

Lecture 24: Drag and Lift

Boundary Layer Separation and Drag

The pressure gradient in a boundary layer is usually negative, ie pressure decreases with friction drag along the length of a conduit as per the head loss laws we have examined so far.

ie $\frac{dP}{dx} < 0$ Typical pressure gradient with friction in uniform or accelerating flow

However, in the case of decelerating flow such as in the divergent zone of a non-streamlined shape, pressure can increase in the downstream direction. This depends on energy exchanges as per Bernoulli's equation:

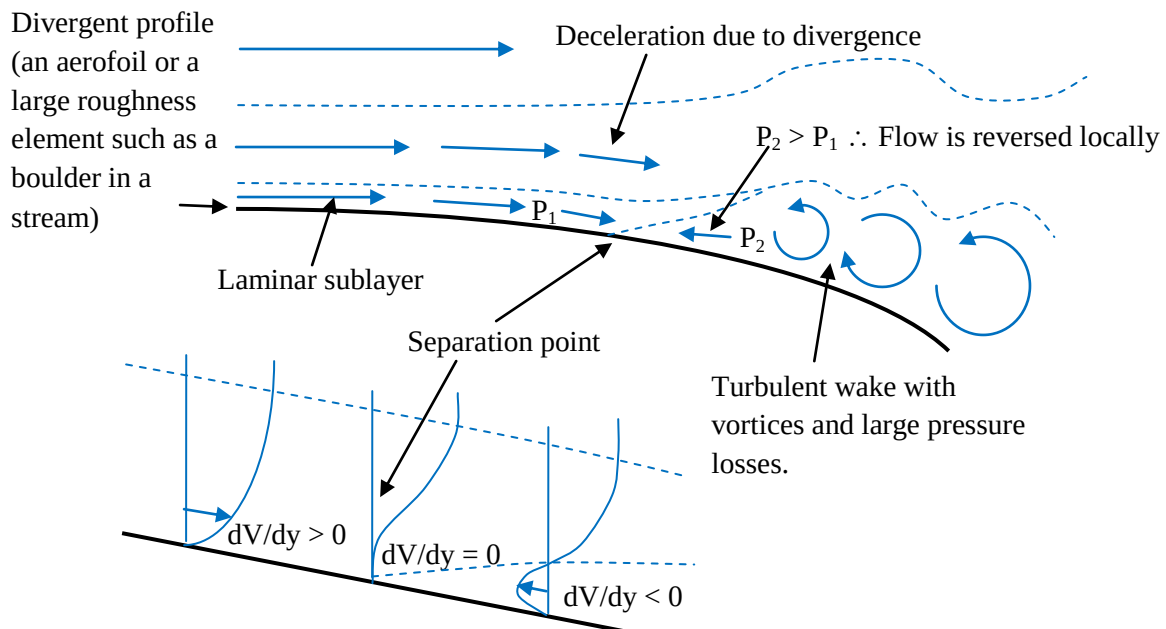
$$\frac{V_1^2}{2g} + \frac{P_1}{\rho g} = \frac{V_2^2}{2g} + \frac{P_2}{\rho g} + h_f \quad (\text{for constant } z)$$

$$\text{So } \frac{(V_1^2 - V_2^2)}{2g} - h_f = \frac{(P_2 - P_1)}{\rho g}$$

$$\text{Or } \frac{\Delta P}{\Delta x} = \frac{\rho(V_1^2 - V_2^2)}{2\Delta x} - \rho g S_f \quad (h_f / \Delta x = \text{friction slope } S_f)$$

$$\text{So } \frac{\Delta P}{\Delta x} > 0 \quad \text{when} \quad \frac{\rho(V_1^2 - V_2^2)}{2\Delta x} > \rho g S_f$$

This is known as an *adverse pressure gradient*, since the higher pressure downstream presses back against the flow and causes it to flow backwards. A separation point arises, below which the flow is partly retrograde with complex dynamic boundary layers and high levels of turbulence.



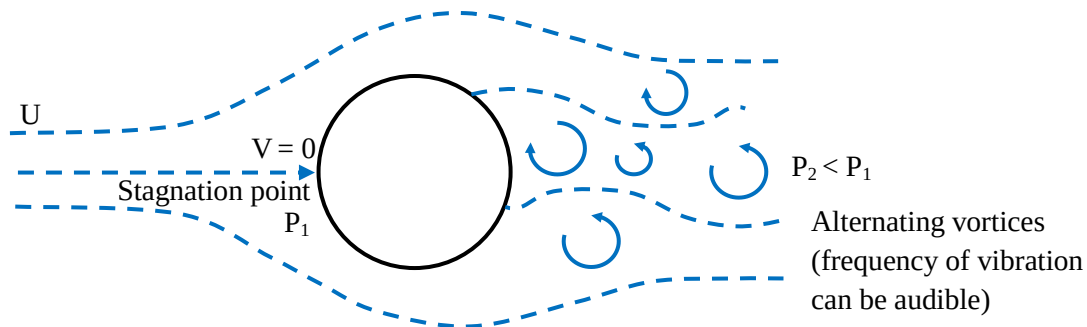
The exact separation point depends on the rate of divergence. The pressure gradient may remain negative if the value of V_2^2 is sufficiently high. The effect of *streamlining* is to force the separation point further along the profile, by reducing the adverse pressure gain to less than the friction pressure loss for as far as possible. This also has the effect of minimising total pressure losses and hence reduces *form drag*.

For example reducing the angle of attack in an aerofoil maintains a higher downstream flow velocity with relatively low pressure loss and reduced separation effects.

A video http://www.youtube.com/watch?v=OSmC2o_EGmg&feature=related shows the effect of angle of attack on wake separation in an aerofoil, with dynamic stall effects.

Form Drag

Form drag is the effect of the profile of a submerged body breaking the flow and causing differing degrees of boundary layer separation. The resulting net pressure losses produce relatively high drag coefficients, typically 10x greater than the skin friction coefficient C_D for the same Reynolds number.



The pressure P_1 is known as the *stagnation pressure*: a complete conversion of kinetic energy into pressure where the velocity in the stagnation streamline is reduced to zero.

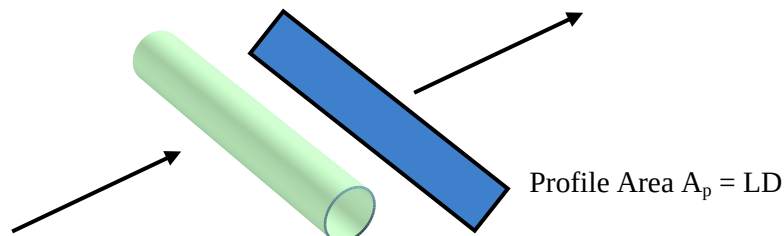
$$P = \rho \frac{U^2}{2} \quad \text{Stagnation Pressure}$$

The pressure downstream of the object will be lower due to combined skin friction and form drag. Adverse pressure gradients collapse quickly after the separation point and the net pressure change is always negative. The (unknown) difference in net pressure is formulated as a *form drag coefficient*:

$$\Delta P = C_D \rho \frac{U^2}{2} \quad \text{Form Drag Pressure}$$

Total Drag: Published Drag Coefficients

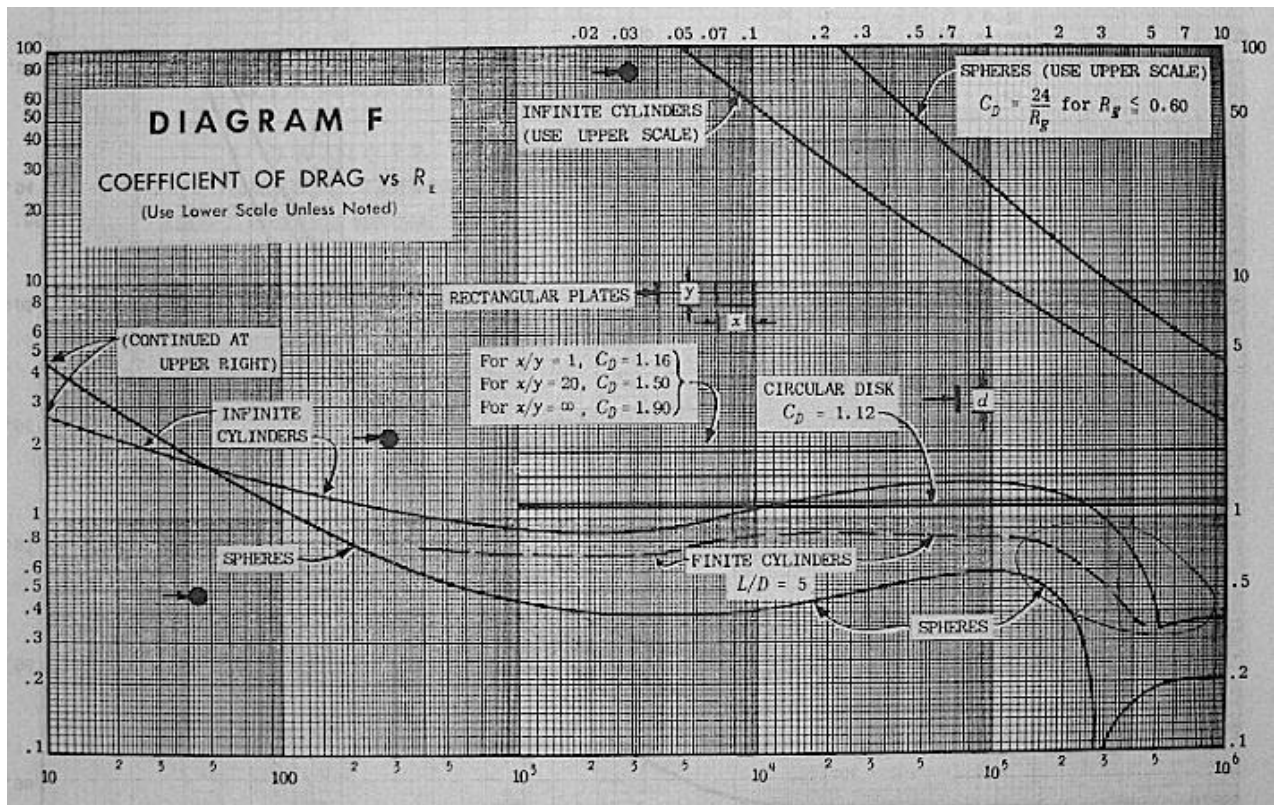
In practice it can be difficult to separate skin drag from form drag, and the two are usually combined in measurements of total drag force. A total drag coefficient is used, multiplying pressure by the *profile area* A_p of the object in the flow to obtain a total drag force:



$$\text{Drag} = C_D \rho \frac{U^2}{2} A \quad \text{Total Drag Force}$$

Drag coefficients for a range of submerged objects are provided in Giles (1983) Diagram F:

Shapes such as cylinders, flat plates, ovoid sections, angles, projectiles of various kinds as well as aerofoils and hydrofoils, have variable drag coefficients as a function of Reynolds number:



Total drag coefficient C_D against Reynolds No $Re = VD\rho/\mu$ for a range of submerged objects of principal dimension D . (Giles, 1983)

Wakes behind blunt shapes typically separate as paired vortices that oscillate at a characteristic frequency of vibration.

$$\frac{fD}{V} = 0.198 \left(1 - \frac{19.7}{Re} \right) \quad \text{Vortex shedding frequency Strouhal number}$$

Where f = frequency of vortex shedding, D = diameter, V = velocity, Re = Reynolds number

Videos <http://www.youtube.com/watch?v=AJgEa2dbJU&NR=1> and <http://www.youtube.com/watch?v=P8VcZzgdfSc&NR=1> provide some insights into vortex shedding.

Structures must be designed so that their characteristic frequency of vibration (a function of masses and structural lengths) is different to the characteristic frequency of vortex shedding at normal windspeeds, for both structural and amenity (user comfort) considerations. A structural member may not collapse but may be a source of noise pollution or unwanted movement.

A classic example of structural failure due to harmonic wind frequency is the Tacoma Narrows bridge collapse. This large-span cable suspension bridge vibrated at the same frequency as the wind vortex street (a few seconds per cycle at that scale) and was destroyed in a moderate wind.

A video <http://www.youtube.com/watch?v=HxTZ446tbze> shows the collapse of the Tacoma Narrows bridge.

Aerodynamics and Lift

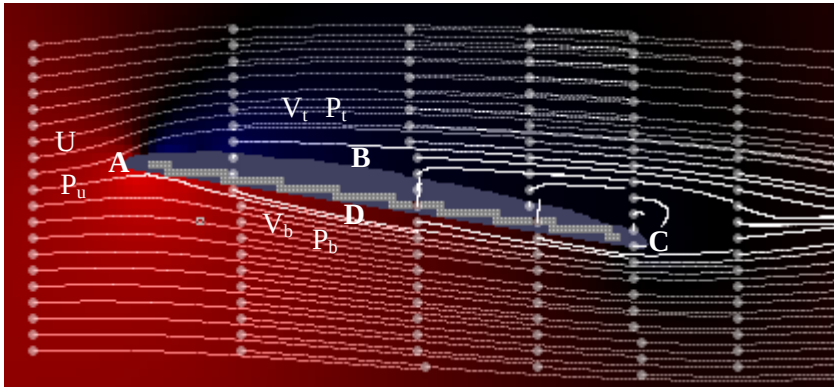
The pressure loss that occurs in a non-streamlined body, producing drag, is also responsible for lift on a streamlined body. The difference between the two forces “Lift” and “Drag” is simply the extent to which the components of a single pressure force are distributed between vertical and horizontal.

Streamlining effectively moves the zone of low pressure from directly behind a submerged object to a position more or less above the object. The component of the pressure that is then directly above the wing generates lift, while the component that remains behind the wing continues to produce drag. The figure below shows low pressure in dark colours, higher pressures in shades of red.

The pressure difference between the top and bottom chord of the wing can be derived from the Bernoulli equation, where the higher velocity on the upper chord is assumed to be due to a longer chord length: Assuming a common travel time t :

$$ABC = V_t t \quad ADC = V_b t$$

$$\frac{V_t}{V_b} = \frac{ABC}{ADC}$$



Taking two initially equal streamlines t and b of velocity U and pressure P_u that separate into top and bottom flows,

$$\frac{U^2}{2g} + \frac{P_u}{\rho g} = \frac{V_t^2}{2g} + \frac{P_t}{\rho g} = \frac{V_b^2}{2g} + \frac{P_b}{\rho g}$$

So

$$P_b - P_t = \rho \frac{(V_t^2 - V_b^2)}{2} = \rho \frac{V_b^2}{2} \left(\left(\frac{ABC}{ADC} \right)^2 - 1 \right)$$

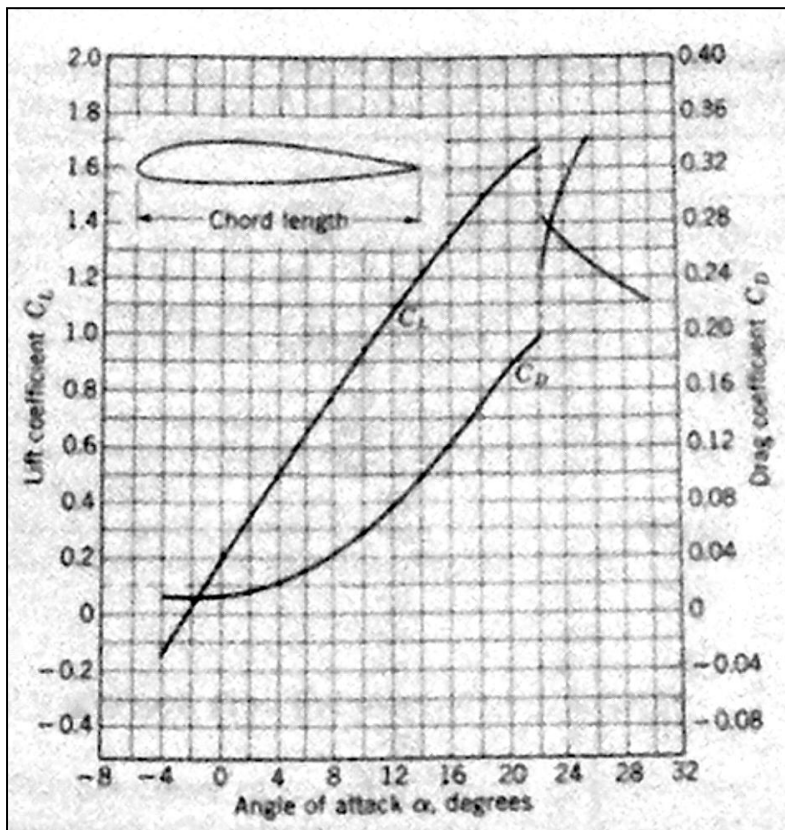
Here V_b can be taken as U , the aircraft speed.

The pressure difference is multiplied by the wing area to produce a lift force. This has the same form as the drag equation except with a *lift coefficient* in the large brackets. As with all such coefficients the lift coefficient C_L is a complex Reynolds-dependent unknown that can only be obtained by experiment:

$$Lift = C_L \rho \frac{U^2}{2} A_w \quad \text{Aerodynamic Lift Force Equation (N)}$$

Where A_w is the vertical projected area of the wing with an effective average pressure difference.

The lift coefficient depends not only on the wing shape and Reynolds number but also on the angle of attack of the aerofoil or hydrofoil: ie the angle of the chord to the uniform velocity vector U :



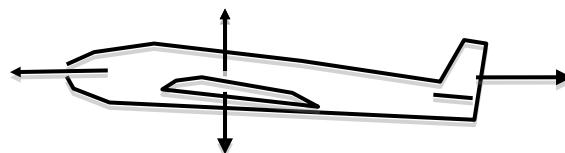
Lift and Drag coefficients for a typical aerofoil. Note stalling behaviour, a sudden drop in lift with increased drag at @ $\alpha = 22^\circ$. (Source: Streeter and Wylie, 1983).

There is generally an ideal angle of attack for a given cruising speed, giving the best combination of lift and drag for the wing. The total aircraft drag is determined by the aircraft profile area and Reynolds number, not the wing alone, and is proportional to the ideal lift.

Aerodynamic stability depends on the distribution of mass of an aircraft relative to its thrust, lift and drag vectors, much as a boat's stability is determined by centres of buoyancy and mass.

An aircraft's climb rate depends on the balance of lift to weight forces. To climb, rather than lift the nose, the thrust is increased to increase speed and hence lift. To descend, the thrust is reduced. The tailplane is not normally aerodynamically loaded, ie, there is no reliance on tail lift other than for raising and lowering the angle of attack (outside of aerobatic or manoeuvre flight).

At cruising altitude, **Lift = Weight** and **Thrust = Drag**



Divide one by the other: **Weight/Thrust = Lift/Drag**

This means the weight and cruising thrust must balance in the same proportion as the lift and drag forces. It's a remarkable result that the optimal cruising ratio of lift to drag also gives an optimal ratio of load to engine power.

Reading Streeter and Wylie 1981 (Reading 10) Section 5.7
Review Learning Outcomes for Section 5

Tutorial Drag Forces and Aerodynamics