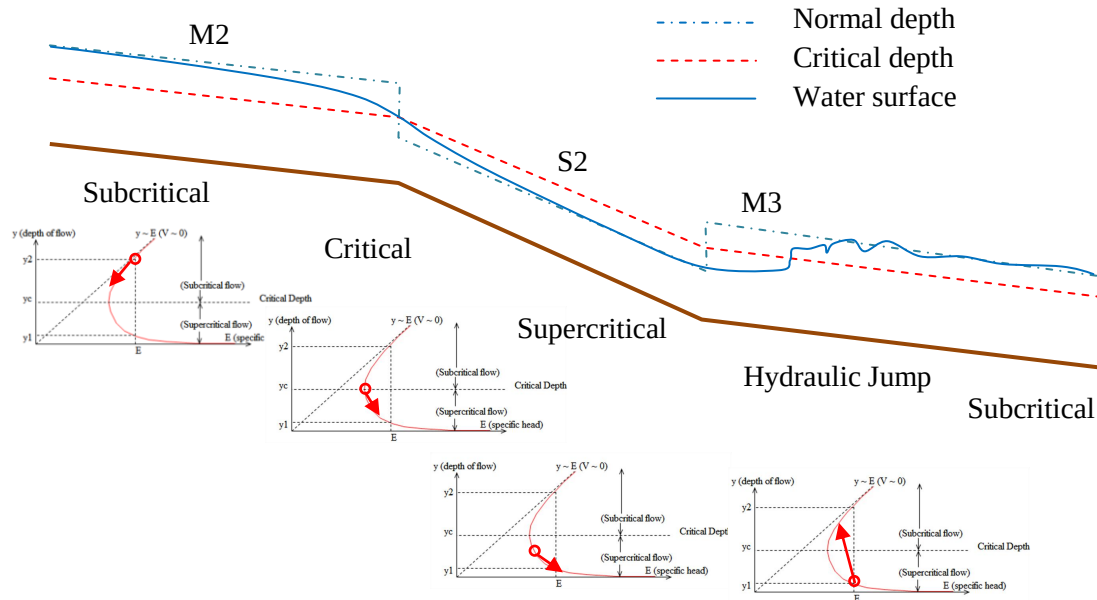


Lecture 27: Non-Uniform Open Channel Flow

Gradually and Rapidly Varying Surface Profiles

An understanding of specific head and its effect on normal and critical depth provides useful insights into a range of open channel flow types. A typical set of transitions is shown below:



Water surface profiles have been classified for the range of typical behaviours (eg Giles p184). The M2 and S2 types shown here are mild and steep drawdowns, which are fairly common in practice, while the M3 is an unstable supercritical flow with shallow bed slope, rapidly losing energy until a hydraulic jump is initiated by some roughness element. This also is fairly common in practice.

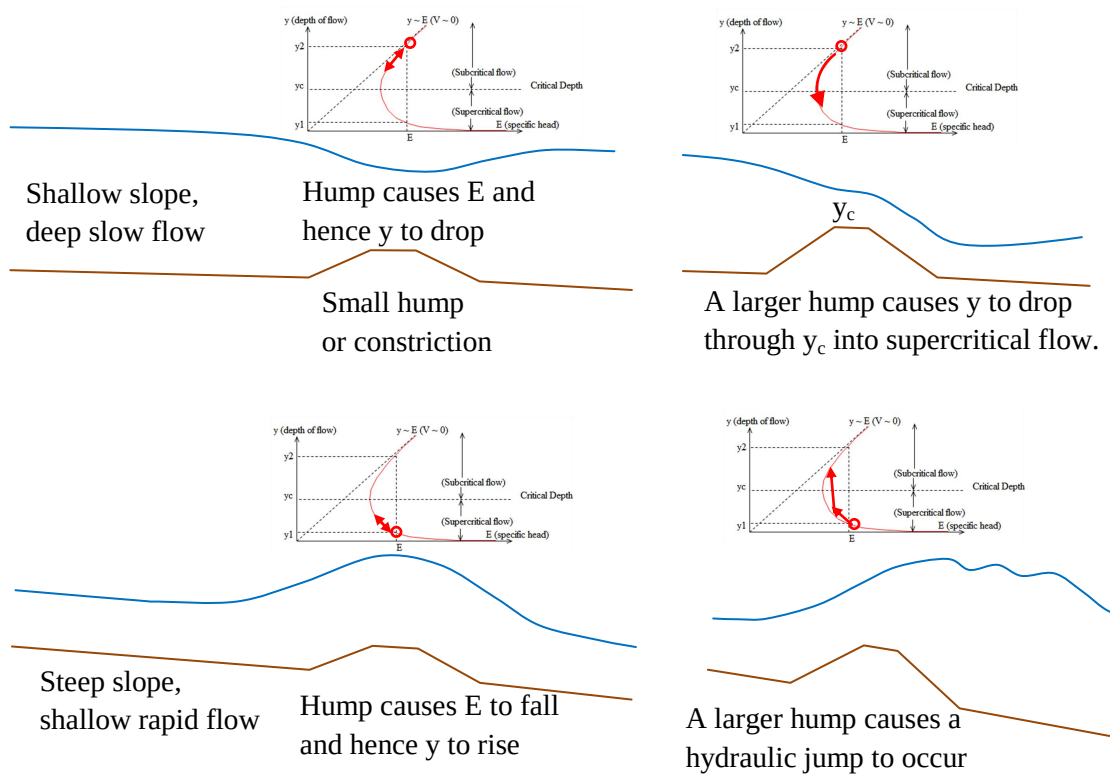
Sudden changes in slope from mild to steep produce control points where the flow depth is critical. These are often used as the start point for water surface profile calculations, since the critical depth is a known depth that can then be projected and recalculated upstream and downstream.

The most common transitions arise due to changes in the capacity of a channel, due to narrowing or adverse bed slope (humps): Note that in the most common (subcritical) flow profiles, there is an unexpected effect in which humps and narrow sections cause the flow depth to *decrease*, while wide sections and troughs cause the flow depth to *increase*. Anecdotal stories are told in which inexperienced engineers designed their channels to produce a lower flow level by excavating a deeper bed, and were surprised to cause floods when they had expected to mitigate them. The specific head diagrams (overleaf) explain this effect.

A hump or narrowing of channel bed will cause a subcritical flow level to decrease (drop), while a pit or widening will cause the subcritical flow level to increase (rise).

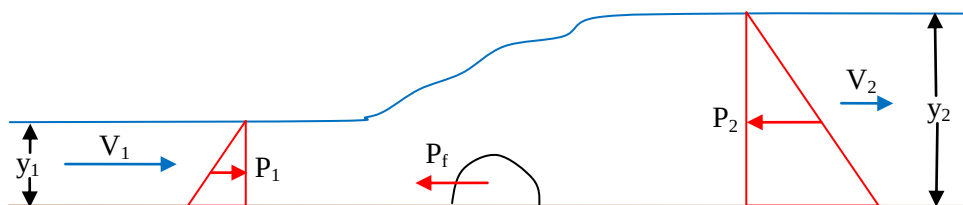
Gas Flow Analogy

Interestingly the behaviour of open channels in this regard is almost exactly analogous to the behaviour of compressible flows, where the depth represents the pressure of the flow. Thus gas conduit contractions at subsonic velocities cause a drop in pressure, while enlargements cause an increase in pressure. At supersonic velocities, the reverse occurs.



Most water surface profiles are smooth and gradually varying, requiring only energy loss methods to calculate the gradual variation in depth; a hydraulic jump is an exception, and requires an estimate of momentum as well as of friction loss:

Hydraulic Jumps



P_f is the force per unit width exerted by an obstruction on the flow, triggering a hydraulic jump. A jump may occur spontaneously, in which case $P_f = 0$. Jumps may be strong, with significant local energy dissipation, or weak, with shallow undulating waves that are undesirable in practice due to the damage such waves can cause as they propagate.

Balancing hydrostatic pressures with the forces due to the obstruction and changes in momentum, we have, per unit width of a rectangular channel of width W :

$$\frac{\sum F_x}{W} = \rho g \frac{y_1}{2} \cdot y_1 - \rho g \frac{y_2}{2} \cdot y_2 - P_f = \frac{\dot{m}}{W} \Delta \mathbf{V} = \frac{\rho Q}{W} (V_2 - V_1)$$

Rearranging,

$$\begin{aligned} \frac{-P_f}{\rho g} &= \frac{q}{g} (V_2 - V_1) + \frac{y_2^2}{2} - \frac{y_1^2}{2} = \frac{q}{g} \left(\frac{q}{y_2} - \frac{q}{y_1} \right) + \frac{y_2^2}{2} - \frac{y_1^2}{2} \\ &= \left(\frac{q^2}{gy_2} + \frac{y_2^2}{2} \right) - \left(\frac{q^2}{gy_1} + \frac{y_1^2}{2} \right) \end{aligned}$$

In non-rectangular sections, the average flow depth and total obstruction force are used:

$$\sum F_x = \rho g \bar{y}_1 A_1 - \rho g \bar{y}_2 A_2 - P_f = \rho Q (V_2 - V_1)$$

or

$$\frac{-P_f}{\rho g} = \frac{Q}{g} (V_2 - V_1) + \bar{y}_2 A_2 - \bar{y}_1 A_1$$

$$= \left(\frac{V_2^2 A_2}{g} + \bar{y}_2 A_2 \right) - \left(\frac{V_1^2 A_1}{g} + \bar{y}_1 A_1 \right) \quad \text{as before.}$$

The centroid depth $\bar{y}$ is the depth to the centroid of area of the channel, ie for a rectangular section this is half the depth of the channel. The centroid depth of a cross section can be calculated from the centroid depth for each segment:

$$\bar{y}A = \sum \bar{y}_i A_i \quad \text{for prismatic or surveyed cross-sections.}$$

eg
$$\bar{y}A = \frac{kh^3}{3} + \frac{h^2 W}{2} \quad \text{for a trapezoidal channel of bed width } W \text{ and } 1:k \text{ side slopes.}$$

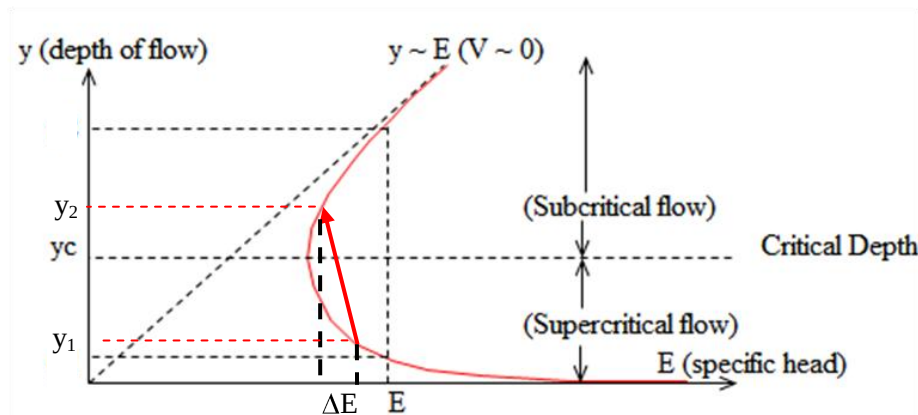
Pipes and culverts flowing partly full are a special case: a jump will fill a pipe, creating additional pressures which must be accounted for.

The quantity $\left(\frac{V^2 A}{g} + \bar{y}A \right)$ is known as the *Momentum Function* M of the flow at that depth.

Thus
$$\frac{-P_f}{\rho g} = M_2 - M_1 = 0 \quad \text{for the spontaneous occurrence of a hydraulic jump.}$$

y_1 and y_2 are known as *conjugate depths*. They are not the same as alternate depths on the specific head diagram, because of the significant headloss that occurs due to turbulence in a jump:

$$\Delta E = \left(y_1 + \frac{V_1^2}{2g} \right) - \left(y_2 + \frac{V_2^2}{2g} \right)$$



For rectangular channels a formula relates the conjugate depths to the Froude number above the jump. This is sometimes used as a simplified method for prismatic or surveyed channels:

$$\frac{y_2}{y_1} = \frac{\sqrt{1 + 8Fr_1^2} - 1}{2} \quad \text{Conjugate depth ratio for rectangular channels only}$$

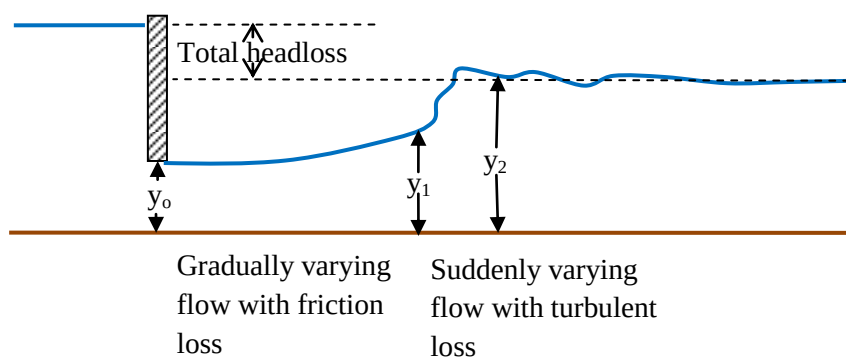
Given a known initial depth y_1 it is thus possible to calculate the upstream and downstream momentum functions and hence the depth y_2 behind a hydraulic jump. The solution is non-linear and is usually carried out by trial and error methods:

1. For a given y_1 , Calculate V_1 , A_1 , centroid depth $\bar{y}_1$ and momentum function
2. Calculate $M_2 = M_1 - P_f/\rho g$
3. For a trial value of y_2 calculate V_2 , A_2 , centroid depth $\bar{y}_2$ and momentum $M_2 = \left(\frac{V^2 A}{g} + \bar{y} A \right)$
4. If M_2 (step 2) is more (less) than M_2 (step 3), increase (decrease) the trial y_2 and repeat from 3
5. Finally calculate total headloss (ΔE) through the jump

The obstruction force P_f is typically estimated using drag coefficients and the projected area of the obstruction:

$$P_f = C_D \rho \frac{V_1^2}{2} A_p$$

In cases where $P_f = 0$ the jump is initiated by energy losses leading up to the jump, such that the flow depth gradually increases until a point is reached where momentum and pressure forces are in balance. Locating this point requires a calculation of friction loss and resulting depth change (see Standard Step Method, below) from a point of known depth, such as a sluice gate:



A short video showing a laboratory-generated hydraulic jump is available at:

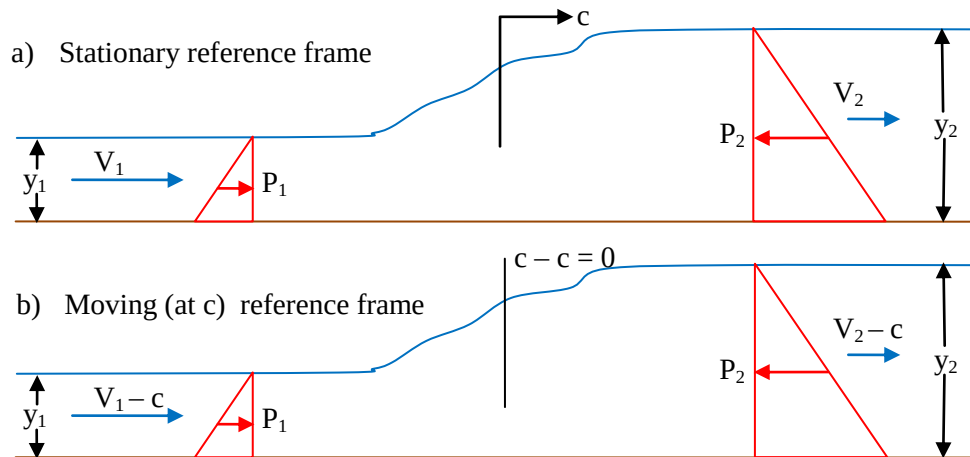
<http://www.youtube.com/watch?v=f7LA0tXNWRg&feature=related>

Surges

Surges (sometimes called bores or bore waves) are travelling hydraulic jumps that may move upstream or downstream depending on the tide and state of flow. These are often triggered by tidal fluctuations in major coastal rivers and can occur in two modes twice daily.

A surge is in fact the more common form of hydraulic jump, where the jump is not localised by an obstruction in the flow. The very precise relationship between y_1 and y_2 means that it is unusual or difficult for a jump to form at a specific location; rather it will hunt upstream or downstream, possibly without success, for a location where the conjugate depths match and produce a solution of $M_1 = M_2$. In practice it is common to force a jump to occur at a specific location by placing an obstruction (energy dissipator) there.

A surge is modelled in a moving reference frame by subtracting the surge speed c (a wave speed conventionally positive in the downstream direction) from the flow velocities V_1 and V_2 to give their velocities relative to the surge. The moving surge behaves exactly as a stationary surge would, within the moving reference frame:



Surge in a) a stationary reference frame and b) a reference frame moving with the wave speed. In b) the relative flow velocities provide an equivalent stationary hydraulic jump where $Fr_1 = (V_1 - c)/c$

Note that a surge is a temporary storage, ie fluid accumulates in the surge, so the continuity equation may appear not to hold: in fact the missing flowrate is actively stored in the surge and does not appear downstream until the surge is dissipated.

A video of the regular tidal bore on the Amazon river, known as the Pororoca, is available at <http://www.youtube.com/watch?v=4ZuZiLuHM1A>

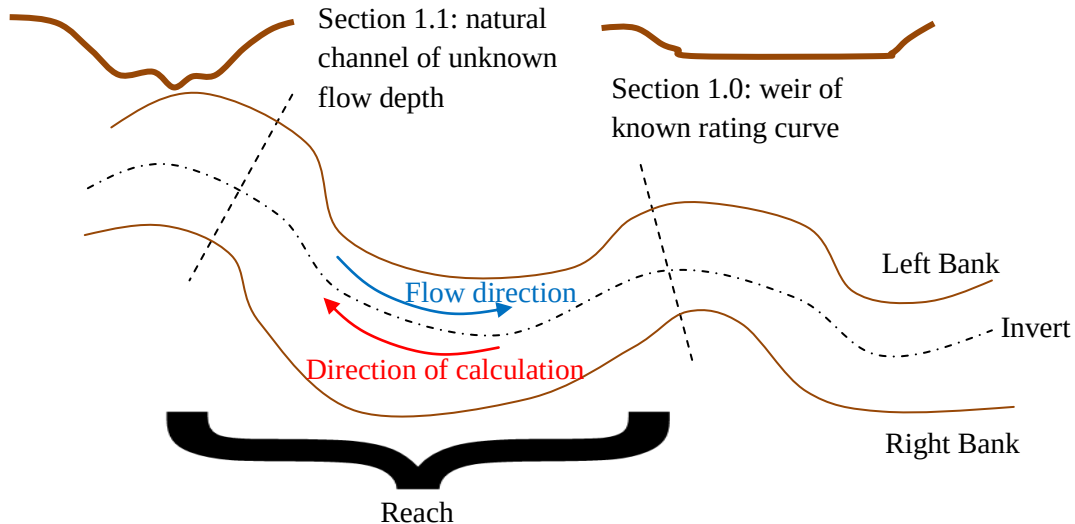


Thought Experiment: The Sink-Jump

Returning to your sink or bathtub laboratory, turn on the tap and observe what happens when the water hits the basin. Keep the plughole open, and try running the tap at a moderate flowrate. You'll see it deflect in all directions in a shallow jet-like flow, with a fairly high velocity out to a radius of a few cm, at which point it suddenly transitions into a deeper flow that lacks the jet characteristics and forms the natural depth of flow in the sink at that flowrate. This is the simplest kind of hydraulic jump. Turn up the flowrate and note how the jump changes with flowrate and water level. At what point does the jump drown, and how does this compare to the outflow transition from weir to orifice?

Gradually Varying Flow Calculations: The Standard Step Method

The *Standard Step Method* is an example of an Integral Method (see Section 4) which relates energy changes (a conserved quantity and property of flow systems) to the differences in flow depth at two ends of a control volume. The control volume is the length of channel between two surveyed cross-sections, commonly known as a *reach*:



A reach is described by the length of the main channel, the radius and length of any bends in the reach, and survey data including roughness of the two end cross-sections.

For subcritical flow the calculation proceeds from downstream to upstream, starting from a point of known depth for the flow rate under consideration. Weirs or riffles are common control points as the critical depth is easy to calculate and may in some cases be available as a measured rating curve.

For supercritical flow the calculation proceeds in the other direction, from upstream to downstream. Care must be exercised with potential numerical instabilities in supercritical calculations. A hydraulic jump may also need to be located at the transition from supercritical to subcritical, and the calculation at that point integrated with subcritical calculations further downstream. These more complex procedures are usually handled with software tools such as HEC-RAS rather than by hand calculation.

The standard step method can be derived from the Bernoulli equation:

$$H = \frac{V^2}{2g} + y + z \quad \text{where the kinetic energy correction factor is assumed to be 1.0}$$

Differentiating with respect to L , the distance along the channel:

$$\frac{dH}{dL} = \frac{V}{g} \frac{dV}{dL} + \frac{dy}{dL} + \frac{dz}{dL}$$

A number of terms can be simplified:

$$\frac{dH}{dL} = -S_f, \text{ the friction slope}$$

$$\frac{dz}{dL} = -S_o, \text{ the bed slope}$$

The slope of velocity with L can be reconstructed by partial differentials, using the relations $V = Q/A$ and $y = A/B$:

$$\frac{dV}{dL} = \frac{dV}{dA} \frac{dA}{dy} \frac{dy}{dL} = -\frac{Q}{A^2} B \frac{dy}{dL} = -\frac{V}{A} B \frac{dy}{dL} = -\frac{V}{y} \frac{dy}{dL}$$

B is the surface width and B/A is formulated as $1/y$ where y is the flow depth.

Thus
$$-S_f = -\frac{V^2}{gy} \frac{dy}{dL} + \frac{dy}{dL} - S_o$$

Or
$$\frac{dy}{dL} = \frac{S_o - S_f}{1 - V^2/gy} = \frac{S_o - S_f}{1 - Fr^2} \quad \text{where } Fr \text{ is the Froude number.}$$

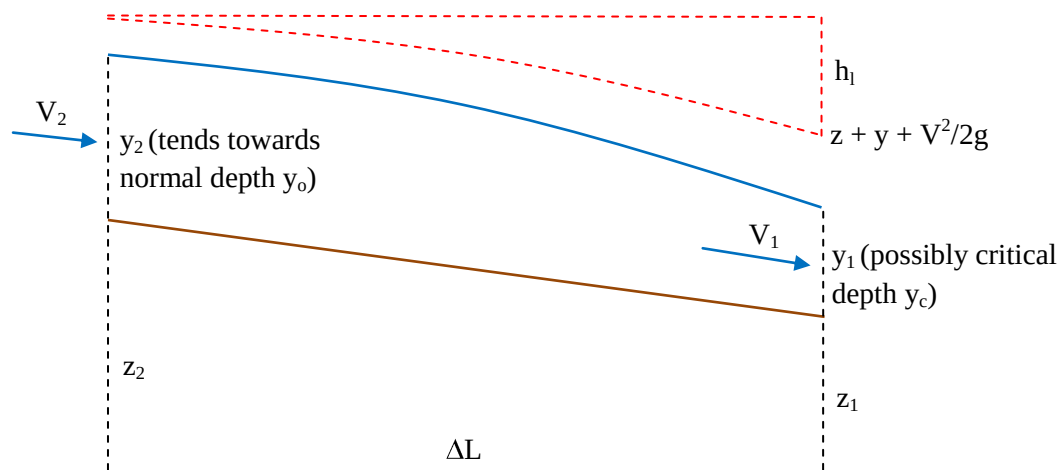
The term dy/dL represents the slope of the water surface relative to the channel bed in the downstream direction. If dy/dL is positive, the depth is increasing downstream. Since the calculation usually proceeds upstream (ie the opposite direction), care must be taken with the sign.

This equation gives a good indication of the range of flow profile types for subcritical ($Fr < 1$) and supercritical ($Fr > 1$) flows. $S_f < S_o$ when the flow depth $y > y_o$ (normal depth), $S_f > S_o$ when $y < y_o$.

Most surveyed channels use reaches of at least 20 – 50 m in length: it is expensive (and usually unnecessary) to survey a channel to a finer resolution than this. Such large steps increase the potential for errors in extrapolating from a slope, so an average slope is calculated at adjacent sections and the water surface extrapolated between them.

This averaging method is sufficiently accurate in practice given the uncertainties in the data and is known as the *standard step method*: the method is usually implemented by guessing a flow depth y_2 at the upstream point and calculating the distance to that point. Revise the estimate of y_2 until the known distance to the surveyed point is calculated to a close tolerance (< 1 m).

Solving Bernoulli's equation with sections numbered from 1 to 2 in the upstream direction:



$$\frac{V_2^2}{2g} + z_2 + y_2 = \frac{V_1^2}{2g} + z_1 + y_1 + h_1$$

But $z_2 - z_1 = S_o \Delta L$ and $h_1 = S_f \Delta L$

so
$$\frac{V_2^2}{2g} + S_o \Delta L + y_2 = \frac{V_1^2}{2g} + y_1 + S_f \Delta L$$

Solving for the length of the reach gives

$$\Delta L = \frac{\left(\frac{V_2^2}{2g} + y_2\right) - \left(\frac{V_1^2}{2g} + y_1\right)}{S_f - S_o} \quad \text{Standard Step Formula}$$

An average friction slope is calculated for use in this equation, as explained above. The friction slope is derived from Manning's equation, which gives, for non-uniform flow:

$$Q = AR^{2/3} \frac{\sqrt{S_f}}{n}$$

So
$$S_f = \left(\frac{nQ}{AR^{2/3}}\right)^2$$

Having assumed a y_2 we can calculate S_{f1} and S_{f2} at cross sections 1 and 2, and compute the average:

$$S_f = \frac{S_{f1} + S_{f2}}{2} = 0.5 \left(\left(\frac{n_1 Q}{A_1 R_1^{2/3}}\right)^2 + \left(\frac{n_2 Q}{A_2 R_2^{2/3}}\right)^2 \right)$$

Again, a hand calculation method can be implemented as a spreadsheet:

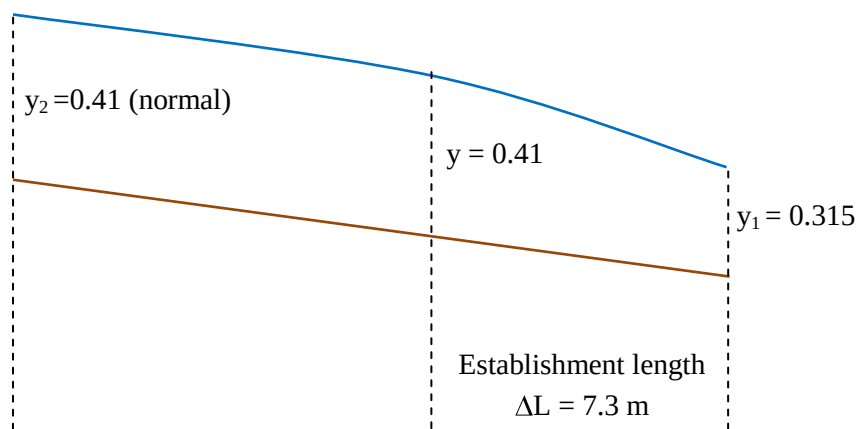
Hand Calculation Method for Standard Step (Subcritical Prismatic Channels)

1. Collate channel survey data including cross-sections, channel lengths, bend radii and roughness. Ensure survey includes a suitable downstream control point (a weir, tidal outfall or stream junction with known rating curve)
2. Identify or calculate the flow depth y_1 for the given discharge at the control point.
3. Calculate A_1 , P_1 , R_1 and S_{f1} for cross section 1 at a depth of y_1 .
4. Assume a flow depth y_2 at the next upstream survey cross section. y_2 may be greater or less than y_1 depending on the type of flow profile, but a good first estimate might be $y_2 = y_1$.
5. Calculate A_2 , P_2 , R_2 and S_{f2} for cross section 2 at a depth of y_2 .
6. Calculate the mean $S_f = (S_{f1} + S_{f2})/2$ and use this to calculate ΔL from the Standard Step Formula
7. If ΔL is significantly different to the actual reach length between sections 1 and 2, revise the estimate of y_2 and repeat from step 5. This may require either an increase or decrease in y_2 .

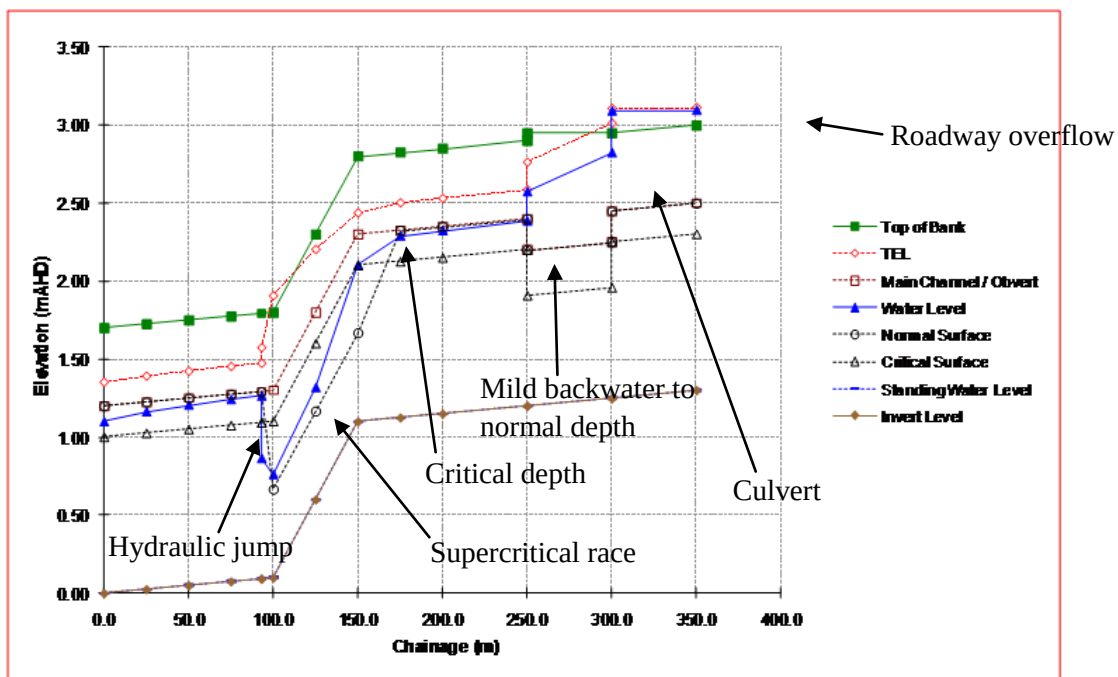
The same trapezoidal channel is used in the spreadsheet below. The control point is assumed to be a outfall flowing at the critical depth previously calculated. As the channel slope is mild ($<$ critical slope) we can expect the flow depth to increase towards normal depth in the upstream direction. The spreadsheet uses two columns to calculate the two cross sections:

Section	Increasing upstream		1 (weir)	2			
Reach Length	L	m	0	20	(control point has zero length)		
Flowrate	Q	m ³ /s	1	1			
Channel Base Width	W	m	1	1			
Bank Slope	k	m/m	4	4			
Roughness	n		0.03	0.03			
Bed Slope	So	m/m	0.005	0.005			
Actual Flow Depth	y	m	0.315	0.41			
Flow Area	A	m ²	0.7119	1.0824			
Wetted Perimeter	P	m	3.597557	4.380947			
Hydraulic Radius	R	m	0.197884	0.24707			
Velocity	V	m/s	1.404692	0.923873			
Friction Slope	Sf	m/m	0.0154	0.004955			
Mean Friction Slope	Sf	m/m	0	0.010178	(mean slope and step length		
Step Length	ΔL	m	0	7.326848	are attributed to section 2)		

In this case we see the water surface goes from critical depth to normal depth within 7.3 m of the control point. Since the channel cross-section is constant, no further adjustment is needed and the flow is established as uniform flow at normal depth:



More complex problems can also be solved by spreadsheets, or analysed using commercial software:



Spreadsheet solution for a design channel including mild, steep and pipe (culvert) sections.

Fixed-Reach Method

An alternative to the standard step method is the fixed-reach method, which computes headlosses over the surveyed reach length and calculates upstream water levels that are consistent with the losses. The same survey and roughness data can be used as for the standard step method.

Again starting from a known downstream control depth the channel profile is used to calculate the flow area and hence the velocity at that downstream point:

$$A_1 = [\text{geometric calculations based on flow depth } y_1 \text{ in the channel cross section at point 1}]$$

$$V_1 = \frac{Q}{A_1}$$

Still using the known depth and the known bed (invert) level, the total head is calculated:

$$H_1 = \frac{V_1^2}{2g} + y_1 + z_1$$

Moving to the next surveyed station upstream at elevation z_2 , as ever we initially assume or guess the flow depth, say $y_2 = y_1$, and use that to compute area, velocity and the new station's head:

$$A_2 = [\text{geometric calculations based on flow depth } y_2]$$

$$V_2 = \frac{Q}{A_2}$$

$$H_2 = \frac{V_2^2}{2g} + y_2 + z_2$$

As with the standard step method, two friction slopes are calculated from Manning's equation and used to create an average friction slope over the whole reach:

$$R_1, R_2 = [\text{geometric calculations based on flow depths } y_1, y_2]$$

$$S_{f1} = \left(\frac{nQ}{A_1 R_1^{2/3}} \right)^2$$

$$S_{f2} = \left(\frac{nQ}{A_2 R_2^{2/3}} \right)^2$$

$$S_f = \frac{S_{f1} + S_{f2}}{2}$$

The friction slope is used to project the total head from downstream to upstream as the reach length L multiplied by the friction slope: this gives a second estimate of the upstream station's total head:

$$H'_2 = H_1 + S_f L$$

The initial estimate of y_2 is revised (again this can be automated iteratively) until the two estimates of total head at station 2 are in agreement.

In practice additional loss-producing features such as bends and significant changes in flow area must be taken into account. These are standard features of open channel design software such as HEC-RAS, which you are far more likely to use than the historical hand-calculation methods.

A tutorial on the use of the HEC-RAS software (download from <http://www.hec.usace.army.mil/software/hec-ras/>) is provided in the Tutorials online.



Thought Experiment: Build your own flume

A flume is an artificial open channel with transparent sides and variable bed geometry, used in the laboratory for simulating or demonstrating different types of open channel flow behaviours. You will get a chance to use the UNE lab flume during your intensive school. As a take-home exercise, could you construct a simple flume in your own kitchen or bathroom? You might consider cheap clear plastic materials such as storage bins, which can be cut up and reassembled with gaffer tape and silicon bathroom sealant. Again, let us know how you go.

Reading	Chadwick and Morfett 1998 (Reading 12) Section 5.8, 5.10 and 5.11 Kay 2008 (Reading 14) Chapter 5
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Review Learning Outcomes for Section 6

Software	See the online Tutorial “HEC-RAS Software”
Tutorial	Open Channel Hydraulics