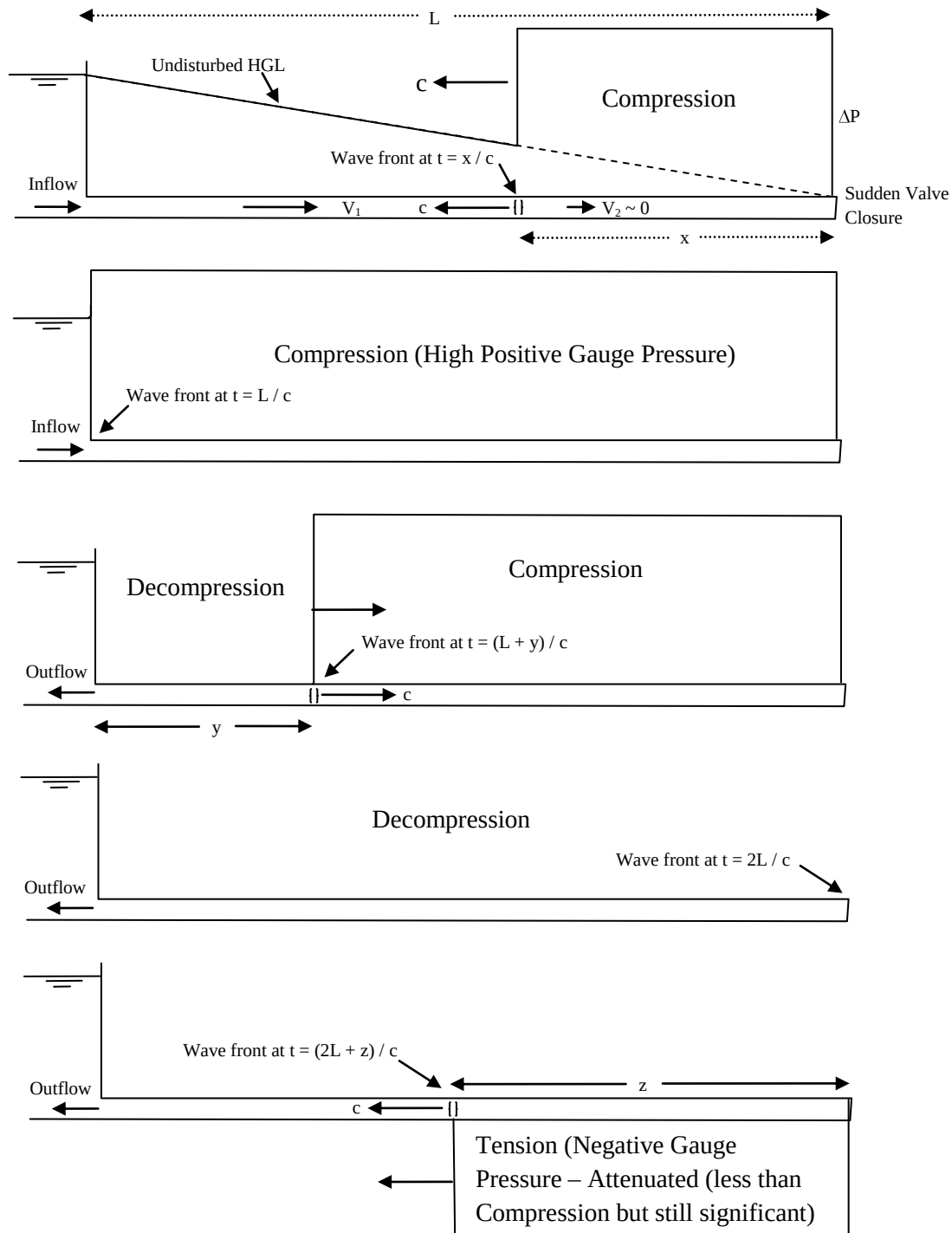


Water Hammer

Any sudden or rapid acceleration of flow in a pipe, as in a control event such as pump on/off or valve close/open, can result in rapid compression and decompression waves that reverberate along the pipe. Uncontrolled, these can result in damage due to high pressure shock or low pressure cavitation.

A rapidly decelerated flow does not slow or stop uniformly along its length. The control event propagates at the speed of sound in the fluid, forming a travelling zone of compression and decompression. When this disturbance reaches control points such as reservoirs, pumps or valves, it reflects and reverses from compression to decompression, generating a negative wave as it travels back the other way:

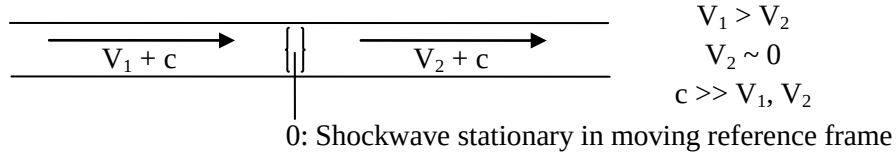


A sudden valve closure event. Flow velocity behind the shock wave is not exactly zero, as the fluid density and resulting volume changes provide a small continuing motion. An oscillating flow is set up and is damped by shear friction.

The water hammer phenomenon is analysed by continuity, compression and momentum equations:

Continuity

We can reduce the flow at the shock wave to steady state by superimposing a reference frame velocity (a Lagrangian method) equal to the celerity c :



Mass continuity holds in the moving reference frame:

$$\dot{m} = \rho Q = \rho V A = \text{constant}$$

$$A = \text{constant} \text{ thus } \rho V = \text{constant}$$

$$\rho_1(V_1 + c) = \rho_2(V_2 + c) \quad \rho_1 < \rho_2$$

Compression

Density changes are related to pressure changes by the bulk Modulus K :

$$\rho_2 - \rho_1 = \frac{m}{V_2} - \frac{m}{V_1}$$

$$= \frac{m}{V_2} \left(1 - \frac{V_2}{V_1} \right)$$

$$= \rho_2 \left(\frac{V_1 - V_2}{V_1} \right) \quad V_1 > V_2$$

$$= \rho_2 \varepsilon_v \quad \text{where } \varepsilon_v \text{ is the volumetric strain.}$$

$$= \rho_2 \frac{\Delta P}{K}$$

So $\rho_2 - \rho_1 = \rho_2 \frac{\Delta P}{K}$

$$\rho_2 - \rho_2 \frac{\Delta P}{K} = \rho_1$$

Or $\rho_2 = \frac{\rho_1}{1 - \frac{\Delta P}{K}} = \frac{K \rho_1}{K - \Delta P}$

Combining Continuity and Compression:

$$\rho_1(V_1 + c) = \frac{K \rho_1}{K - \Delta P} (V_2 + c) \quad \text{or} \quad (V_1 + c) = \frac{K(V_2 + c)}{K - \Delta P}$$

Momentum

Pressure forces are related to mass flowrate and velocity changes as per Newton's second law:

$$P_1 A + \rho_1 Q_1 V_1 = P_2 A + \rho_2 Q_2 V_2$$

This applies in the reference frame of the wave front, ie Q is the rate at which fluid is delivered to or from the front:

$$Q_1 = (V_1 + c)A \quad Q_2 = (V_2 + c)A$$

Note that the actual compression is small (K is very large $\sim 2 \times 10^9$), so $\rho_1 \sim \rho_2$.

Note also that $c \gg V_1, V_2$, so $(V_1 + c) \sim (V_2 + c) \sim c$

We thus have $\Delta P = \rho[(V_1 + c)V_1 - (V_2 + c)V_2]$

$$\sim \rho[cV_1 - cV_2]$$

$$= \rho.c\Delta V$$

This is a very significant result: For a sudden velocity change from V_1 to zero, there is a step pressure change equal to:

$$\Delta P = \rho.cV_1$$

c is the speed of sound in the fluid medium: We can work this out from continuity, compression and momentum. We now have:

$$\frac{K(V_2 + c)}{K - \rho.c\Delta V} = (V_1 + c)$$

$$\text{ie} \quad KV_2 + Kc = KV_1 + Kc - \rho.c\Delta V.V_1 - \rho.c^2\Delta V$$

$$\text{Or} \quad \rho\Delta Vc^2 + \rho\Delta V.V_1c - K\Delta V = 0 \quad : \quad \text{A quadratic equation}$$

The middle term is small by comparison to the others (c^2 and K are very large), so we have a simplifying relation for c :

$$\rho\Delta Vc^2 = K\Delta V$$

$$\text{Or} \quad c = \sqrt{\frac{K}{\rho}} \quad \text{Speed of Sound in a Fluid Medium (m/s)}$$

This relation holds to a very good approximation (solve the quadratic equation to prove it!)

For water, $K \sim 2 \times 10^9$, $\rho \sim 1000$, thus $c = 1000\sqrt{2} = 1414 \text{ m/s}$

This means the potential pressure shock of a water hammer is very large. For a ΔV of only 1 m/s:

$$\underline{\Delta P = \rho.cV_1 = 1000(1414) = 1.4 \text{ MPa}}$$

Celerity Modifiers

In most circumstances c is less than the theoretical maximum.

- Air entrainment is very significant: as little as 1% air bubbles by volume can reduce the celerity to 400 m/s, near the speed of sound in pure air.
- Elastic pipe walls (steel, PVC) reduce the celerity and in effect absorb some of the shock:

$$c = \sqrt{\frac{K}{\rho \left[1 + \frac{K.D}{E.t} \right]}} \quad \text{Sound Speed in elastic pipes (derivation beyond scope of fluid mechanics)}$$

D = pipe diameter

E = Pipe elastic modulus

t = Pipe wall thickness

Note that $D/t \sim 100$ (most elastic pipes are thin-walled compared to their diameter)

And since $E \sim 200 \times 10^9$, $K/E \sim 1/100$

$$\text{Hence } c = \sqrt{\frac{K}{2\rho}} \sim 1000 \frac{\sqrt{2}}{\sqrt{2}} = 1000 \text{ m/s} \quad \text{First-guess sound speed in industrial elastic pipes}$$

Even with the velocity modifier the very high pressure-velocity relation still holds:

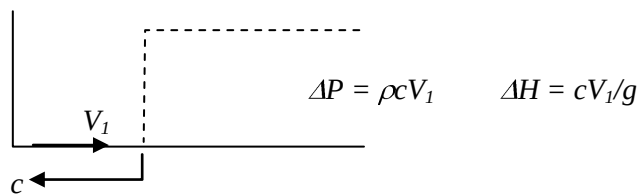
$$\Delta P = \rho.cV_1 = 1000(1000) = 1.0 \text{ MPa per } 1 \text{ m/s } \Delta V$$

A 1 MPa shock is likely to be the largest pressure stress encountered in the system.

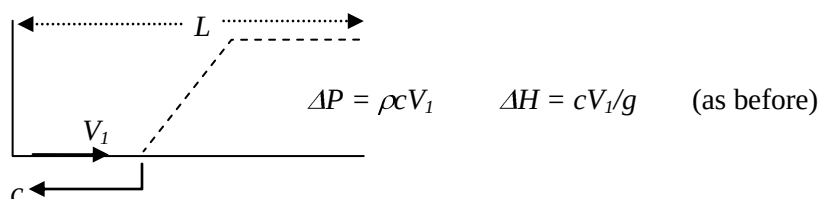
Surge tanks are designed to absorb compression by taking fluid out of the compression zone, however these should not be regarded as normalising the pressure shock, they are an added safety.

Water hammer phenomena must be avoided by ensuring that control operations are carried out slowly – pumps are brought online slowly, valves are slowly opened and closed.

Sudden closure: $t_c \sim 0$ (t_c = closure time)



Fast closure: $t_c < 2L/c$

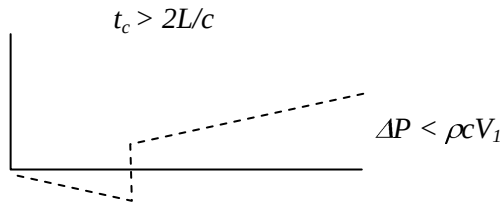


Fast closure (defined as an operation in less than the time it takes the shockwave to propagate and return (return time $t_R = 2L/c$) only has the effect of softening the onset of the pressure shock – but at 1000 m/s it is not softened by much. The maximum pressure of $\Delta P = \rho c V_1$ is still obtained.

In both sudden and fast closure (actually just degrees of the same thing) the negative return shock may be as damaging as the compression shock, or worse, due to cavitation effects (see *Section 5, Turbomachinery*).

Slow Closure:

A closure time greater than the return time ($t_c > 2L/c$) has the effect of mixing the compression shock wave with the return wave while the compression is still increasing towards its potential maximum. The maximum pressure is reduced as the return wave partly cancels the compression.



Given a maximum design pressure that is safe for the system, a suitable $t_c > t_R$ can be selected:

Assuming an approximately constant flow deceleration during the closure, from V_1 to zero in the interval t_c (as yet unknown), we have:

$$\frac{dV}{dt} = \frac{V_1}{t_c} \quad \text{Average or assumed constant deceleration during } t_c.$$

The rate of pressure change in response to this deceleration can be derived from the pressure velocity law:

$$\frac{dP}{dt} = \rho \cdot c \frac{dV}{dt} = \rho \cdot c \frac{V_1}{t_c} \quad (\text{after differentiating } \rho c V_1 \text{ by time})$$

The pressure change rate can be integrated to give a maximum pressure at $t_R = 2L/c$:

$$P_{max} = \frac{dP}{dt} t_R = \rho \cdot c \frac{V_1}{t_c} t_R = \rho \cdot c \frac{V_1}{t_c} \cdot 2 \frac{L}{c} = 2\rho \frac{V_1}{t_c} L$$

The pressure then declines through successive counter-shocks. The minimum tension pressure can be conservatively assumed to equal $P_{min} = -P_{max}/2$ as the return wave has only one pipe length, not two, to develop fully.

P_{max} is thus independent of celerity and can be controlled by selection of a sufficiently large t_c . Given a design (system maximum) pressure P_{des} and safety factor $S \sim 1.5$,

$$P_{max} = 2\rho \frac{V_1}{t_c} L = \frac{P_{des}}{S}$$

Thus a closure time is found:

$$t_c = \frac{2S\rho \cdot V_1 L}{P_{des}} = \frac{3\rho \cdot V_1 L}{P_{des}} \quad \text{Closure time for safe design system pressure}$$

Examples: For $V_I = 1$ m/s, $L = 50$ m and $P_{des} = 100$ kPa,
 $t_c = 3(1000)50/100,000 = \underline{1.5 \text{ seconds}}$

For $V_I = 2$ m/s, $L = 1000$ m and $P_{des} = 100$ kPa,
 $t_c = 3(1000)2(1000)/100,000 = \underline{60 \text{ seconds}}$

NOTE: THE CLOSURE TIME IS A CONSERVATIVE APPROXIMATE METHOD. A MORE PROFESSIONAL APPROACH INVOLVES SEPARATION OF THE UNDERLYING HARMONIC DIFFERENTIAL EQUATION INTO TWO ORDINARY DE'S USING THE METHOD OF CHARACTERISTICS. SEE STREETER AND WYLIE (1986) FOR EXAMPLE