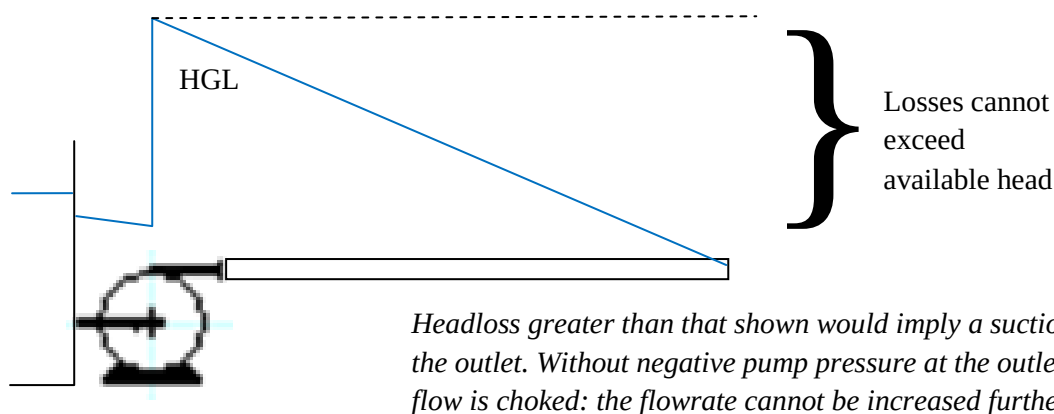


Lecture 22: Application of Pipe Friction Model

Application of three-regime model of pipe headloss

The three phase model is applied by *iterative* methods (equivalent to trial and error), since the regime (laminar, transitional, turbulent) is often unknown and must be determined by an initial calculation of flow velocity based on an initially assumed friction factor. Furthermore the turbulent regime uses an *implicit* equation for friction factor (the solution $1/\sqrt{f}$ appears on both sides of the equation) that can only be solved by iterative methods.

A typical design problem is to identify a pipe diameter that will convey a given flowrate without causing excessive headloss. Losses cannot exceed the available head relative to the datum and atmospheric pressure at the outlet:



The iterative method is to assume an initial value of f (0.02 is a good start), use this to calculate the velocity V and/or D for a given headloss, then recalculate f based on the resulting Reynolds number. This is repeated until successive guesses converge, ie when the same input f emerges as output.

More direct methods are possible depending on the actual knowns and unknowns: eg for a pipe of known diameter and flowrate but unknown headloss, use Q and D to calculate V , f and h_f .

With modern calculators and iterative controls in spreadsheets it is easy to write iterative formulas which essentially solve themselves by trial and error. Even a cheap scientific calculator can accept a calculated answer as an input to a formula, which can then be iterated by repeated execution.

Example: Determine a theoretical diameter for a pipeline which is to convey $0.057 \text{ m}^3\text{s}^{-1}$ of oil a horizontal distance of 300 m, if the head loss is not to exceed 15.9 metres. At the operating temperature the specific gravity of the oil is 0.9 and the viscosity is 1.43 Pa.s. Assume laminar flow, and check with your result that this is the case.

Using the Darcy-Weisbach equation we can equate headloss with pipeline length and diameter for a given velocity and friction factor:

$$h_f = \frac{fLV^2}{2gD} = 15.9 \text{ m at } L = 300 \text{ m.}$$

Assuming laminar flow we can use the laminar form of the friction factor:

$$f = \frac{64}{Re} \quad Re < 2000 \quad \text{where } Re = \frac{VD\rho}{\mu}$$

We know the flowrate Q but not the velocity:

$$Q = 0.057 \text{ m}^3/\text{s} = V\pi D^2/4$$

$$\text{thus } V = 0.057 \times 4 / \pi D^2 = 0.0726 / D^2$$

To identify the diameter, assemble unknowns:

$$h_f = \frac{fLV^2}{2gD} = \frac{64\mu}{VD\rho} \cdot \frac{LV^2}{2gD} = \frac{64\mu L}{2g\rho} \cdot \frac{V}{D^2} = \frac{64\mu L}{2g\rho} \cdot \frac{0.0726}{D^4} = 15.9 \text{ m}$$

$$\mu = 1.43 \text{ Pa.s}$$

$$\rho = 900 \text{ kg/m}^3$$

$$\text{Thus } D^4 = \frac{64(1.43)300(0.0726)}{2(9.8)900(15.9)} = 0.00711 \text{ m}^4$$

$$\text{and } D = 0.00711^{0.25} = 0.290 \text{ m}$$

$$V = 0.0726/D^2 = 0.86 \text{ m/s}$$

Check the Reynolds Number to ensure the flow is laminar: $Re = 0.86(0.29)900/1.43 = 157 < 2000$
thus flow is laminar, assumptions are good and no second iteration is required:

$$f = 64/Re = 64/157 = 0.41$$

Example: Repeat the above exercise for a flow of $5.7 \text{ m}^3\text{s}^{-1}$. Assume a smooth pipe ($\varepsilon = 0$).

We still have

$$h_f = \frac{fLV^2}{2gD} = 15.9 \text{ m for } L = 300 \text{ m}$$

$$\text{and } V = 5.7 \frac{4}{\pi D^2} = 7.26 / D^2$$

$$\text{so } h_f = \frac{f(7.26)^2}{2gD^5} = 15.9 \text{ m}$$

$$\text{ie } f = 159 \frac{2gD^5}{300(7.26)^2} = 0.019 D^5 \text{ or } D = \left(\frac{f}{0.0197} \right)^{1/5}$$

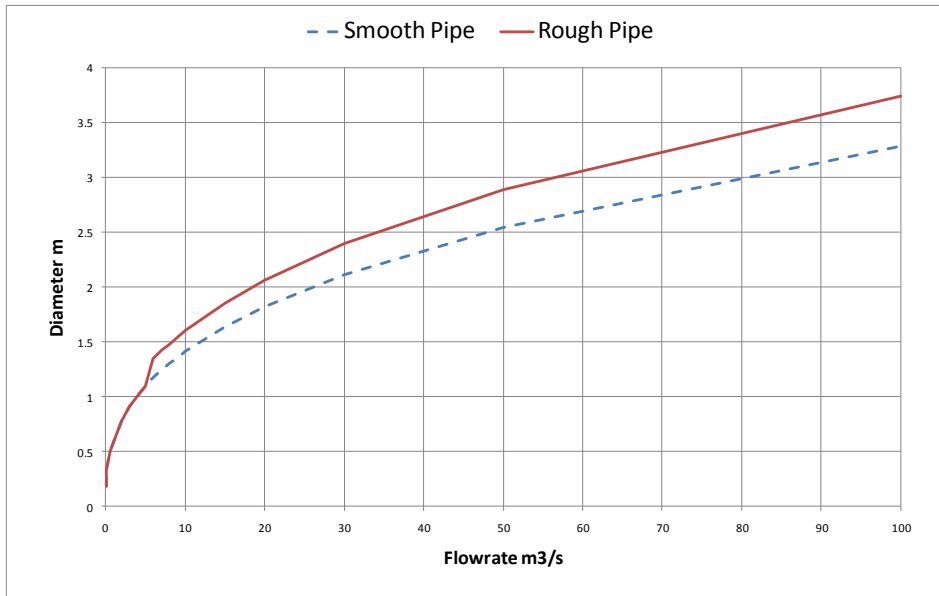
We can now assume a value of f (start with $f = 0.02$) and use it to calculate D , then use D to calculate V and Reynolds number, then use the Reynolds number to re-calculate f , and repeat to convergence. The value of f is selected from the appropriate phase depending on the value of R :

Initial f	f_1		GUESS	0.02
Improved guess f	f_2		Copy appropriate value	0.039811
Diameter	D	m	$= (f/0.0197)^{1/5}$	1.151085
Velocity	V	m/s	$= 7.26/D^2$	5.4792592
Reynolds number	R		$= VD\rho/\mu$	3969.4991
Laminar Friction Factor	f_{la}	$R < 2000$	$= 64/R$	0.0161229
Transitional Friction Factor	f_{tr}	$2000 < R < 4000$	$= 0.316/R^{0.25}$	0.039811
Turbulent Friction Factor	f_{tu}	$R > 4000$	$= (1/-2\log(2.51/(R \cdot f_2^{0.5})))^2$	0.04003

After 3 iterations the phase is transitional to turbulent and $f \sim 0.04$
 $\Rightarrow D$ is now $\sim 4x$ the previous solution (the flow is 100x)!

(Spreadsheet solution may be written from the formulas shown with formula iteration on)

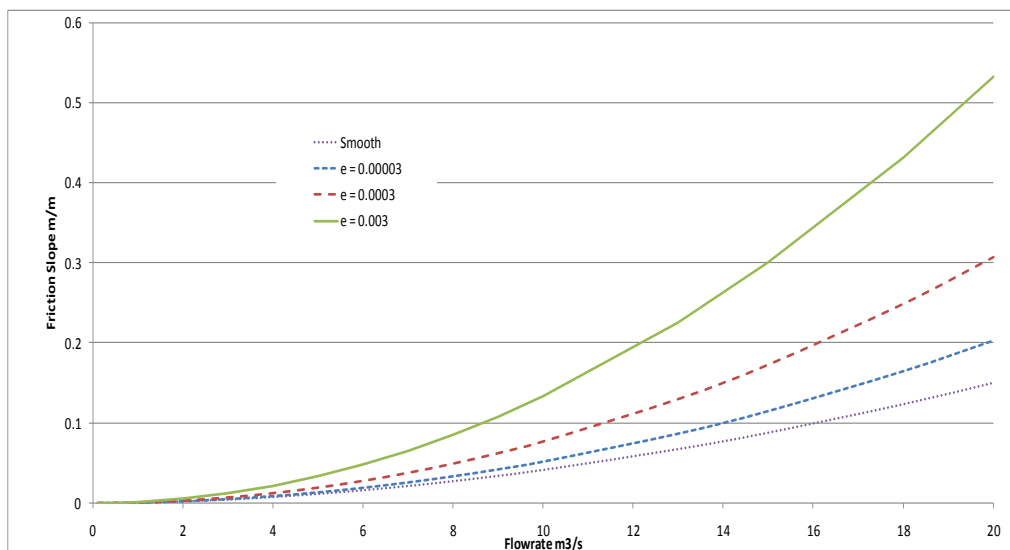
Comparing these two solutions, the small increase in diameter for a large increase in flowrate at the same headloss is worth noticing. At the turbulent transition for rough pipes, there is a sudden increase in friction factor and hence diameter for a given headloss.



Comparison of smooth and very rough ($\varepsilon = .06$ m) pipes for a constant friction slope $S_f = 0.05$ m/m (Heavy Machine Oil $\rho = 900$ kg/m³, $\mu = 1.43$ Pa.s)

Note from the Moody diagram that there is a *complete* or *fully developed turbulence* range at high values of R . In this range f tends to constant values for a given ε/D , ie for fixed diameter and roughness. Headloss in fully developed turbulence for a given pipe is a function of velocity head only.

The operating range of a typical industrial or drainage pipe is in fully developed turbulence.



Friction slope for 1m pipes of variable roughness (Water $\rho = 1000$ kg/m³, $\mu = 0.001$ Pa.s).

Graphical Method

As an alternative to iterative calculations, many still prefer to use the Moody diagram iteratively to determine a value of f . The technique is otherwise much the same, ie for given Q assume a value of Diameter, calculate Velocity and Reynolds number and use this to look up f . Use f to calculate headloss and check against available head. Adjust diameter and repeat to convergence.

Minor Losses; Inlets, outlets, bends, expansions/contractions, valves, pits.

In addition to pipe friction losses, which are uniformly distributed along the length of a pipe, there are losses at any point where the flow changes in direction or magnitude, ie wherever there is a reaction force acting on the fluid. This includes bends, joints, sudden or smooth expansions and contractions, valves, junctions, inlets and outlets to name a few basic controls in typical pipelines.

These *minor losses* (as they are collectively called) are usually estimated as a function of downstream kinetic energy, ie the second velocity term in the Bernoulli equation.

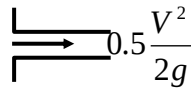
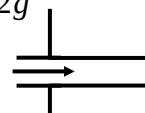
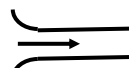
$$h_m = k \frac{V_2^2}{2g} \quad \text{Minor loss equation}$$

The loss coefficient k depends on the specific configuration. In some cases k can be calculated, but more usually these are based on measurement and calibration in the design process. It is usual to provide loss coefficients with manufactured pipeline parts.

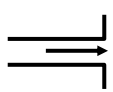
Loss Type

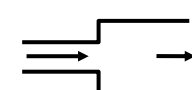
Average Head Loss (m) (Giles, 1982)

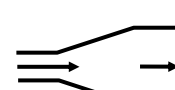
From reservoir or tank to pipe (entrance loss)

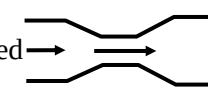
– flush connection.....	$0.5 \frac{V^2}{2g}$	
– projecting connection.....	$1.0 \frac{V^2}{2g}$	
– rounded connection.....	$0.05 \frac{V^1}{2g}$	

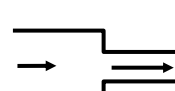
From pipe to reservoir or tank (exit loss)

– all connections.....	$1.0 \frac{V^2}{2g}$	
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Sudden Enlargement	$\frac{V_1^2 - V_2^2}{2g}$	
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Gradual Enlargement (see Table for K)	$K \frac{V_1^2 - V_2^2}{2g}$	
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Venturi Meters, Nozzles and Orifices	$\left(\frac{1}{c_v^2} - 1 \right) \frac{V^2}{2g}$	c_v as published 
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Sudden Contraction (See Table for K_c)	$K_c \frac{V_2^2}{2g}$	
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Elbows, fittings and valves	$k \frac{V_2^2}{2g}$	k as published
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Elbows, fittings and valves	k	Contraction		Gradual Enlargement K (Angle of Expansion)							
		D_1/D_2	K_c	4°	10°	15°	20°	30°	50°	60°	
45o bend	0.35 to 0.45	1.2	0.08	0.02	0.04	0.09	0.16	0.25	0.35	0.37	
90o bend	0.50 to 0.75	1.4	0.17	0.03	0.06	0.12	0.23	0.36	0.50	0.53	
Tee Junction	1.50 to 2.0	1.6	0.26	0.03	0.07	0.14	0.26	0.42	0.57	0.61	
Gate Valve	0.18 to 0.25	1.8	0.34	0.04	0.07	0.15	0.28	0.44	0.61	0.65	
Check Valve	3.0	2.0	0.37	0.04	0.07	0.16	0.29	0.46	0.63	0.68	
Angle Valve	5.0	2.5	0.41	0.04	0.08	0.16	0.30	0.48	0.65	0.70	
Globe Valve	10.0	3.0	0.43	0.04	0.08	0.16	0.31	0.48	0.66	0.71	
		4.0	0.45	0.04	0.08	0.16	0.31	0.49	0.67	0.72	
		5.0	0.46	0.04	0.08	0.16	0.31	0.50	0.67	0.72	

Fluid acceleration tends to be more efficient than deceleration, so loss factors for expansions tend to be larger than for equivalent contractions.

Contractions can be calculated by a contraction coefficient, which depends on the type of fluid. For water at 10-15°C Weisbach determined a contraction formula:

$$h_c = \left(\frac{1}{C_c} - 1 \right) \frac{V_2^2}{2g}$$

Weisbach Contraction Head Loss

A ₂ /A ₁	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C _c	0.624	0.632	0.643	0.659	0.681	0.712	0.755	0.813	0.892	1.000

A₂/A₁ the ratio of downstream and upstream pipe areas.

Software **See the online Tutorial “FluidFlow3 Software”**

Tutorial **Dimensional Analysis and Pipe Flow**