

Lecture 26: Uniform Open Channel Flow continued

Manning's Channel Geometry

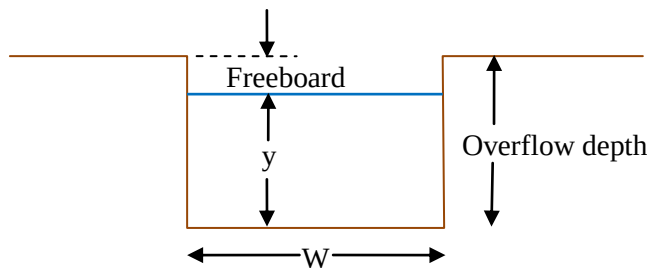
The most common application of Manning's equation is to determine the flow depth for a given discharge Q . The value of Q is a hydrologic rather than hydraulic question, ie it depends on the flow-producing capacity of the catchment or the flow requirements for particular operating conditions. So for a given or known value of Q we usually need to know how deep a channel will flow, whether it will overflow and cause flooding and erosion, etc.

This usually requires an iterative procedure, since the flow area A and wetted perimeter P in Manning's equation are non-linear functions of flow depth. It is seldom possible to rearrange the equation to isolate flow depth y as a single unknown.

The usual procedure is to guess a value of y , use that value to calculate A , P and hence Q , then compare the calculated value to the known Q . If the calculated value is too small, guess a larger value of y and repeat until the solution matches the known value of Q . This can easily be automated in spreadsheets or modern calculators.

For some simple channel geometries it is possible to isolate y as a function of Q . The most common types of channel geometry, with formulae for A , P and y if possible, are set out below:

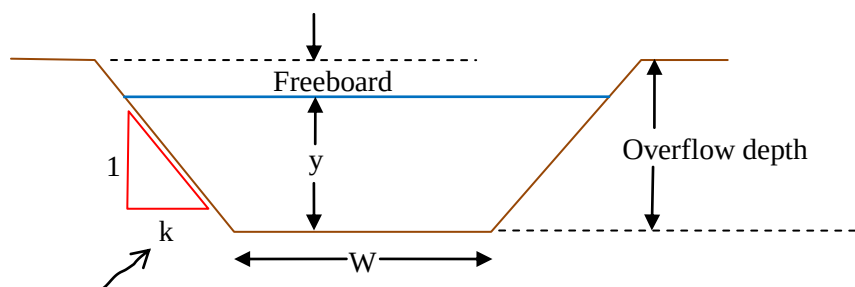
Rectangular channels:



Flow Area $A = Wy$
Wetted Perimeter $P = W + 2y$

Flow Depth	$y \approx \left(\frac{Q.n}{W\sqrt{S_o}} \right)^{3/5}$	Approximate. Exact for wide, shallow floodplains
------------	--	--

Trapezoidal channels:

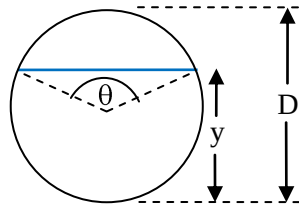


Bank slope: a maximum slope of 1 in 4 ($k \geq 4$) is a safety limit to allow a child to escape.

Flow Area $A = Wy + ky^2$
Wetted Perimeter $P = W + 2y\sqrt{1+k^2}$
Flow Depth: No direct solution

(If the channel has different slopes on the two banks then the two values of k must be included)

Part-full Pipes:



$$\theta = 2 \cos^{-1} \left(\frac{2y}{D} - 1 \right) \quad \text{where } \cos^{-1} \text{ is the inverse cosine, or arcsine}$$

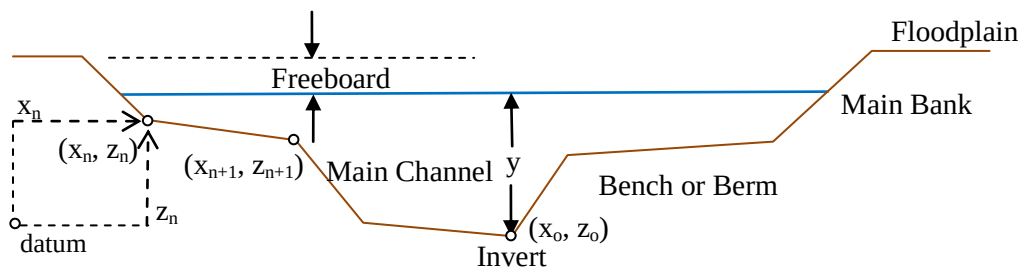
Flow Area $A = \frac{D^2}{4} \left(\pi - \frac{\theta}{2} + \cos \left(\frac{\theta}{2} \right) \sin \left(\frac{\theta}{2} \right) \right)$

Wetted Perimeter $P = D \left(\pi - \frac{\theta}{2} \right)$

Flow Depth: No direct solution

Pipes flowing partly full discharge a maximum flow rate when $y \sim 0.9D$.

Surveyed Channels



Surveyed channels are described by offsets and elevations from a survey datum, that is x - and z -values at survey points where the bank slope changes significantly. The area and wetted perimeter of each station are calculated separately and added together to give the total area and wetted perimeter:

For a given line segment n of the surveyed channel:

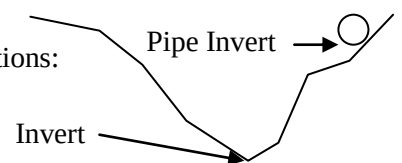
Flow Area: $A_n = (x_{n+1} - x_n) \left(y + z_0 - \left(\frac{z_n + z_{n+1}}{2} \right) \right)$ $A = \sum A_n - C_A$

Wetted Perimeter: $P_n = \sqrt{(x_{n+1} - x_n)^2 + (z_{n+1} - z_n)^2}$ $P = \sum P_n - C_P$

C_A and C_P are corrections at the two ends of the flooded cross section where $z > y + z_0$. These can be calculated by locating the horizontal positions of the water's edge and calculating A_n and P_n to those points. z_0 is the *invert* elevation, the lowest point in the cross section.

Manning's equation is solved by summing the *conveyance* of individual sections:

$$Q = \sum \frac{A_n^{5/3} \sqrt{S_o}}{P_n^{2/3} n_n}$$



The roughness n_n is variable from survey section to section, a realistic additional control that can match the variation in roughness across a natural stream.

Hand Calculation Method for Normal Depth

A 6-step iterative method to solve for normal (steady, uniform) flow depth as a function of flowrate is set out below. Spreadsheets can be set to perform iterative calculations of this type.

1. Estimate the flowrate Q for a given design setting eg recurrence hydrology.
2. Collate the available data on channel cross section, slope and roughness.
3. **Guess a flow depth y .** This will depend on your judgement of the flowrate and channel capacity but any value around 1 metre will do.
4. Calculate area and wetted perimeter at that flow depth (see formulae above for particular channel types).

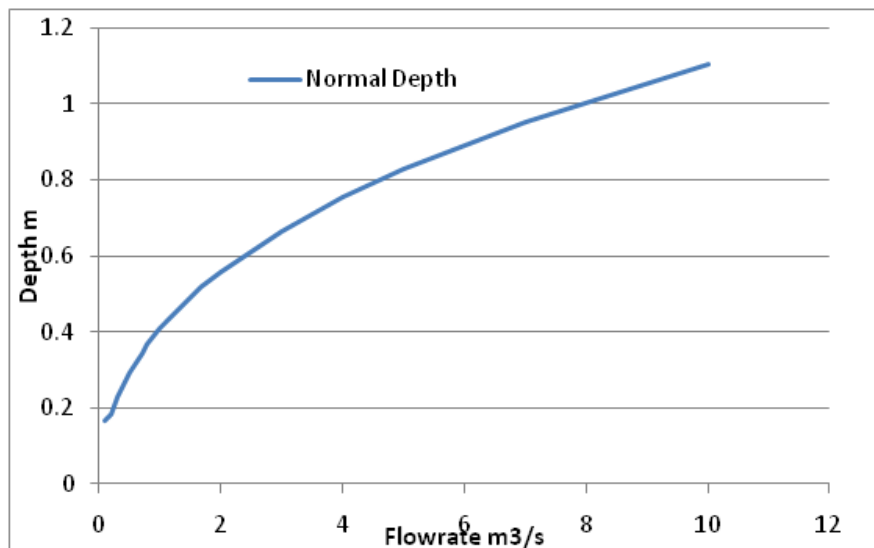
$$5. \text{ Calculate the discharge } Q_o = \frac{A^{5/3}}{P^{2/3}} \frac{\sqrt{S_o}}{n}$$

6. Compare the calculated Q_o to the known flowrate Q . If it is less (or more) than the known flow rate, increase (or decrease) your estimate of flow depth y and repeat from step 4.

A spreadsheet solution for a trapezoidal channel can be reworked from the equations shown:

Flowrate	Q	m ³ /s	1	
Channel Base Width	W	m	1	
Bank Slope	k	m/m	4	
Roughness	n		0.03	
Bed Slope	So	m/m	0.005	
Normal Depth	y_o	m	0.41	adjust to match Q=Q_o
Normal Area	A	m ²	1.0824	=W y _o + k y _o ²
Normal Wetted Perimeter	P	m	4.380947	=W + 2 y _o (1 + k ²) ^{1/2}
Normal Flowrate	Q _o	m ³ /s	1.004534	=A ^{5/3} S _o ^{1/2} / n P ^{2/3}

A solution of $Q_o = Q$ gives the normal depth $y = y_o$.



Calculated rating curve for the trapezoidal channel of this example. This conforms well to measured rating curves. Natural streams can be calibrated to a high degree of accuracy by varying the roughness parameter across the profile of the cross-section.

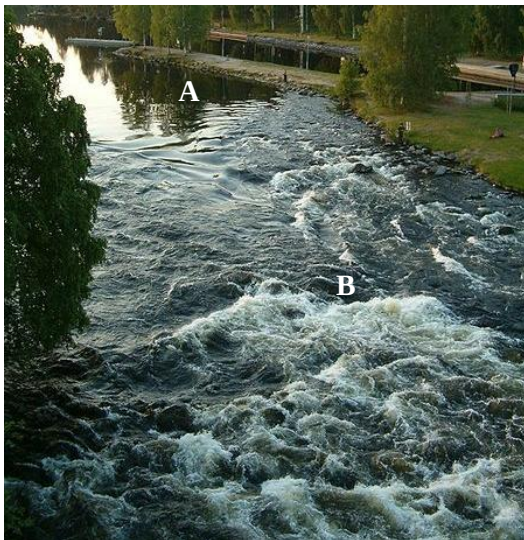
Critical Depth; Subcritical and Supercritical Flow

In addition to the normal depth y_o , which is the depth to which a steady uniform flow will establish, a second important depth is the *critical depth* y_c . This is the depth at which a deep, tranquil flow will transition to a shallow, rapid flow, a phenomenon visible at any weir overflow or riffle head in a natural stream.

Critical depth also marks the depth at which a surface wave speed is just equal to the flow velocity, so *standing waves* form or are swept downstream. Complex patterns of such waves form in the rapid or *supercritical* flow downstream of a critical point.

The main control on flow depth is the bed slope S_o : steep gradients tend to flow fast and shallow, with a normal depth less than critical depth, producing rapid supercritical flow. Roughness is another control, as smoother channels tend to develop supercritical flow. Both tendencies are consistent with Manning's equation.

Supercritical flow is strong, turbulent and highly erosive. People and equipment can be washed away. Generally only hard substrates such as igneous rock, willow roots and well laid reinforced concrete are strong enough to sustain supercritical flow. Erosion results in an eventual lowering of the gradient and a possible drop to subcritical, deep slow flow. Designers usually seek to avoid supercritical flow due to the potential for damage or injury, except for weir walls or other well-contained races.



Subcritical and supercritical flow showing (A) the development of standing waves near critical depth and (B) minor hydraulic jumps located above submerged obstructions.

Surface Wave Speed and Critical Flow

The speed of a surface wave is a complex derivation and is not given here. However it is easily recognisable as a form of the dimensionless Froude number (Section 5):

$$c = \sqrt{gA/B} \quad \text{Surface wave speed,}$$

A/B is the ratio of flow area to surface width, ie the average depth of flow across its width

Comparing this to the flow speed V we obtain the Froude number:

$$Fr = \frac{V}{c} = \frac{V}{\sqrt{gA/B}}$$

Critical flow occurs by definition where $V = c$ or $Fr = 1$.

For supercritical flow $V > c$ and $Fr > 1$.

For subcritical flow $V < c$ and $Fr < 1$.

As a first principle it is possible to calculate the Froude number and check that it is well less than 1, say a maximum of 0.8 for a 20% safety factor.

In other circumstances it is useful or essential to calculate the critical depth y_c . This is done by considering the *specific head* of a flow, defined as the total head relative to the channel bed:

Specific Head

As with confined (pipe) flows, the Total Energy Line or TEL is equal to the Hydraulic Grade Line (HGL or free surface) plus the velocity head $V^2/2g$.

Note that the elevation z and TEL H are not constant, as the flow is driven by a non-zero hydraulic gradient: the bed slope S_o and the friction slope S_f are defined as:

$$\text{Bed Slope: } S_o = \frac{z_1 - z_2}{L}$$

$$\text{Friction Slope: } S_f = \frac{H_1 - H_2}{L} \quad \text{Where } H = z + y + \frac{V^2}{2g}$$

In steady uniform flow, since $d/dx = 0$ we have $V_1 = V_2$ and $y_1 = y_2$ hence $H_1 - H_2 = z_1 - z_2$. As a result we have $S_o = S_f$ ie the friction slope conforms to the bed slope. This indicates that *energy dissipation gradients tend towards the natural slope of the landscape*, an important condition in geomorphology for the natural formation of landscapes through dissipative processes.

$$S_f = S_o \quad (\text{Steady, Uniform Open Channel Flow})$$

The sign convention for slope is that a downhill bed slope is positive, and friction slope is always positive. Typical bed and friction slopes are very small, of the order of 0.1% for deep, slow flow, thus a horizontal measurement of distance (from maps or plan elevations) is a sufficiently accurate estimate of the true length of a channel bed.

In open channel flow the Bernoulli equation simplifies to a measure of total energy *relative to the bed*:

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z - h_f = \text{constant}$$

At the free surface $P = 0$ and $z = z_n + y_n$ where z_n and y_n are the bed elevation and depth at point n .

$$\frac{0}{\rho g} + \frac{V_1^2}{2g} + z_1 + y_1 - h_f = \frac{0}{\rho g} + \frac{V_2^2}{2g} + z_2 + y_2$$

But $h_f = \text{friction headloss} = S_f L = S_o L = z_1 - z_2$ (steady, uniform flow)

$$\text{Hence } \frac{V^2}{2g} + y = \text{constant} = E \quad \text{Specific Head; Steady, Uniform Flow}$$

The *specific head* is independent of the bed elevation and provides an indication of free-surface transitions such as critical depth and hydraulic jumps. These are naturally occurring flow control points where flow depth is determined by flow energy and momentum relative to the bed.

Specific Head is constant only in steady, uniform flow conditions. The variation of specific head from section to section in non-uniform flow is a useful constraint on the shape of the water surface and controls transition points such as weirs and hydraulic jumps.

Specific Head and Critical Depth

We have seen that the specific head, or total head relative to the channel bed, is constant for steady, uniform flow.

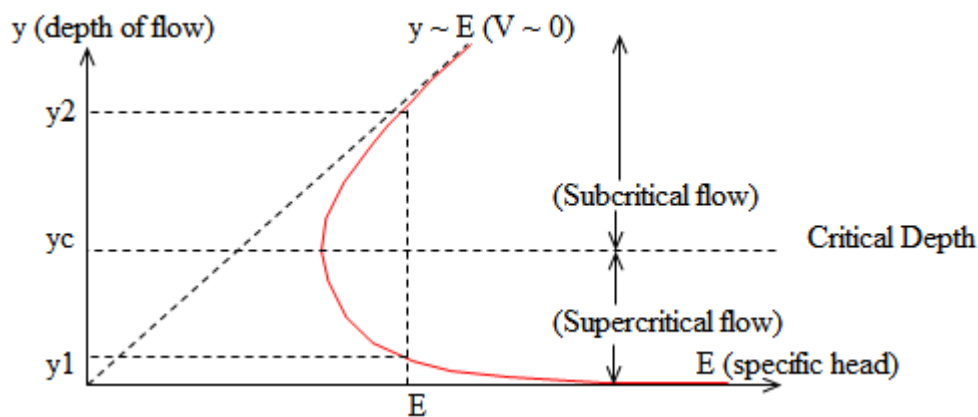
$$E = y + \frac{V^2}{2g} = \text{constant}$$

Steady, uniform flow.

In other circumstances (unsteady or non-uniform flow) the specific head increases or decreases as the flow moves through transitions such as changes in channel width or slope. E is purely a function of discharge and flow depth, as $V = Q/A$ and A is a function of depth:

$$E = y + \frac{Q^2}{2gA^2}$$

Variable for non-uniform flow



This curve arises from the dependence of specific head on flowrate and area, where area is a non-linear function of flow depth (as given above for common channel types).

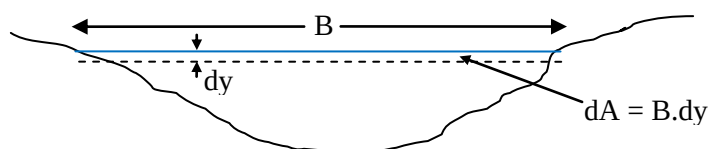
The hyperbolic curve indicates that for any given specific head there are two possible flow depths, y_1 and y_2 , known as *alternate* depths: one in the subcritical regime and one in the supercritical.

A hydraulic jump occurs as a sudden, turbulent transition from supercritical to subcritical, approximately at the alternate depths. A weir flow occurs at the critical depth, where the minimum specific head allows a smooth, even laminar transition from subcritical to supercritical.

We can differentiate to find the rate at which E changes relative to changes in flow depth:

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3} \frac{dA}{dy}$$

But $\frac{dA}{dy} = B$, the width of the surface:



This applies to channels of any cross-section. Thus

$$\frac{dE}{dy} = 1 - \frac{Q^2 B}{gA^3}$$

The critical depth occurs where E is at a minimum: $\frac{dE}{dy} = 0 = 1 - \frac{Q^2 B_c}{gA_c^3}$

or $\frac{Q^2 B_c}{gA_c^3} = 1$

thus $Q_c = \sqrt{\frac{gA_c^3}{B}}$ Critical Depth-Discharge Relation

also $\frac{Q}{A_c} = \bar{V} = \sqrt{\frac{gA_c}{B_c}} = \sqrt{gy}$ ie the mean velocity equates to wave speed at the mean flow depth.

This occurs at only one possible depth, the critical depth, which can be found by a trial and error procedure similar to that for normal depth:

Hand Calculation Method for Critical Depth

An iterative method for critical depth as a function of flowrate is the same as for normal depth.

1. Estimate the flowrate Q for a given design setting.
2. Collate the available data on channel cross section.
3. **Guess a critical depth y_c .** Again a value around 1 will do.
4. Calculate area A and surface width B at that flow depth.
5. Calculate a discharge $Q_c = \sqrt{\frac{gA^3}{B}}$
6. Compare the calculated Q_c to the known flowrate Q. If it is less (or more) than the known flow rate, increase (or decrease) the estimate of critical depth y_c and repeat from step 4.

Formulae for B are easy to obtain for the common channel geometries above:

Rectangular channels: $B = W$

Trapezoidal channels: $B = W + 2ky$

Open Pipes: $B = D \sin(\theta/2)$

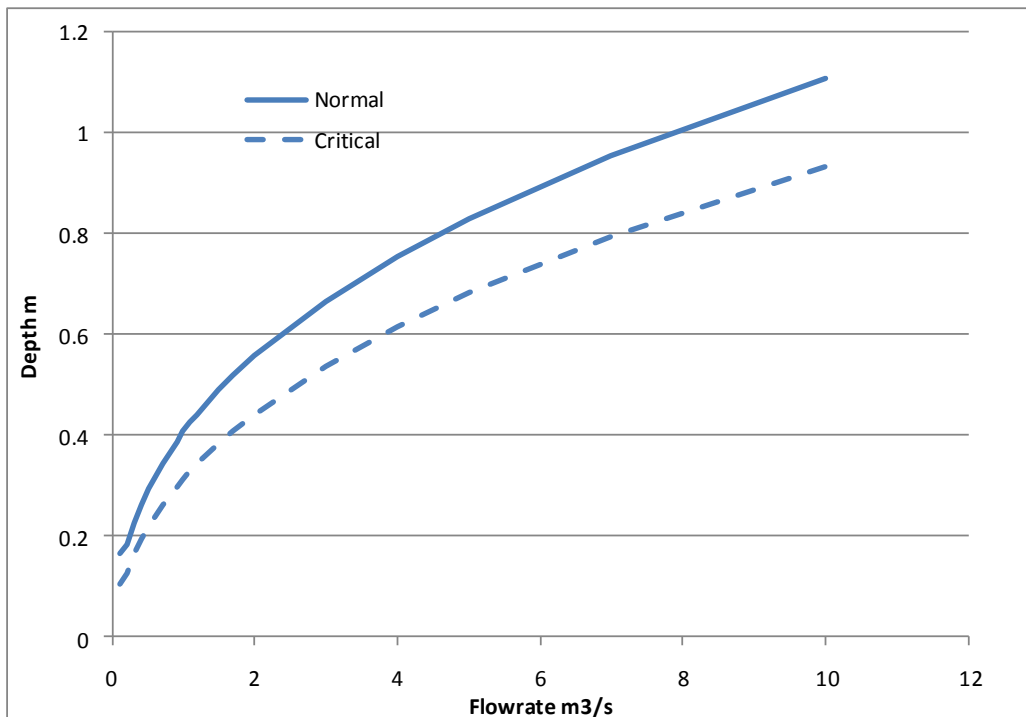
Surveyed Channels: $B =$ the width between water edges as determined geometrically

The spreadsheet solution for a trapezoidal channel can be extended to include critical depth:

Flowrate	Q	m ³ /s	1	
Channel Base Width	W	m	1	
Bank Slope	k	m/m	4	
Roughness	n		0.03	
Bed Slope	So	m/m	0.005	
Normal Flow Depth	y_o	m	0.41	adjust to match Q=Q_o
Normal Area	A	m ²	1.0824	=W y _o + k y _o ²
Normal Wetted Perim	P	m	4.380947	=W + 2 y _o (1 + k ²) ^{1/2}
Normal Flowrate	Q _o	m ³ /s	1.004534	=A ^{5/3} S _o ^{1/2} / n P ^{2/3}
Critical Depth	y_c	m	0.315	adjust to match Q=Q_c
Critical Area	A	m ²	0.7119	=W y _c + k y _c ²
Critical Surface Width	B	m	3.52	=W + 2 k y _c
Critical Flowrate	Q _c	m ³ /s	1.002236	=(g A ³ / B) ^{1/2}

A solution of $Q_c = Q$ gives the critical depth $y = y_c$.

In the above example we see the normal depth is significantly greater than the critical depth. This indicates that the flow will tend to settle into subcritical rather than supercritical flow, which is the ideal design target.



Alternatively a critical bed slope can be calculated, and the actual bed slope designed to be less (shallower) than the critical value:

The critical slope is the slope at which normal depth = critical depth:

$$Q = \sqrt{\frac{g A_c^3}{B_c}} = \frac{A_c R_c^{2/3} \sqrt{S_c}}{n}$$

$$\text{Thus } S_c = \left(\frac{nQ}{A_c R_c^{2/3}} \right)^2 \quad \text{where } A_c \text{ and } R_c \text{ are calculated for } y = y_c.$$

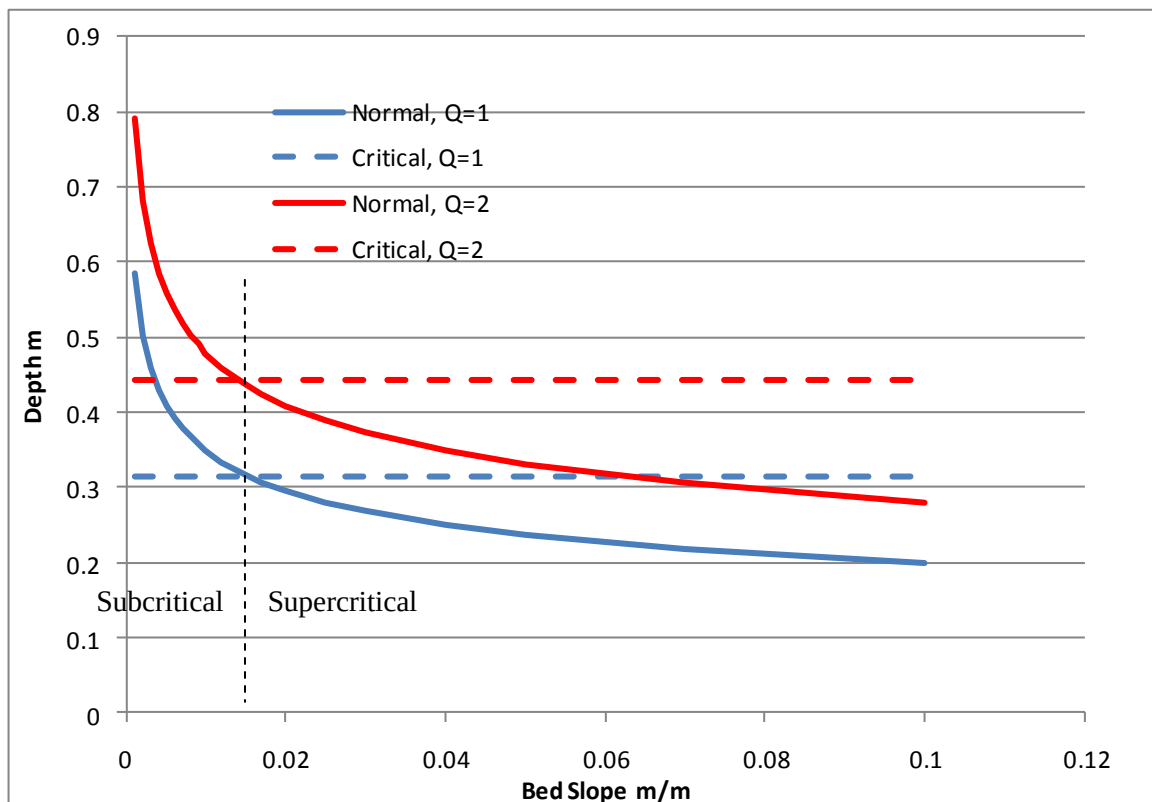
Critical flow is unstable and creates long-cycle waves that can travel upstream and downstream, damaging the banks. It is not a good idea to design a channel with slope equal to the critical slope. Most designs aim for slope at least 20% less than critical slope.

A simple calculation of critical depth is possible in rectangular channels only:

$$y_c = \sqrt[3]{\frac{Q^2}{gW^2}} \quad \text{Critical depth (rectangular channels of bed width W)}$$

This formula is not accurate for prismatic or natural channels.

Note: It is a common error to think that critical depth varies with bed slope. Critical depth is a function of cross-section and flowrate only and is independent of slope. The critical slope is the slope at which normal depth = critical depth. Both normal and critical depth increase with flowrate:



Normal and flow depth as a function of flowrate and slope, showing critical slope (vertical dashed line, around 0.016 m/m in this trapezoidal model). Note that critical slope is nearly the same for all flowrates in a fixed channel profile; in fact critical slope decreases slightly with increasing flowrate.

Reading

Chadwick and Morfett 1998 (Reading 12) Section 5.7 and 5.9