

Supplementary Lecture 4: Applications: Aerodynamics and Propulsion

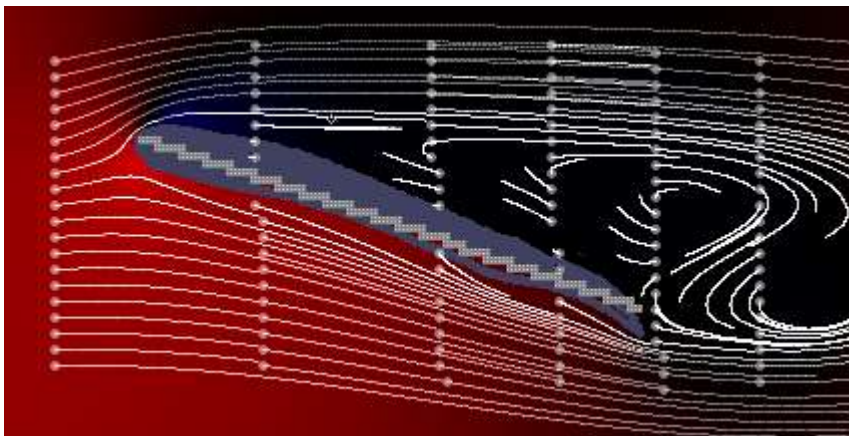
The aerodynamics of aircraft, propellers, jet engines and ultimately spacecraft are among the greatest achievements of fluid mechanics in the 20th century. No first course would be complete without a detailed exploration of all aspects of the subject. The application of the subject however is highly specialist and is routinely handled by advanced computer models. The following lecture is for your interest only.

Stall

A stall is a reduction in lift coefficient that occurs as the angle of attack of a wing or airfoil is increased beyond about 15°, depending on the airspeed and flight attitude (level, banking or pulling out of a dive). 15° is a typical value, but varies with the airfoil design and Reynolds number.

A stall does not mean that the engine has stopped working – but the wing certainly has.

Stall is triggered by excess flow separation that spreads across the upper surface of the wing, usually making a sudden jump from normal tip separation to the more extensive stall separation. This is accompanied by a movement of the low pressure region from above to substantially behind the wing, causing a large increase in drag as well as reducing lift.



A stalled airfoil – note substantial backflow due to separation along the top surface

Most fixed wing aircraft give early signs of stall, as the turbulent separated flow causes buffeting of the tail. Stall warning indicators, triggered by the rising pressure on the upper wing surface, are installed on almost all modern aircraft.

Any initial yaw of the aircraft as it enters stall may result in spin which makes recovery from the stall more difficult. Some aircraft, in particular military fighter jets, are designed with a high natural instability to promote manoeuvrability. High speed stalling is a standard manoeuvre, coupled with thrust vectoring for lift. Performed with sufficient altitude for recovery, there is little risk of a crash.

A *deep stall* (or super stall) is a more dangerous situation that particularly effects T-tail aircraft (such as some liner jets). In a normal stall the nose of the aircraft will drop, reducing the angle of attack and so promoting recovery from the stall. In a deep stall the turbulent wake of the stalled wing blankets the tail stabiliser surface, so the nose rises to the point where no further control is possible. Early T-jet prototypes were unfortunately prone to this type of stall, with the loss of a number of test crews in the US and USSR.

Hydrofoils, Hulls and Wave Drag

Hydrofoils operate in far higher density than aerofoils, so they can be quite small while still generating very large lifts at moderate velocities ($V \ll$ aircraft). Skin friction is comparably higher due to the higher viscosity of water.

With hydrofoils and conventional hulls an additional wave drag due to gravity or *fluid body forces* must be allowed for. The net drag force on hulls is usually modelled by the Froude and Euler Numbers in relation to scale model tests:

A model of scale $1:L_p/L_m = L_r$ (length ratio) is towed at a velocity V_m , requiring a towing force F_m .

The Euler number gives a basic ratio of forces to velocities:

$$Eu_r = \frac{P_r}{\rho_r V_r^2} = 1 \quad \text{Euler similitude of pressures}$$

Thus $F_r = P_r A_r = \rho_r V_r^2 A_r$ Ratio of model and prototype forces

ρ_r is the ratio of model lab to ocean or river environment fluid densities (usually = 1)

A_r is the ratio of surface or profile areas: $A_r = L_r^2$

V_r is the ratio of velocities. For kinematic similitude at towing speed this can be determined by the Froude number:

$$F_r = \frac{V_r}{\sqrt{g_r L_r}} = 1 \quad \text{Froude similitude of velocities.}$$

ie $V_r = \sqrt{L_r}$ ($g_r = 1.0$ in terrestrial labs)

Thus $F_r = \frac{F_m}{F_p} = \rho_r V_r^2 A_r = L_r L_r^2 = L_r^3$

ie model and prototype forces are in the ratio of the *cube* of the length ratio:

$$F_p = \frac{F_m}{L_r^3} \quad \text{Hull or aerofoil drag force derived from scale model tow testing}$$

L_r is a ratio of lengths: a model scale of 1:30 gives an L_r of $1/30 = 0.033$ and $L_r^3 = 3.7 \times 10^{-5}$. The towing force required for the model will be 1/37,000 of the thrust required for the prototype: a conveniently small force.



Naval hydrofoil in action (<http://www.ocw.mit.edu>)

Turbulence and Friction Controls on Ships and Aircraft

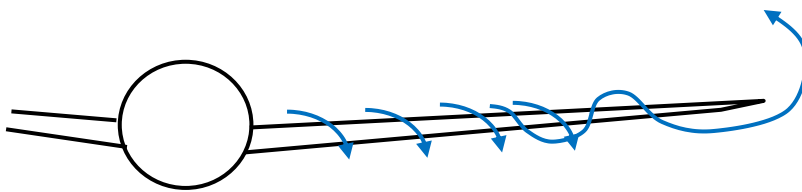
Modern developments in fluid machinery including ships, aircraft and spacecraft were heavily influenced by the Prandtl boundary layer model, since it provided a relatively easy calculation of forces and velocity distributions.

Increased computing power has greatly improved on the design process. The Navier-Stokes equations, a complex system of differential equations that are applicable to compressible and unsteady flows, are more accurate than the Prandtl model. With the right choice of computing framework they also provide a pretty good model of turbulence and related dynamic phenomena.

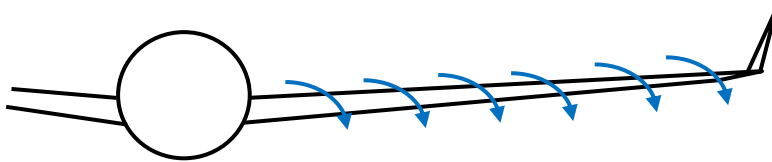
Among the advances made possible by Navier-Stokes computing are a range of design modifications that control turbulence and friction in ships and aircraft.

Wing-tip vortices: Vortices form along a wing and tend to migrate outwards to the tip, adding up and generating a transverse momentum that throws the vortex sideways.

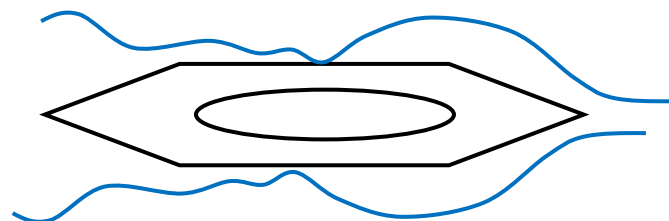
The tip vortex comes down on the wing tip, reducing lift at dangerous moments such as the final approach.



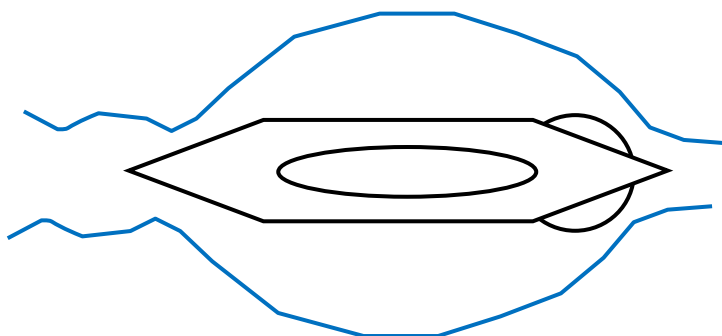
It was found by computational fluid dynamics (CFD) modelling that adding a fin at the end of the wing eliminates the tip vortex in large aircraft:



Similarly the development of a bulbous bow for large ships was developed through CFD: the wake is thrown wider, possibly missing the hull entirely and reducing form drag:



Conventional bow:
wake returns and
impacts on hull

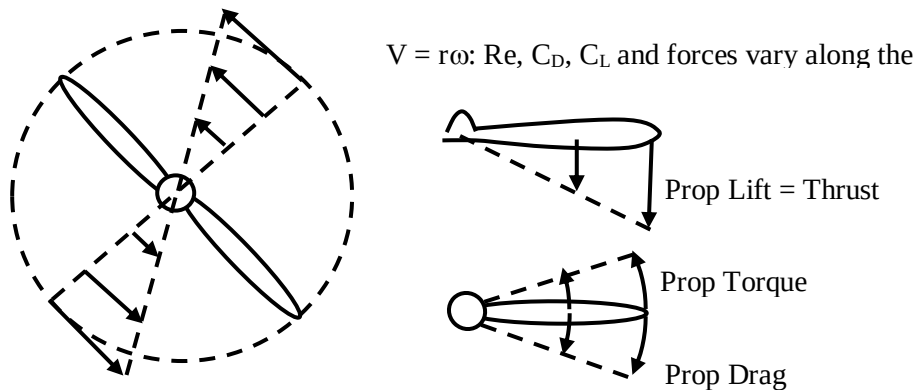


Bulbous bow: wake
impacts further back
or misses hull

Propulsion

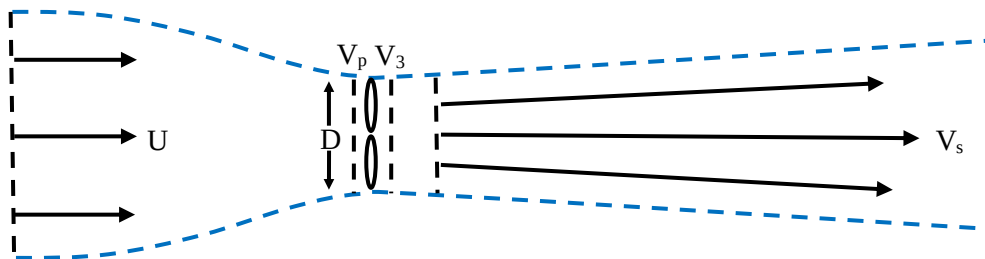
Propellers, and to some extent jet turbines, behave as aerofoils with lift and drag coefficients that vary along the length of the wing due to rotation about the engine axis. The rotation produces a linear velocity gradient along the blades, with varying Reynolds number and lift/drag coefficients.

Propellers are not uniform in cross-section or in angle of attack. The variation is designed to yield optimal lift-drag characteristics in climb, cruise and descent. Modern turboprop engines have blades that can be gimballed around their axis to vary the overall angle of attack.



Due to the non-uniformity of the blade, propeller thrust is described in bulk terms by the net change in fluid momentum. The propeller's reference frame is the aircraft, so the initial velocity is the airspeed U . The flow accelerates to V_p just before the propeller. The slipstream velocity V_s is measurable in a wind-tunnel or by test observations and provides a direct estimate of the thrust and efficiency:

The slipstream velocity needs to be measured a small distance downstream from the propeller, as there is a pressurised zone giving high local velocities V_3 immediately behind the blades.



$$\text{Thrust: } F = \dot{m}\Delta V = \rho Q(V_s - U) = \rho V_p \frac{\pi D^2}{4} (V_s - U)$$

Bernoulli's equation can be used to show that

$$V_p = \frac{(V_s + U)}{2}$$

$$\text{So } F = \rho \frac{\pi D^2}{8} (V_s - U)(V_s + U) = \rho \frac{\pi D^2}{8} (V_s^2 - U^2) \quad \text{Propeller Thrust}$$

The useful power of the propeller is force x airspeed:

$$\text{Power} = F \cdot U$$

A larger power input is required to power the flowrate at the propeller:

$$\text{Input} = F \cdot V_p$$

$$\text{The efficiency of the propeller is thus } \eta = \frac{\text{Power}}{\text{Input}} = \frac{U}{V_p} = \frac{2U}{U + V_s} \quad \text{Propeller Efficiency}$$

At cruising speed, where Thrust = Drag,

$$Thrust = \rho \frac{A_p}{2} (V_s^2 - U^2) = C_D \rho A_A \frac{U^2}{2} = Drag$$

where A_p and A_A are profile areas of the propeller and the aircraft respectively.

Thus $A_p V_s^2 = C_D A_A U^2 + A_p U^2$

Or $V_s = U \sqrt{C_D \frac{A_A}{A_p} + 1}$

The aircraft drag coefficient is generally constant, in the order of 0.5 to 1.0, decreasing slightly with velocity. Hence the slipstream velocity is determined by aircraft velocity and body/prop area ratio. Placing this into the expression for efficiency:

$$\eta = \frac{2U}{U + V_s} = \frac{2}{1 + \sqrt{C_D \frac{A_A}{A_p} + 1}} \quad \text{Alternate expression for propeller efficiency}$$

U cancels out and the efficiency is a function of C_D and the area ratio. As C_D decreases with velocity, efficiency increases slightly with airspeed. Proportionally larger propellers are also more efficient.

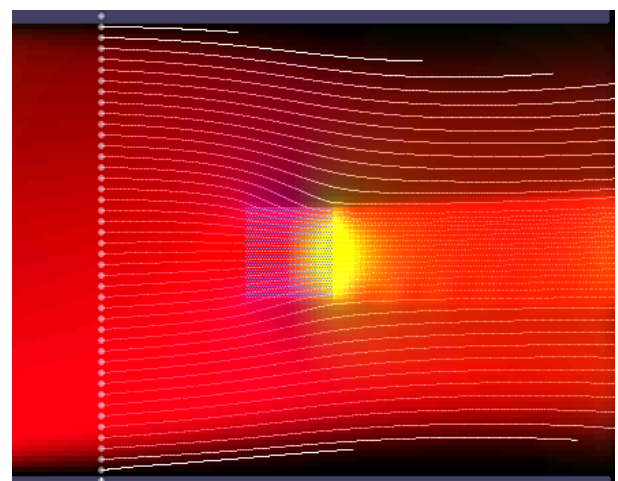
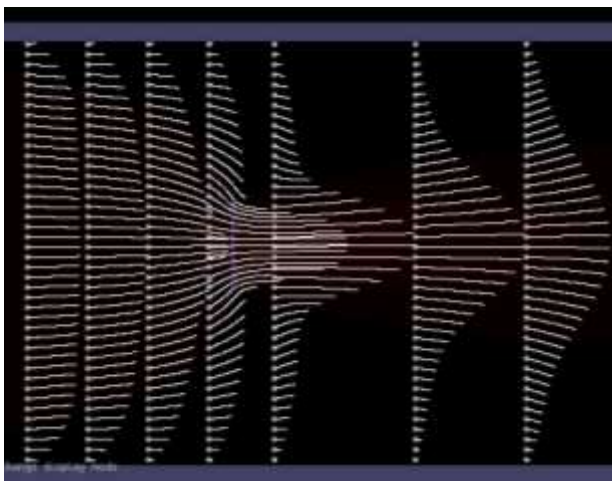
Thrust can also be expressed as a function of propeller lift or *thrust coefficient* and angular velocity ω :

$$Thrust = C_T \rho A_B \frac{(\bar{r} \omega)^2}{2} \quad \text{Alternate expression for propeller thrust}$$

where C_T is a thrust coefficient ~ 1.0 @ 10° AoA, A_B is the total blade area of the propeller (profile area of both blades) and $\bar{r}$ is a dominant blade radius. C_T and $\bar{r}$ are determined by experiment.

Usefully, thrust is proportional to the square of the prop speed (as is the engine torque to overcome prop drag). Again, C_T decreases with increasing prop speed so the proportionality declines slightly.

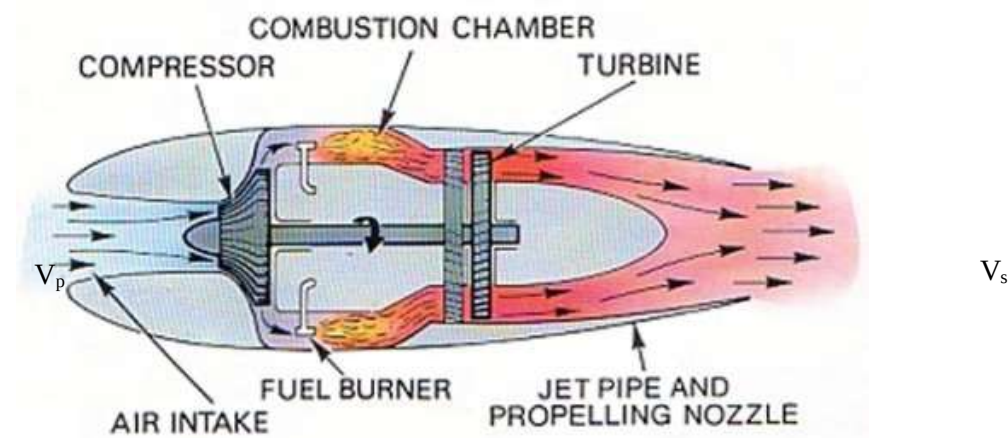
A model propeller in a CFD wind tunnel:



Note the contraction of the air tube and the small range of the exit velocity V_3 (bright yellow). The average velocity at V_p is very nearly the mean of V_s and U . (This low-speed simulation does not strongly distinguish U and V_s at the velocity display's colour resolution).

Jet Engines

In a conventional jet engine a high speed turbofan with multiple blades is fed burning air-fuel mixture, greatly accelerating the force of the flow.



Source: www.freemansgarage.com

Air enters the intake and is compressed by the 1st fan in the system (the compressor). Fuel is injected and ignited in the combustion chamber, expanding to produce pressure. The hot gas exits through the turbine, which rotates due to the hot gas rushing past it. The turbine is connected to the compressor by a shaft, which powers the compressor. The system is self-sustaining as long as fuel is ignited. Initial startup requires an electric motor to turn the shaft until the engine starts.

The thrust and efficiency equations for a propeller still apply, but must be modified to include the mass flow rate of fuel:

$$F = \dot{m}\Delta V = \rho Q(V_s - U) + \dot{m}_f V_s = \rho Q(V_s(1+r) - U) \quad \text{Jet Thrust}$$

where r is the fuel mix ratio,

$$r = \frac{\dot{m}_f}{\rho Q}$$

The flowrate is again estimated by the intake speed:

$$Q = V_p \frac{\pi D^2}{4} = (V_s + U) \frac{\pi D^2}{8}$$

The engine efficiency is similarly modified:

$$\eta = \frac{FU}{FU + \rho Q(1+r) \frac{(V_s - U)^2}{2}} \quad \text{Jet Efficiency}$$

Rocket Engines

Spacecraft must be capable of developing enormous, sustained thrust both in atmospheric and vacuum conditions. The work of Konstantin Tsiolkovsky, Robert Goddard, Werner von Braun and others owes much to the extraordinary insight of Sir Isaac Newton that any physical action is accompanied by an equal and opposite reaction: a propellant ejected downward with high speed and mass flow rate creates a strong thrust reaction in the opposite (upward) direction.

The main technical problems solved by so-called “rocket science” are the thermal materials science of engines subjected to enormous combustion temperatures, and the control systems technology required for thrust vectoring and orbit alignment.

The thrust delivered by a rocket engine can be derived from the momentum reaction law:

$$\underline{F} = \dot{m}\Delta\underline{V} = M(\underline{g} + \underline{a})$$

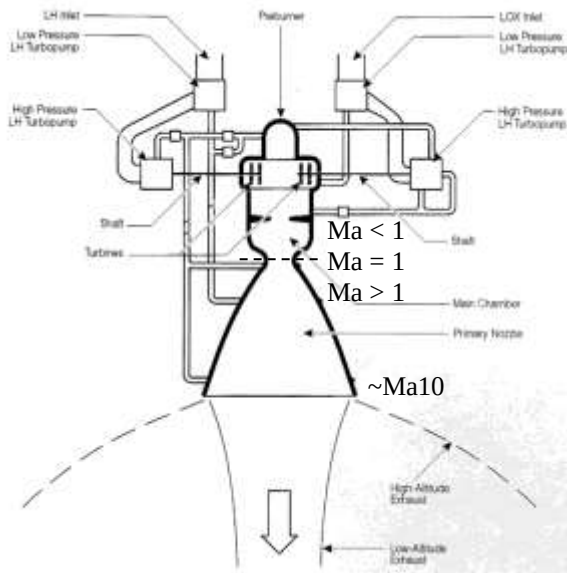
Here $\dot{m}$ is the mass flowrate and M is the mass of the rocket, which decreases as fuel is consumed and launch stages are jettisoned. The thrust $\underline{F}$ applies within the stationary reference frame of the launch vehicle, ie $\Delta\underline{V}$ is the velocity of the exhaust relative to the engine, regardless of the changing velocity of the rocket as it accelerates to orbit velocity. $\underline{g} + \underline{a}$ is the combined gravitational and active acceleration of the rocket.

The mass flowrate and exhaust velocity are provided by burning a solid or liquid fuel with an oxidant, and accelerating the exhaust to supersonic speeds through a converging-diverging *Laval nozzle*. In the original Saturn-V engines of the Apollo program the first-stage fuel was ordinary kerosene burned with liquid oxygen: the resulting high temperatures required the launch platform to be drenched in immense quantities of liquid water, so in fact much of the dramatic lift-off smoke cloud was just steam from this crude evaporative cooling system.

Since the space shuttle program began in the late 1970s, industrial rocket technology has been based on the combustion of hydrogen and oxygen to produce water as a non-polluting propellant. The exhaust cloud is pure steam, but superheated and accelerated in the reaction chamber and exhaust nozzle rather than evaporated off concrete with other combustion products.

A converging-diverging nozzle is essential to produce the supersonic speeds (up to Mach 10) required in the exhaust. Subsonic flows accelerate in the converging section of the nozzle, to sonic speed (Mach 1) at the throat, and continue to accelerate at supersonic velocities in the diverging section. “Sonic boom” shock waves occur at the mouth of the nozzle and in a series of slipstream waves often visible as the thrust is tuned to lift-off. The sound energy released by these shock waves is enormous and is a significant unavoidable source of rocket inefficiency.





Conventional liquid H-O rocket engine. Pressure in the accelerated and superheated exhaust drops to near absolute zero while the velocity increases to around Mach 10. (Nasa)

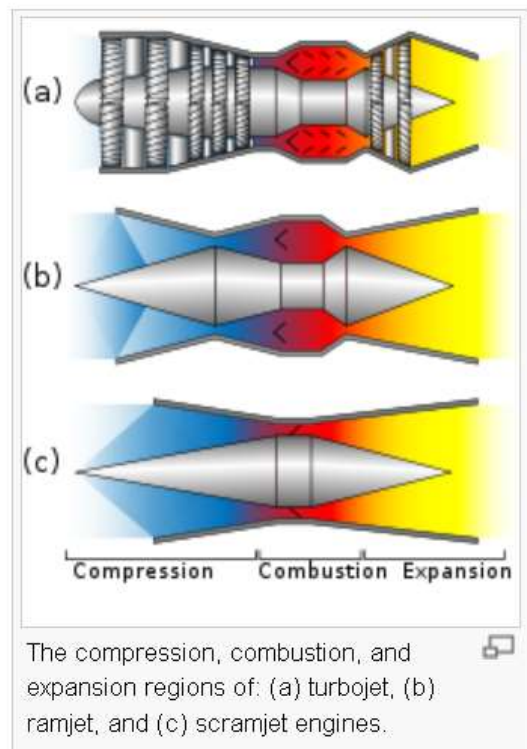
Thought Experiment: Bottle Rocket Science

With what you now know about fluid mechanics in general and propulsion in particular, could you design and build a bottle rocket? You will need to think about the energy stored in compressed air, the momentum exchange of a liquid propellant, the aerodynamics of nose cones and fins, the *ballistics* of a reducing launch vehicle mass, the precision engineering of nozzles, some material science issues related to pressure containment and joint sealing, and some control systems issues relating to the nozzle valve release. Let us know how you go!

Scramjet Engines

A scramjet (**supersonic combustion ramjet**) is a variant of a ramjet airbreathing combustion jet engine in which the combustion process takes place in supersonic airflow. As in ramjets, a scramjet relies on high vehicle speed to forcefully compress and decelerate the incoming air before combustion (hence ramjet), but whereas a ramjet decelerates the air to subsonic velocities before combustion, airflow in a scramjet is supersonic throughout the entire engine. This allows the scramjet to efficiently operate at extremely high speeds: theoretical projections place the top speed of a scramjet between Mach 12 and Mach 24, which is near orbital velocity. For comparison, the fastest manned airbreathing aircraft, the SR-71 Blackbird, has a maximum speed of Mach 3.2.

The scramjet is composed of three basic components: a converging inlet, where incoming air is compressed and decelerated; a combustor, where gaseous fuel is burned with atmospheric oxygen to produce heat; and a diverging nozzle, where the heated air is accelerated to produce thrust. Unlike a typical jet engine, such as a turbojet or turbofan engine, a scramjet does not use rotating, fan-like components to compress the air; rather, the achievable speed of the aircraft moving through the atmosphere causes the air to compress within the nozzle. As such, very few moving parts are needed in a scramjet, which greatly simplifies both the design and operation of the engine. In comparison, typical turbojet engines require inlet fans, multiple stages of rotating compressor fans, and multiple rotating



turbine stages, all of which add weight, complexity, and a greater number of failure points to the engine. It is this simplicity that allows scramjets to operate at such high velocities, as the conditions encountered in hypersonic flight severely hamper the operation of conventional turbomachinery.

Scramjet engines are a type of jet engine, and rely on the combustion of fuel and an oxidizer to produce thrust. Similar to conventional jet engines, scramjet-powered aircraft carry the fuel on board, and obtain the oxidizer by the ingestion of atmospheric oxygen (as compared to rockets, which carry both fuel and an oxidizing agent). This requirement limits scramjets to suborbital atmospheric flight, where the oxygen content of the air is sufficient to maintain combustion. (Source: Wikipedia)

Reading

Streeter and Wylie 1981 (Reading 10) Section 5.7