibrium.

Pressure Vessels

An old cylindrical tank 8 m in diameter is constructed of sheet iron 1.0 mm thick. Due to years of disuse and occupation by wildlife, the tank contains 2.3 m of water overlying 1.2 m of mud, density 2,600 kg/m³. Calculate the maximum stress in the tank. You guess that the old iron sheet has a yield stress of about 300 MPa. How safe is the tank as a agricultural tourist attraction?

In this situation the total maximum pressure in the tank is composed of two distinct loads, due to the water and the mud. We can derive the total pressure from the static pressure formula applied twice:

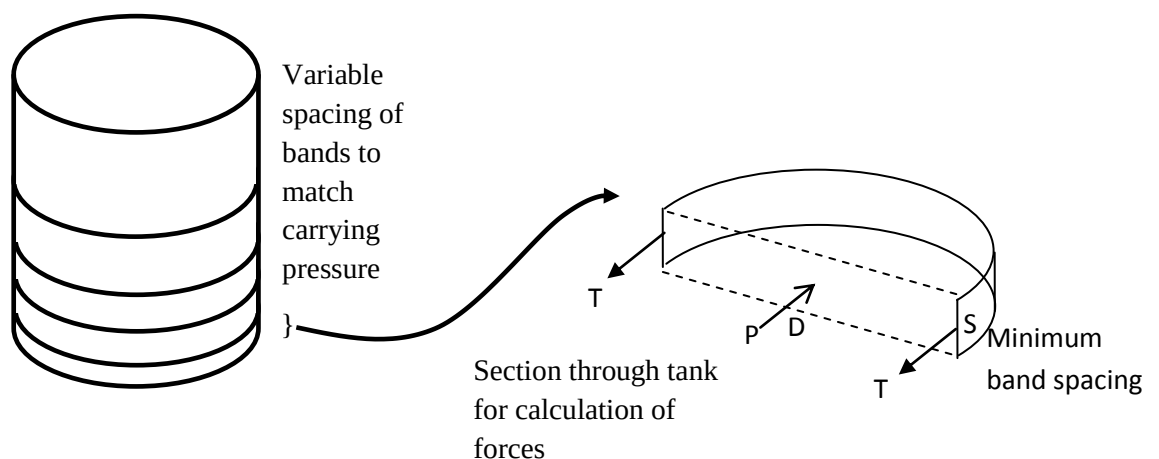
$$P = \sum \rho g H = 1000(9.8)2.3 + 2600(9.8)1.2 = 53.12 \times 10^3 \text{ Pa}$$

From p40 (study guide) we find a formula for the tensile stress in a tank, which we again have to modify for the combined pressure:

$$\begin{aligned} \sigma &= \sum \rho g H \frac{D}{2t} \\ &= 53.12 \times 10^3 (8) / (2 \times 0.001) = 212.5 \times 10^6 \text{ Pa} = 212.5 \text{ MPa} \end{aligned}$$

This is about 70.8% of the maximum yield stress. With families around we wouldn't want it to get above 66%. Tell the DEH sorry, it's gotta go.

A wooden storage vat 6 m in diameter is filled with 7.3m of saline solution, density 1060 kg/m³. The wood staves are bound by flat steel bands, 50 mm wide by 6 mm thick, whose allowable stress is $110 \times 10^6 \text{ N/m}^2$. What is the minimum spacing of bands near the base of the vat, neglecting any initial stress?



Let's first work out the total tensile stress the bands can carry: $110 \times 10^6 \text{ Pa}$ applied over a 50x6 mm area gives a tensile force of:

$$T = 110 \times 10^6 (0.05) (0.006) = 33 \times 10^3 \text{ N}$$

Each band provides twice this much tensile force, which is balanced against the internal pressure acting over the diameter D and band spacing S :

$$2T = PDS$$

The minimum band spacing S can be estimated from the maximum static pressure:

$$2T = \rho g H D S$$

Thus $S = 2T / \rho g H D = 2(30 \times 10^3) / (1060(9.8)7.3(6)) = 0.132 \text{ m}$

The Reef HQ Aquarium in Townsville has ordered a new free-standing, walk-through shark pool, which is to be of 1-inch tensile glass 16 m on each side and 4 m deep. There is a 2m semicircular glass observation tunnel through the tank. The glass walls are designed to withstand pressure to 10m depth. The joint seals at the edges of each glass face are steel-belted rubber strips bolted with stainless steel backing strips (see Figure 1). Similar joints attach each glass wall to the concrete floor.

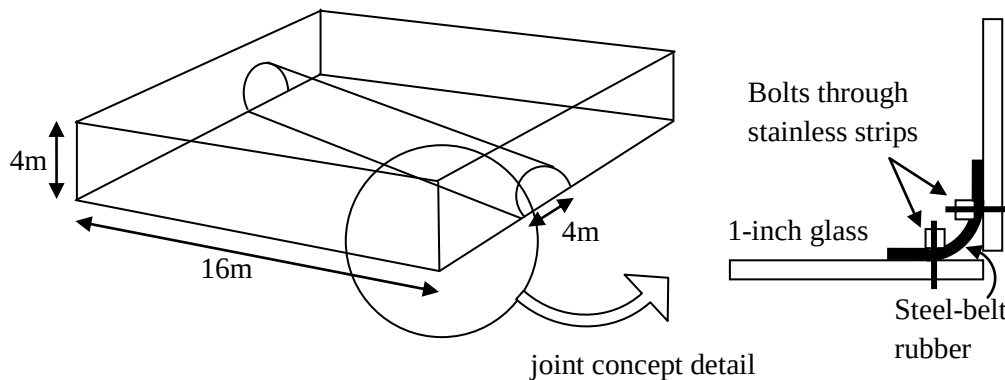


Figure 1: 16x16x4 (1024 m³) shark tank at Reef HQ, Townsville (not to scale – concept only)

- Assuming the stainless joint strips are 2.5 cm wide and the bolts are spaced 15 cm apart, what tensile force must be induced in the bolts by the tightening of the nuts? (Hint: Derive a maximum pressure and equate a bolt force applied to a strip area)
- Assuming the rigid glass walls are suitably designed, and considering only a balance of horizontal forces for the moment, what average distributed tensile force (in kN per metre of length) must be sustained by the belted rubber strips in the wall and floor joints of each face? (Hint: Derive an outward pressure force on each 16x4 face and equate a restraining force distributed over each 16+2x4 m edge length)
- Considering the floor joint as a possible overturning failure point, what average distributed tensile force (in kN per metre as before) must be sustained by the wall joints independent of the floor joint to provide a sufficient restraining moment? (Keep it simple, ie make uniform assumptions). Is this larger or smaller than the distributed force derived in b)?
- Since the tensile joint forces are supplied by the stretching of the belt-rubber strips, sketch a side view of the deformation of a wall face outwards under water pressure. Assume each glass face remains rigidly flat. Does the tank bulge outward more at its base, or at the top?

a) Strip area per bolt = $0.025 \times 0.15 = 0.00375 \text{ m}^2$

Maximum Pressure = $\rho gH = 9800(4) = 39200 \text{ Pa}$

Bolt force = $PA = 39200 \times 0.00375 = 147 \text{ N per bolt}$.

Note: this is a *residual* force, ie it is what is left after the bolts have taken on the structural load of supporting the wall. Structural loads are much larger (see part b) than the injected pressure load.

b) The average pressure in the tank is $\rho gH/2 = 9800(2) = 19600 \text{ Pa}$

The face area is $16 \text{ H} = 16 \times 4 = 64 \text{ m}^2$

The pressure reaction force is $\rho gH/2 \times 16 \text{ H} = 19600 \times 64 = 1254400 \text{ N} = 1.2 \text{ MN}$

The edge length is $16 + 2 \times 4 = 24 \text{ m}$

The average distributed load is $1.2 \times 10^6 / 24 = 52266 \text{ N/m} = 52 \text{ kN/m}$

This is quite a large force to hang from each metre of rubber. A free standing design relying on joint strength alone will require massive joints and may not be optimal. The bolts themselves must carry this structural load in addition to the residual force worked out in a). The structural load per bolt, given a total force of 1.2 MN and 0.15 m spacing, is

Structural load per bolt = $1.2 \times 10^6 / (24/0.15) = 7.84 \text{ kN/bolt}$ which is independent of the residual force needed to retain water pressure.

c) The pressure reaction force is located at $H/3$ above the floor: $y_c = 4/3 = 1.33 \text{ m}$

The overturning moment is:

$$M_o = PAy_c = \rho gH/2 \cdot 16H \cdot H/3 = \rho gH^3 \cdot 8/3 = 9800(4^3)8/3 = 1672533 \text{ Nm} = 1.7 \text{ MNm}$$

Assuming there is a uniform distributed tensile force in the wall joints, the centre of this force distribution will be at $H/2$: $y_r = 4/2 = 2 \text{ m}$

The resisting force must create an equal moment for stability: A uniform force distribution F_d applied over the wall joint length $L = 2 \times H = 8 \text{ m}$ at $y_r = 2 \text{ m}$ gives a resisting moment of:

$$M_r = F_d \times L \times y_r = F_d \times 2H \times H/2 = F_d \times H^2 = 16F_d$$

For stability this is equal to M_o :

$$M_r = 16F_d = M_o = 1.7 \text{ MNm}$$

Thus $F_d = 1.7 \times 10^6 / 16 = 104,533 \text{ N/m} = 104 \text{ kN/m}$

This is exactly twice the force distribution required for horizontal stability. Under the assumption of a uniform distributed load in the wall joints, the floor joint stress is reduced by an equivalent amount to maintain horizontal force balance, taking its length into account, and as a result the wall joint stresses will distribute non-uniformly to compensate for the reduced stretching of the floor joint.

A solution of this is not required, but provides a useful point to note: The wall stresses are higher than the floor stresses, which indicates the probable failure point to produce the probable overturning failure mode, starting with a tear at the top of a wall joint. With suitably designed joints (not required in the answer), that's not going to happen.

d) The ideas in the conclusion of c) show that the wall bulges outward at the top of the wall, where the distributed tensile stresses are highest.

Fisheries researchers are trying to design a portable on-ground fish enclosure consisting of an inflatable waterproof plastic ring attached to a sealed plastic floor. The inflatable ring is designed to float as the enclosure is filled, raising the outside edge of the floor into a wall which bulges into a semi-circular curve. At 1 m depth the enclosure should look as per Figure 2. The designers are not sure how thick the inflatable ring should be, ie what thickness d is needed:

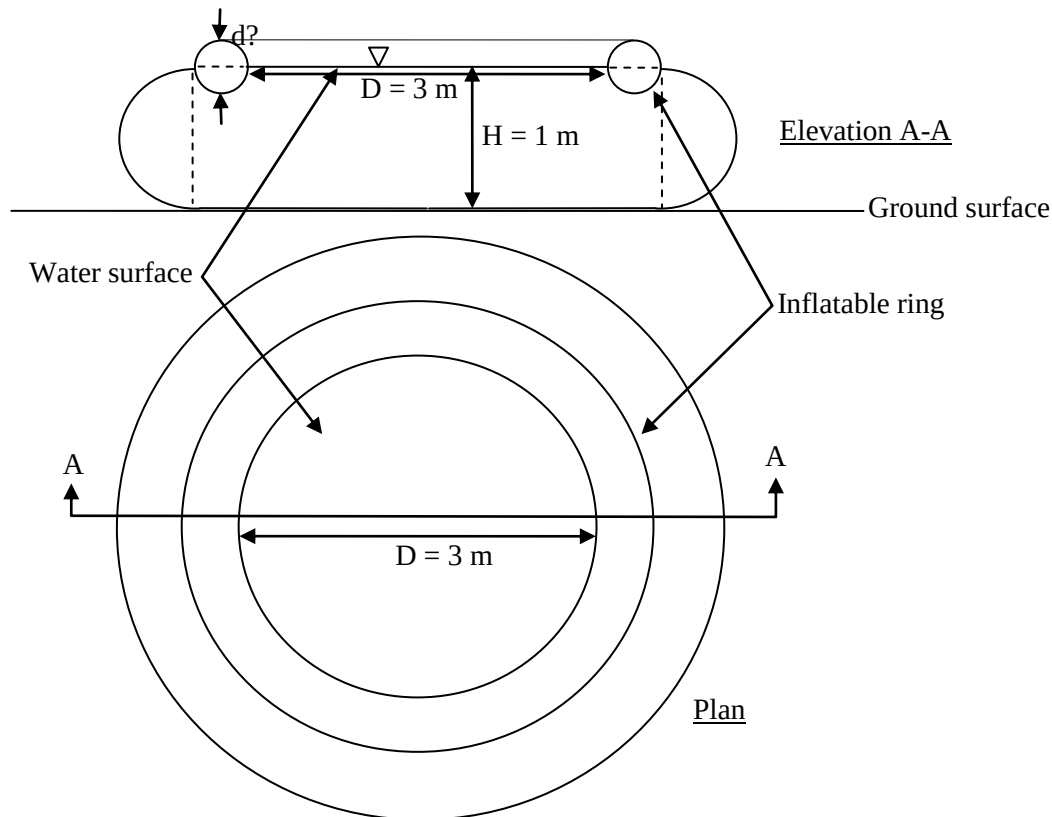


Figure 2: Proposed inflatable fish enclosure consisting of a buoyant rubber ring which floats on the water surface, thereby supporting a flexible waterproof bag that contains the water.

- When full, what total weight force of water is supported by the buoyancy ring, ie not directly supported by the ground? (Vertical dashed lines are a suggested delineation). Do you need to make an assumption about the ring thickness d , ie a first guess?
- Given your answer in a), what ring thickness d will provide sufficient buoyancy to support that unsupported mass? (horizontal dashed lines ...) Do you need to correct your answer in a) based on the revised estimate of d ? Is this starting to seem ridiculous or impossible?
- A bit depressed by this calculation, and knowing these things do work as they have one at home (a children's wading pool), the designers assume a 30 cm thick ring and try to sketch what the resulting "bag of water" should look like. How does it affect the depth of the design? What really is holding up all that water?

a) Taking the cross section A-A as a free body diagram, all the water outside the vertical dashed lines has no visible means of support other than the buoyancy of the ring. The volume, mass and weight of the unsupported water is found by wrapping the area of a 1m diameter semicircle around a circle (the “volume of revolution” method) which is 3 m across plus twice the unknown diameter d.

Let’s assume a thickness $d = 0.3$ m (just a guess, seems reasonable, others assumed a diameter of 1 m to start with).

Then the diameter of the outside circle is $D_o = 3 + 2(0.3) = 3.6$ m

The circumference of the 3.6 m outside circle is $C_o = \pi D_o = \pi 3.6 = 11.3$ m

The area of the unsupported semicircle is $A_o = \pi H^2/8 = \pi 1^2/8 = 0.393$ m²

The unsupported volume is $V_o = A_o C_o = 0.393(11.3) = 4.4$ m³

The unsupported weight force is then $\rho g V_u = 9800(4.4) = 43558$ N

This is 4.4 metric tonnes mass, which is a lot.

b) Come to think of it (as designers are meant to say *before* the unit is built) the buoyancy ring has to displace the same volume of water that it is supporting. Since the inner circle has a lesser circumference than the outer circle, so the ring has to have a larger displacement area than the unsupported area.

That could be impractical, but let’s go through with the calculation anyway.

Let’s correct that calculation with $d = 1.1$ m at least (how much larger?)

Table 1: Put it into a spreadsheet to speed things up

Thickness d	0.3	1.1
Outside Diameter Do	3.6	5.2
Outside Circumference Co	11.30973	16.33628
Outside Area Ao	0.393	0.393
Outside Volume Vo	4.441322	6.415243
Outside Weight Force	43524.96	62869.38

So now the weight force is 62869 N. Is the ring buoyant enough for that?

A 1.1 m thick ring at half its depth displaces an area $A_d = \pi d^2/8 = \pi 1.1^2/8 = 0.475$ m²

The diameter of the displacement ring centre is $D_d = 3 + d = 4.1$ m

The circumference of the ring centre is $C_d = \pi D_d = \pi 4.1 = 12.9$ m

The volume displaced is $V_d = A_d C_d = 0.475(12.9) = 6.12$ m³ compared to 6.42 m³

And the displaced weight is $W = \rho g V = 9800(6.12) = 59976$ N compared to 62869 N

The displacement volume has to be the same as the unsupported volume. With this first guess it's a bit less. Trial and error methods quickly show a diameter of between 1.12 and 1.13 m would work (Table 2) even though this is ridiculous:

Table 2: Trial and error calculation for thickness d:

Thickness d	0.3	1.1	1.12	1.13
Outside Diameter Do	3.6	5.2	5.24	5.26
Outside Circumference Co	11.30973	16.33628	16.46195	16.52478
Outside Area Ao	0.393	0.393	0.393	0.393
Outside Volume Vo	4.441322	6.415243	6.464591	6.489265
Outside Weight Force	43524.96	62869.38	63352.99	63594.8
Displaced Area	0.035343	0.475166	0.492602	0.501437
Displaced Centre Diameter	3.3	4.1	4.12	4.13
Displaced Circumference	10.36726	12.88053	12.94336	12.97478
Displaced Volume	0.366409	6.120388	6.375922	6.50604
Unsupported Volume	-4.07491	-0.29485	-0.08867	0.016775

Some of you solved the underlying quadratic equation for an exact solution. Nice work!

c) The design can work with a 30 cm ring, but the shape is more like a lozenge, flattened near the base (see photo). The depth correspondingly reduces, to a bit less than 0.5 m at best on flat ground. The effect is that the bulge of the bag around the unsupported edge is smaller, reduced geometrically to the same volume of displacement as the buoyancy ring. In fact all of the horizontal forces and most of the vertical ones are provided by tension in the bag, which stretches to provide an elastic force. The buoyancy ring is more important when the bag is filling as it raises the edge to keep pace with the rising water level.



Wading pool: the designer's mistake was to sketch a circular bag shape and assume circular geometries. A circular profile is actually impossible, of course.

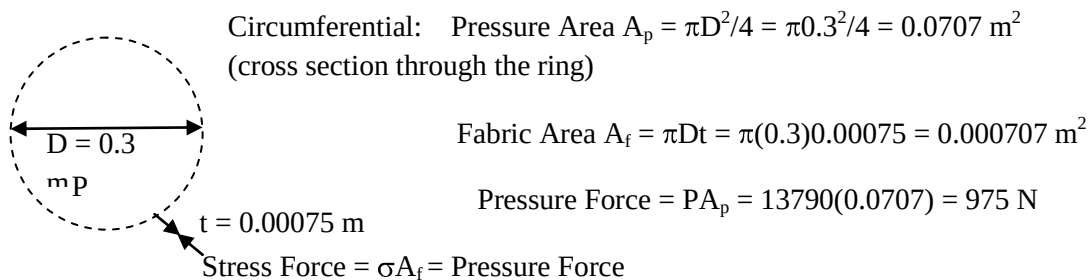
A thin-walled hollow steel sphere 3 m in diameter holds gas at 1.5 MPa. If the steel has an allowable stress of 60 MPa what minimum wall thickness is required?

- a) If the 30 cm inflatable ring in (Fisheries researchers, above) is inflated to 2 PSI (about the pressure of a typical airbed) what is the tensile stress in the fabric in both longitudinal and circumferential directions? The fabric is 0.75 mm thick. If the tensile strength of the fabric is 11 MPa what is the safety factor against rupture in both directions? Assuming the ring is accidentally overfilled, can you predict in which direction it will rupture?
- b) Calculate the compressive stress in the walls of the glass tunnel in the Reef HQ Pool from Q1. The tunnel is of 1 inch (2.5cm) tempered glass. If the glass has a 100 MPa compressive strength what is the safety factor?

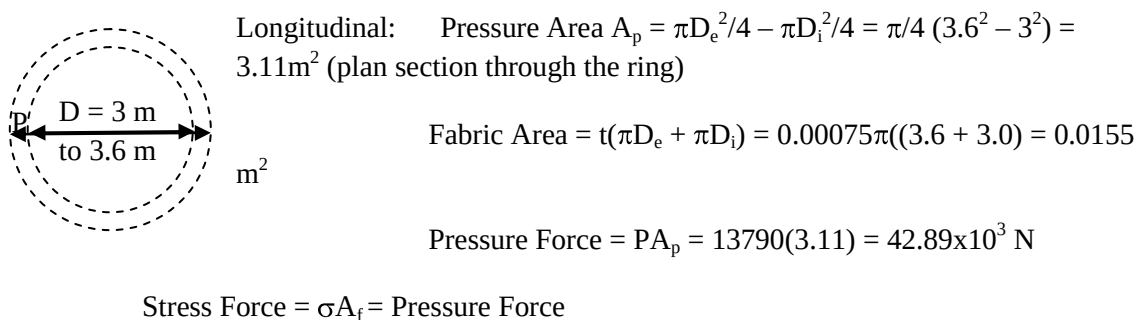
a) Convert PSI to Pa : $\text{PSI} \times 6895 = \text{Pa}$

2 PSI is equivalent to $2 \times 6895 = 13790 \text{ Pa}$

The ring is cross-sectioned in two directions: Circumferentially (through the ring at right angles) and longitudinally (around the ring):



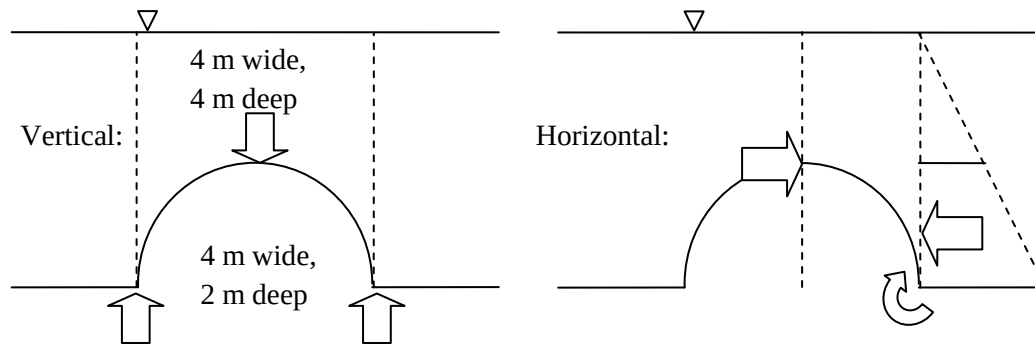
Thus Circumferential Tensile Stress = $\sigma_c = P A_p / A_f = 975 / 0.000707 = 1379000 = 1.38 \text{ MPa}$



Thus Longitudinal Tensile Stress = $\sigma_l = P A_p / A_f = 42.89 \times 10^3 / 0.0155 = 2767000 = 2.77 \text{ MPa} = 2 \sigma_c$

The safety factor on 40 MPa plastic is $11 / 2.77 = 3.971$ –rupture risk is small in normal use. If overfilled the fabric will rupture longitudinally (ie a long rip around the big circumference) because the stresses are always higher in that direction.

b) The 4 m tunnel has a 2m radius, and is thus in varying hydrostatic pressure from 2 to 4 m deep:



(Note: the diagram on the assignment brief showed a 2 m diameter – my mistake. Fortunately you all realised that this would give a tunnel only 1 m high – claustrophobic in a shark tank!)

There are no circumferential stresses as the tunnel is open to atmosphere at both ends. Longitudinal stresses depend on the direction because of the varying pressure. Careful thought would suggest that an analysis involving the vertical weight and horizontal moment, as per structural analysis (as opposed to enclosed pressure vessel theory) would be more appropriate. The tunnel is only approximately a pressure vessel.

As a structural problem, longitudinal compressive stresses must be evaluated from two directions: horizontal and vertical. The two directions can be treated separately as alternative worst cases.

Vertical: Per metre length of tunnel, the tunnel supports a volume of water equal to a $4 \times 4 \times 1 \text{ m}^3$ block less the volume of the tunnel:

ie a vertical volume of water $4 \times 4 \times 1 - \pi 4^2/8 \times 1 = 9.72 \text{ m}^3$

This volume of water has a weight of $9800(9.72) = 95220 = 95 \text{ kN}$

This weight is supported by the base of the glass tunnel, ie two strips of glass 2.5 cm wide and 1 m long, an area of $2 \times 0.025 \times 1 = 0.05 \text{ m}^2$

The maximum vertical compressive stress on the glass is thus $95 \times 10^3 / 0.05 = \underline{\underline{1904000 = 1.9 \text{ MPa}}}$

The vertical safety factor is $100 / 1.9 > 50 \text{ OK}$

Horizontal: The average pressure on the tunnel between 2 and 4 m depth is $P = \rho g H_{\text{average}}$

ie $P_{\text{average}} = 9800(2 + 4)/2 = 9800(3) = 29400 \text{ Pa}$

The net horizontal pressure force on a 1 m length of tunnel is $F_p = P_{\text{average}} A = 29400(2 \times 1) = 58800 \text{ N}$

This force applies at the pressure centre depth $h_p = h_c + I/Ah_c$

Where h_c is the mean tunnel depth 3 m, A is the profile area of a 1 m length of tunnel $A = 2 \text{ m}^2$, and I is the moment of inertia of the 1 m length of tunnel, $I = bh^3/12 = 1(2^3)/12 = 0.667 \text{ m}^4$

Thus $h_p = 3 + 0.667/(2(3)) = 3.11 \text{ m}$. The pressure height is $y_p = 4 - 3.11 = 0.89 \text{ m}$ from the floor.

Assuming an overturning moment failure about the toe of the tunnel due to crushing of the top of the tunnel, the pressure force moment about the toe is $F_p y_p = 58800(0.89) = 52330 = 52 \text{ kNm}$

This is supported by compressive forces in a 1 m strip at the top of the tunnel, ie a compressive force F_c at a lever height of 2m:

$$2F_c = F_p y_p = 52 \text{ kNm}$$

Thus $F_c = 52 \times 10^3 / 2 = 26 \text{ kN}$

The area of the 1 m, 2.5cm glass strip is 0.025 m².

The maximum horizontal compressive stress on the glass is thus $26 \times 10^3 / 0.025 = \underline{\underline{1045000 = 1.05 \text{ MPa}}}$

The horizontal safety factor is $100 / 1.05 \sim 100 \text{ OK}$