

Lecture 23: Introduction to Turbulence

Turbulent Friction

Friction in general has the effect of transferring momentum or applying force between surfaces or bodies in relative motion, ie wherever there is a velocity difference. This generates heat and as such is a thermodynamic as well as mechanical phenomenon.

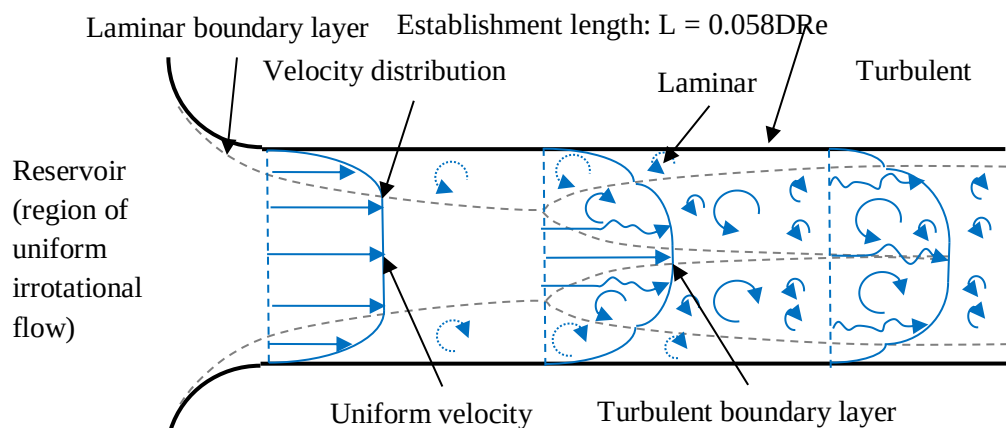
Viscous friction in fluids occurs as a result of momentum transfer between regions of fast and slow-moving fluid. The slow layers slow down the fast, and vice versa. This is analogous to a *diffusive* process, ie momentum appears to diffuse into the slower region, much as a solute will mix into a solvent by diffusion along a concentration gradient. In a viscous fluid, momentum diffuses along the velocity gradient.

As with ordinary diffusion there are two mechanisms for momentum transfer: laminar and turbulent. Laminar flow is a relatively gentle and hence slow mechanism; turbulent flows are rapid and effective. A laminar solution diffuses slowly, by molecular motions; turbulence creates rapid diffusion by bulk fluid motion. Turbulent flows will mix completely and are very effective at exchanging heat.

Turbulent friction has a different mechanism to viscous friction: laminar friction occurs between layers of fluid sliding against each other. Turbulent friction occurs when momentum is transferred as parcels of fluid moving *across* the flow in velocity fluctuations. The momentum of the bulk lateral flow creates drag, so *it is the density rather than the viscosity that dominates turbulent friction*.

Eddies, and the components of turbulence in general, arise due to instabilities in the rotational components of laminar flow at higher velocity gradients. Rotation due to the relative velocity of adjacent fluid layers tends to magnify and break up into vortices, backwashes and random lateral movements. This tends to occur when the momentum forces in the flow become much larger than the viscous forces, so that the damping effect of viscosity decreases and instabilities begin to amplify.

The Reynolds number is a ratio of momentum to viscous forces and is thus a natural scale for turbulence. Its recurrence throughout the theory of laminar and turbulent flow is indicative of its importance.



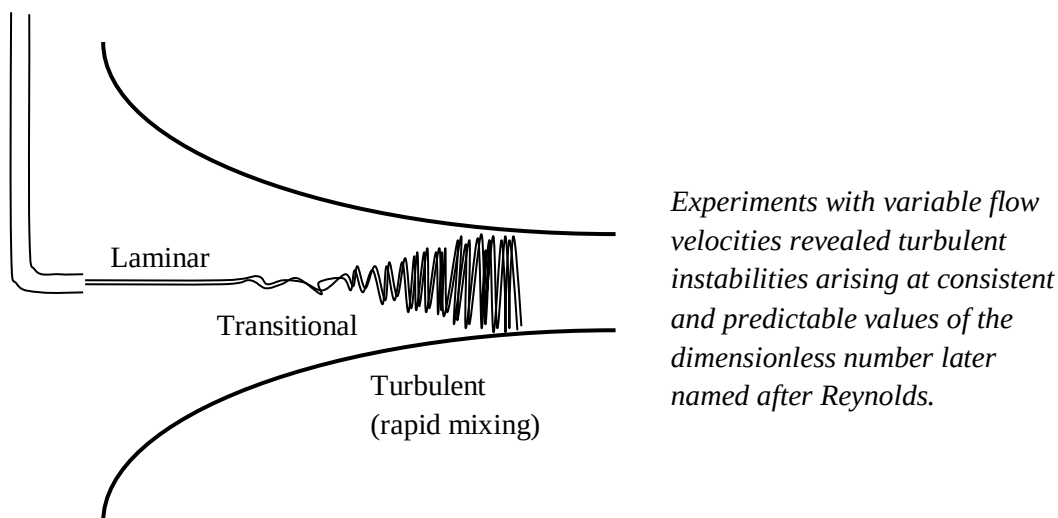
The “spirals” indicated are an attempt at illustrating the development and direction of rotation. Laminar rotation is a microscale phenomena due to any non-uniform velocity distribution – adjacent layers in relative motion undergo a continuous deformation that is analogous to a rotation. Turbulent rotation is due to bulk rotational motion in eddies and vortices, arising partly due to instability of laminar rotation. In either laminar or turbulent zones the direction of rotation reverses at the centreline where shear stress = 0.

A video <http://www.youtube.com/watch?v=NplrDarMDF8&feature=related> shows the flow structure of a mixing flow in a turbulent pipe.

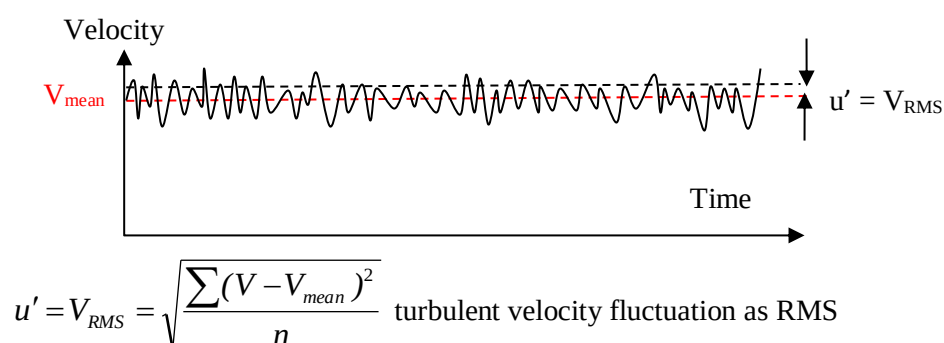
Turbulence remains a poorly understood phenomenon, not well described by any known theory (See Stewart, 1989, Reading 11). The following discussion reflects a standard model with acknowledged limits. The value of this model is that it provides a flexible approach to fluid friction with applications in a wide range of friction phenomena including pipe flow, drag, lift, and propulsion.

The Reynolds Model

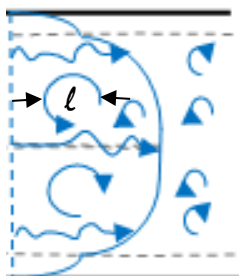
Osborne Reynolds was the leading researcher in fluid turbulence for many years. His experiments involved careful injection of dye into laminar flows, with observation of the development of velocity fluctuations:



Reynolds realised that turbulent velocity fluctuations around the average velocity occur at random and can be described by an average *fluctuation velocity* u' . This tends to increase with increasing Reynolds number and distance from a wall or boundary. The fluctuation velocity is now usually measured as a *root-mean-square* velocity, ie the square root of the mean of squared velocity variations:



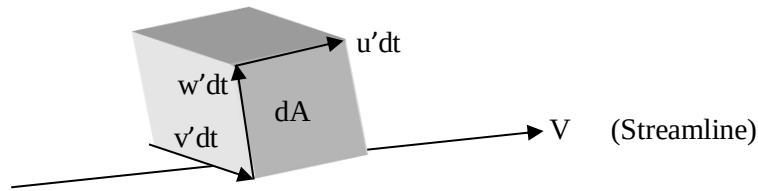
where n is the number of data points of velocity measured in a given direction. The formula gives the square root of the mean of squared deviations from mean velocity, a statistical concept similar to a standard deviation.



Reynolds and later Prandtl also developed the idea of a *mixing length*, based on the average size of turbulent motions or eddies. The mixing length ℓ estimates the size of fluid parcels in lateral motion and as such is a measure of bulk momentum transfer, ie turbulent friction as defined. Mixing length also

tends to increase with Reynolds number and wall distance. The mixing length can be observed by tracer methods.

Reynolds (1884) analysed velocity fluctuations in three dimensions, u' (along streamlines), v' and w' (lateral to streamlines) and derived a simple model of turbulent shear along corresponding planes:



In a short time interval dt , velocity fluctuations create mixing lengths $u'dt$, $v'dt$ and $w'dt$. This defines a volume and hence mass undergoing acceleration from V to $V + u'$ along the streamline:

$$dm = \rho dV = \rho v' dt dA$$

The acceleration creates an elemental shear force:

$$dF = dma = \rho dV \frac{dV}{dt}$$

The turbulent shear stress on the element is this shear force divided by the elemental area. Noting that dV includes both dt and dA we have

$$\tau_t = \frac{dF}{dA} = \rho v' dV$$

But dV , the change in velocity in time dt , is just the fluctuation velocity u' :

$$\tau_t = \rho v' u' \quad \text{Reynolds' Turbulent Shear Model}$$

An equivalent shear stress along the second lateral plane (darkest shading) uses w' in place of v' .

The fluctuating velocity distribution creates an additional laminar shear stress modelled as per any Newtonian fluid:

$$\tau_l = \mu \frac{dV}{dy} = \mu \frac{u'}{v' dt} = \mu \frac{u'}{\lambda} \quad \text{Fluctuation Viscous Shear}$$

where λ is a mixing length. The ratio of turbulent to viscous shear stress is a Reynolds number:

$$\frac{\tau_t}{\tau_l} = \frac{\rho \lambda v'}{\mu} \quad \text{Fluctuation Reynolds Number}$$

The emergence of a Reynolds number here is an indication of the internal validity of this derivation, ie it correctly produces the fundamental experimental dimensionless number for laminar-turbulent transitions, more or less exactly as required.

Unfortunately Reynolds' model is difficult to calibrate experimentally due to the combined (large) uncertainties of u' , v' and λ . Further progress depended on a more elaborate mixing length model and experimentation carried out by Prandtl.

The Prandtl Model

Prandtl (c 1900) performed experiments to determine reasonable estimates of mixing length based on observable disturbances in flow regimes. He used this to derive an eddy-based model of turbulence that leads to a theoretical derivation of the logarithmic velocity distribution in turbulent pipes:

Prandtl based his derivation on Boussinesq's proposal (1877) that there are two forms of viscosity:

$$\tau = (\eta + \mu) \frac{dV}{dy}$$

Where η (eta) is a dynamic eddy viscosity and μ (mu) is laminar viscosity as previously defined. η is a property of the flow regime while μ is a property of the fluid.

Laminar viscous flow: $\eta = 0$

Turbulent flow: $\eta \gg \mu$

So
$$\tau_t = \eta \frac{dV}{dy} = \rho \nu' u'$$

Prandtl used a toroidal (donut-shaped) eddy model to reason geometrically and by continuity that a velocity fluctuation should be proportional to velocity gradient and mixing length:

$$u' \sim \lambda \frac{dV}{dy}$$

Similar reasoning suggested a correlation between fluctuations in different directions:

$$u' \sim v' \sim \lambda \frac{dV}{dy}$$

So
$$\tau_t = \eta \frac{dV}{dy} = \rho \nu' u' = \rho \lambda^2 \left(\frac{dV}{dy} \right)^2$$

And
$$\eta = \rho \lambda^2 \frac{dV}{dy} \gg \mu.$$

Prandtl's and later Von Karman's (c 1912) experiments provided estimates of λ :

Prandtl: $\lambda = ky \quad k \sim 0.4$

Von Karman:
$$\lambda = k \frac{\left(\frac{dV}{dy} \right)}{\left(\frac{d^2V}{dy^2} \right)}$$

Prandtl's model is usually accepted as being simpler and more tractable than Von Karman's, an easy decision given their very similar results.

k is now known as the *Von Karman Constant*, measured originally at about ~ 0.4 . This is no longer thought to be a fixed constant, but the value of 0.4 is still used.

Thus
$$\tau_t = \rho (ky)^2 \left(\frac{dV}{dy} \right)^2 = \eta \frac{dV}{dy} \quad k \sim 0.4 \quad \text{Prandtl Model}$$

Logarithmic Velocity Distribution

We have

$$\tau_t = \rho(ky)^2 \left(\frac{dV}{dy} \right)^2 = \eta \frac{dV}{dy}$$

Or
$$\sqrt{\frac{\tau_t}{\rho}} = ky \frac{dV}{dy}$$

This has the units of velocity : $(\text{shear}/\text{density})^{1/2} = ((\text{N}/\text{m}^2)/(\text{kg}/\text{m}^3))^{1/2} = (\text{m}^2/\text{s}^2)^{1/2} = \text{m/s}$ and is known as the *shear velocity*, u^* :

$$u^* = \sqrt{\frac{\tau_t}{\rho}} = ky \frac{dV}{dy} \quad \text{Shear velocity}$$

Let's assume u^* is constant: rearranging,

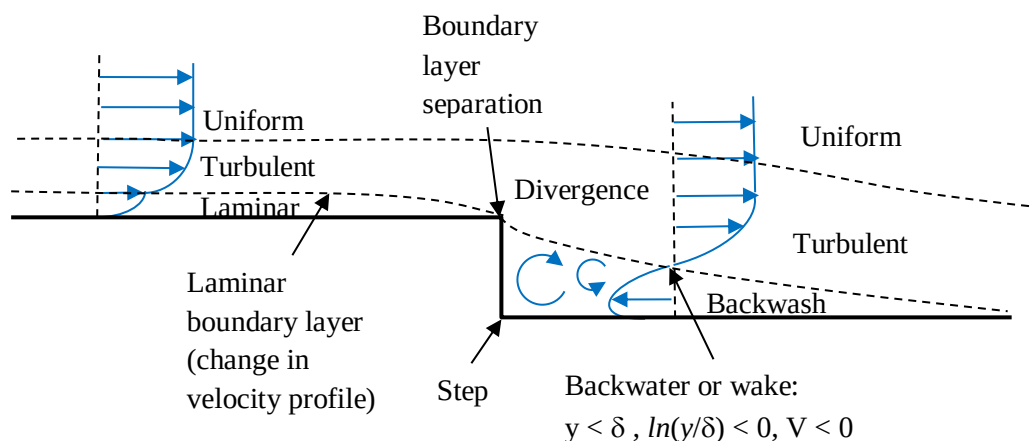
$$\frac{u^*}{k} \frac{dy}{y} = dV$$

Integrating,
$$\frac{u^*}{k} \ln\left(\frac{y}{\delta}\right) = V + c$$

or
$$V = \frac{u^*}{k} \ln\left(\frac{y}{\delta}\right) + c \quad k \sim 0.4 \quad \text{Logarithmic Velocity Distribution}$$

The values of u^* and the integration constants δ and c must be determined by experiment. In particular δ and c are needed to model the laminar-turbulent transition at the laminar boundary layer, and the even more complex transition of wake formation or *boundary layer separation*: the velocity is negative where the flow reverses in permanent eddies due to roughness, steps or divergence.

Broadly speaking any non-streamlined shape has divergence where the flow spreads and becomes subject to adverse pressures causing backwash, ie a backwater or wake:

**Reading**

Streeter and Wylie 1981 (Reading 10) Section 5.3 to 5.5
Stewart 1989 (Reading 11)

