

Lecture 25: Uniform Open Channel Flow

Open channel theory deals with the behaviour of fluids flowing in unconfined conduits, ie where there is a free surface at atmospheric pressure dominated by gravity effects. Flow in open channels is initiated and sustained by a hydraulic gradient, which combines the slope of the channel bed with the slope of the free surface.

Accurate solution of open channel systems is complex and depends on the availability of data on the cross-section shape and roughness of the channel bed as well as the hydrology of the system or landscape over a wide range of conditions. Open channel problems are typically solved using advanced hydraulic software in conjunction with hydrologic modelling of flowrates under different climate or operational assumptions.

This section provides a brief introduction to the basic concepts of open channel flow, and includes two methodologies most commonly used in open channel modelling:

- Steady, uniform flow using the *Manning Equation*.
- Steady, gradually varying non-uniform flow using the *Standard Step Method*.

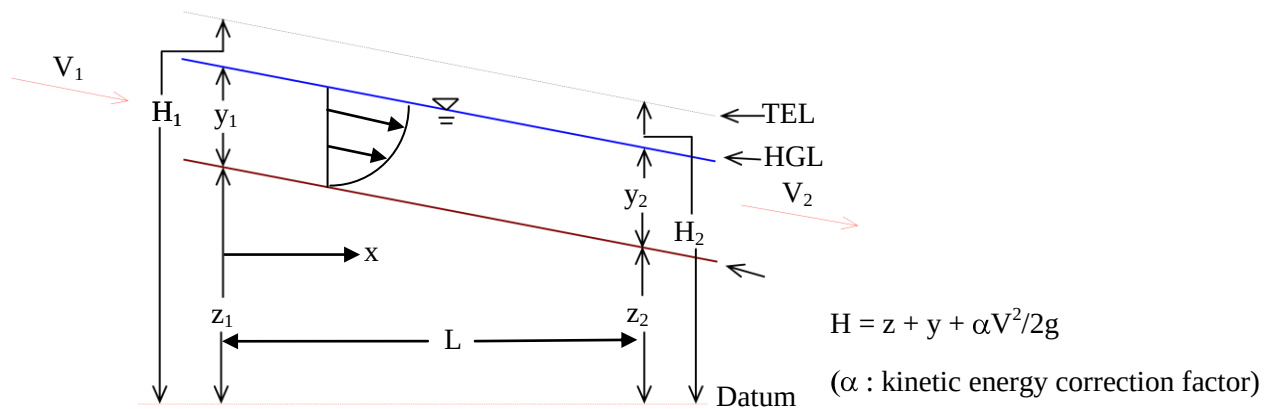
The section also includes an introduction to steady, rapidly varying flow in hydraulic jumps and surges. A supplementary lecture on drainage design is also provided.

World standard open channel modelling software, now known as HEC-RAS (Hydraulic Engineering Corps River Analysis System), developed by the US Army Corps of Engineering is available as a free download (<http://www.hec.usace.army.mil/software/hecras/>). A tutorial on the use of this software is provided online.

Modelling of unsteady non-uniform open channel flow, using the Saint-Venant equations for example, is relatively advanced and is not covered in this course.

Uniform Steady Flow in Open Channels

In uniform steady flow, an open channel develops a *constant depth* relative to the channel bed: The diagram shows flow depth (y), Total Head (H) and channel bed elevation (z) for a uniform, steady open channel flow.



In terms of rates and derivatives, we have:

$$\begin{aligned} \frac{d}{dx} &= 0 && \text{Uniform flow condition: } V_1 = V_2, y_1 = y_2 \\ \frac{d}{dt} &= 0 && \text{Steady flow condition: } V(t) = V(t+\Delta t) \\ \frac{d}{dy} &\neq 0 && \text{Variable vertical flow profile: } V(y) \neq V(y+\Delta y) \end{aligned}$$

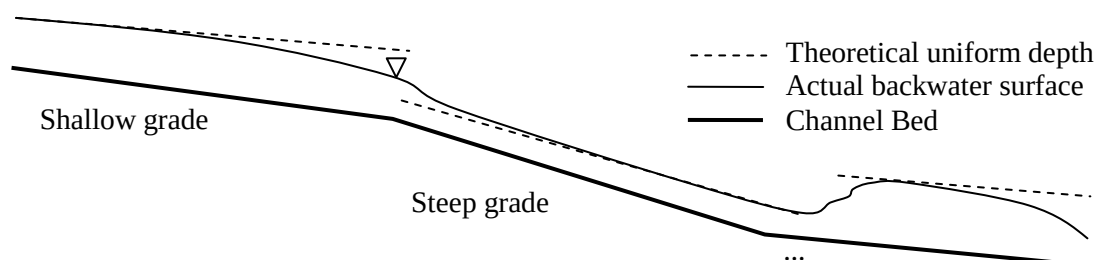
The third condition indicates that flow conditions such as pressure and velocity vary with depth in open channels, regardless of the uniformity of flow in the horizontal direction. Vertical velocity profiles depend on the type of flow regime (laminar or turbulent in various ranges) and may be parabolic, hyperbolic, logarithmic, linear or adverse (backwash eddies) as described in Section 5.

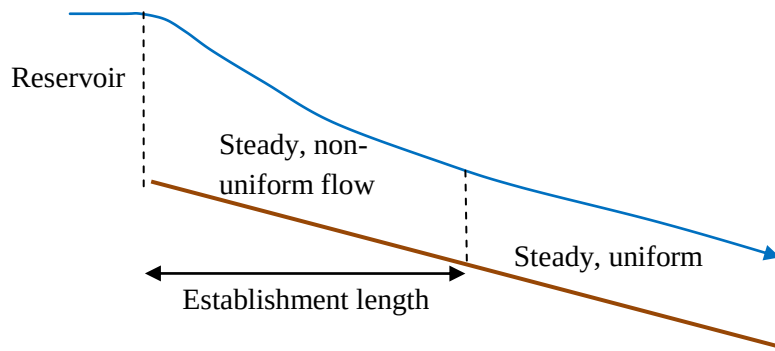
Steady, uniform open channel flow is dependent on two conditions:

1. A channel of constant uniform cross-section throughout the channel length.
2. An establishment length in which the friction slope tends towards the bed slope.

The first condition indicates that, strictly speaking, only artificial channels with uniform constructed cross-sections can develop steady uniform flow. In practice the steady uniform assumption applies adequately to long natural channels if cross-section changes are not too great.

The second condition is more important: flow near to control points such as weirs or sluice gates is not uniform but undergoes a gradual transition to uniform as the flow progresses. In practice the uniform assumption applies only to long channels away from controls, and the water surface goes through “backwater curves” from one point of uniform flow to the next.





There are no specific guidelines for how long or uniform a channel must be before the steady, uniform assumption is acceptable. In practice a first estimate of flow conditions is often based on the steady uniform assumption regardless of profile uniformity and establishment length.

Secondary Flows

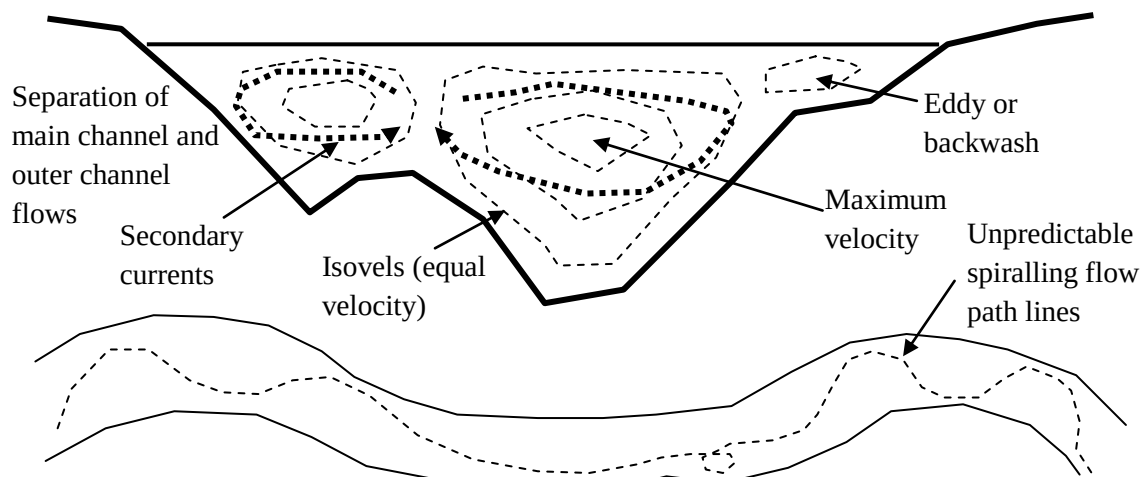
In practice almost all open channel flows are non-uniform, and all are subject to internal or *secondary flows* such as eddies and vortices. It is possible for a designer to work with an assumption of uniform flow to simplify calculations, but it would be unwise to ignore the effects of secondary flows.

In real channels the flow velocity varies across the width and depth of the channel. Usually the maximum velocity occurs at the centre of the channel and at a depth about 1/4 of the depth of the channel, but the maximum point can actually appear almost anywhere depending on the changing symmetry of the cross-section and the dynamic interactions of adjacent streamlines.

The flow velocity near the bank can be retrograde (eddies) due to adverse pressure gradients and flow separation points caused by obstructions in the shallow flow near the bank.

The main flow can be circulatory, again due to pressure gradients arising from velocity gradients. These can set up cross-currents that cause the flow to roll over on itself, so that a current does not simply flow downstream, it turns over in a loose spiral motion.

This spiralling motion becomes very pronounced as a flow moves through bends: A centrifugal force in conjunction with pressure gradients creates a very strong rolling action, downwards on the outside of the bend and upwards on the inside, creating cross-currents that are strong enough to erode the outer bank and deposit sediment on the inner bank. The long-term effect is to create a meander, where the bend travels laterally across the landscape, eroding and depositing a new channel bed as it goes. In extreme cases on level floodplains the bend can pinch itself off, creating an ox-bow lake.



The Chezy and Manning Equations

Open channel flow profiles are extremely complex and depend on conditions upstream and/or downstream of a given measurement point.

A number of models of the *average* velocity in open channel flow have been developed. The previous section on real fluids provides some insights into this, however in practice the variety of conditions that are encountered means that the choice of model for a particular set of conditions depends on data that may be practically unavailable or very expensive to obtain.

An equation developed by Antoine Chezy (1776) is the first and most durable historical attempt at a single-equation model of average velocity in open channels:

$$V = C\sqrt{RS_o} \quad \text{where } R \text{ is hydraulic radius and } S_o \text{ is bed slope.} \quad \text{Chezy Equation}$$

$$R = \frac{\text{Flow Area } A}{\text{Wetted Perimeter } P} \sim \text{depth } y \text{ in wide rectangular channels}$$

The Chezy coefficient C is dependent on Reynolds number and bed roughness. Estimating the coefficient for significant managed rivers and canals of his period was one of Chezy's major contributions.

Subsequent models use the Chezy equation with a range of formal estimates of C :

$$C = \sqrt{\frac{8g}{f}} \quad \text{Real Fluid Shear Stress Model: See Section 5}$$

$$C = \frac{R^{1/6}}{n} \quad \text{Manning (1890)}$$

$$C = \frac{87}{1 + \frac{m}{\sqrt{R}}} \quad \text{Bazin (1897)}$$

In the last two equations m and n are empirical estimates of roughness, while f is a friction factor derived from shear stress theory and experiment as per Section 5.

Of these models the *Manning Equation*, first suggested by Phillippe Gauckler in 1867, and further developed by Robert Manning in 1890, has been almost universally adopted (other than in French-speaking countries) as a standard estimate of average velocity in open channel flow: Multiplying C into the Chezy formula we have the modern form of the Manning equation:

$$V = R^{2/3} \frac{\sqrt{S_o}}{n} \quad \text{Manning Equation}$$

And

$$Q = VA = AR^{2/3} \frac{\sqrt{S_o}}{n} = \frac{A^{5/3}}{P^{2/3}} \frac{\sqrt{S_o}}{n} \quad \text{Manning Discharge Relation}$$

A typical problem is to solve for the depth of flow for a given discharge Q . However, both the area and the wetted perimeter require an estimate of flow depth, y , so the equation is non-linear and must be solved by iteration. A flow depth that solves the Manning's equation is a point at which uniform steady flow establishes a constant depth and is known as *normal depth* y_o .

The following sections examine the components of Manning's equation in more detail.

Manning Roughness Coefficient n

The roughness coefficient n in the Manning equation is purely empirical and has been estimated for a wide range of conditions. Using the Manning equation requires a certain amount of judgment as to the condition of the channel bed, ie the presence of snags and boulders in natural streams, or the ageing and deterioration of surface materials in constructed channels.

Tables of Manning's n coefficients are a common and essential reference for open channel calculations:

Channel Lining	Manning's n
Glass, plastic, metal	0.01
Dressed Timber, Flush Joints	0.011
Sawn Timber, Uneven Joints	0.014
Cement Plaster	0.011
Vitrified Clay	0.012 - 0.014
Cast Iron	0.013
Concrete Pipes or Box Sections	0.012
Concrete (Trowel Finish)	0.012 - 0.015
Concrete (Formed, no finish)	0.013 - 0.018
Concrete (Guniting)	0.016 - 0.02
Bitumen	0.014
Bricks	0.015
Pavers or dressed stone in mortar	0.015 - 0.017
Random stones or rubble in mortar	0.02 - 0.035
Smooth Rock or Rip-Rap	0.025 - 0.03
Earth (clear)	0.018 - 0.025
Earth (With weeds or gravel)	0.025 - 0.035
Cut Rock	0.035 - 0.045
Short Grass	0.03 - 0.035
Long Grass	0.035 - 0.05
Natural Channels	
Clean and Straight	0.025 - 0.03
Winding, Pools and Shoals	0.033 - 0.04
Very Weedy, Winding and Overgrown	0.075 - 0.15
Dense Willows and Instream Vegetation	0.15 - 0.2
Floodplains	
Bare Cropland	0.03 - 0.04
Mature Crop	0.04 - 0.05
Weeds and Brush	0.05 - 0.07
Brush and Trees (Dead Season)	0.05 - 0.08
Heavy Timber	0.10 - 0.16

Pipes are occasionally modelled as flowing partly full, ie as open channels. A pipe condition assessment is often required to estimate flow depth:

Manning's n per Pipe Condition

Pipe Lining	Good	Normal	Poor
Precast with O Ring Joints	0.06	0.15	0.6
Spun Precast with O Ring	0.06	0.15	0.3
Steel Forms	0.3	0.6	1.5
Rough Form	0.6	1.5	3
Asbestos Cement	0.015	0.03	0.1
UPVC with cemented joints	0.03	0.03	0.03
UPVC with socket joints	0.06	0.06	0.06

Cowan (1956²) suggested a calculation of Manning's roughness for natural channels as a composite of bed material, surface irregularities, cross-sectional changes, obstructions, vegetation and meandering:

$$n = (n_0 + n_1 + n_2 + n_3 + n_4) * m_5 \quad \text{Cowan's Method}$$

Bed Material, n_0

Earth	0.02
Rock	0.025
Fine Gravel	0.024
Coarse Gravel	0.028
Cobble	0.03-0.05
Boulder	0.04-0.07

Surface Irregularity, n_1

Smooth	0
Minor	0.005
Moderate	0.01
Severe	0.02

Cross section change, n_2

Gradual	0
Occasionally abrupt	0.005
Frequently abrupt	0.01-0.015

Obstructions, n_3

Negligible	0
Minor	0.01-0.015
Appreciable	0.02-0.03
Severe	0.04-0.06

² Cowan, W.L. 1956. 'Estimating hydraulic roughness coefficients', *Agricultural Engineering*, 37:7 pp 473-475

Vegetation, n_4	
None	0
Seedlings or dense grass	0.005-0.01
Bushy growths	0.01-0.025
Young trees	0.025-0.05
Brushy, dense instream growth	0.05-0.1

Meandering, m_5	
Minor	1
Appreciable	1.15
Severe	1.3

This assessment is particularly useful for before/after comparisons of stream bed clearing or reconstruction works, as it allows a designer to distinguish the effect on channel roughness of different types of works.

Bed Friction

Erosion of a channel bed is caused by shear stress and turbulence. Generally these only become active when the shear stress exceeds the shear resistance of the bed, which is governed by the cohesiveness and density of the bed material. While the cohesive behaviour of bed materials is outside of the scope of this course, the shear stress is determined by an early derivation from real fluid theory (Section 5), namely the friction slope equation:

$$S_f = \frac{\tau_0}{\rho g R}$$

Assuming uniform flow, $S_f = S_o$ the bed slope, so the shear stress on the bed can be calculated by:

$$\tau_0 = \rho g R S_o$$

Uniform bed shear stress (Pa) given hydraulic radius and bed slope

It is now understood that Manning's equation applies equally well to non-uniform flow, as long as the actual (usually unknown) friction slope is used rather than the bed slope:

$$Q = \frac{A^{5/3}}{P^{2/3}} \frac{\sqrt{S_f}}{n}$$

Assuming the state of flow is known at any point, ie the depth and hence area/wetted perimeter, the friction slope can be solved:

$$S_f = \left(\frac{n Q P^{2/3}}{A^{5/3}} \right)^2$$

Reading

Chadwick and Morfett 1998 (Reading 12) Section 5.1 to 5.6
Kay 2008 (Reading 14) Chapter 5