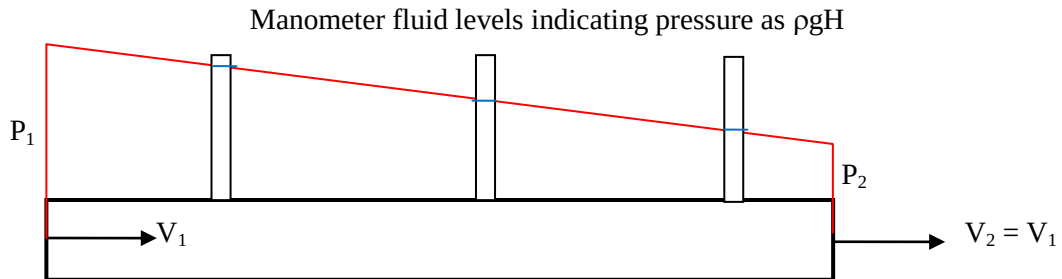


Lecture 21: Pipe Friction and Headloss

General Pipe Friction Head Loss

Pressure measurements in pipes show a uniform rate of pressure loss for constant pipe characteristics such as diameter and surface roughness:



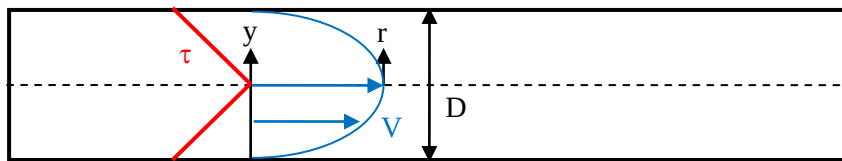
$$\text{Again } \frac{d\tau}{dy} = \frac{d}{dx}(P + \rho gH) = \rho g \frac{dH}{dx} = \text{constant } b \text{ (constant friction slope)}$$

$$\text{Or } \frac{d\tau}{dy} = b \quad \text{so} \quad \tau = by + c = \mu \frac{dV}{dy}$$

$$\text{Thus } V = \frac{b}{\mu} \frac{y^2}{2} + \frac{c}{\mu} y + d \quad \text{A parabolic equation as observed (Raleigh, Hagen, Poiseuille)}$$

In fact a wide range of solutions give the observed constant friction slope. The parabolic model uses the least assumptions and corresponds to laminar flow with no turbulence.

To find the values of b , c and d , a coordinate substitution replaces y with $r/2$ (radius position from centre of pipe, symmetric around the axis):



$$\tau = b \frac{r}{2} + c = 0 \text{ at } r = 0 \text{ so } c = 0$$

$$\text{but } \tau = b \frac{r}{2} = \mu \frac{dV}{dr}$$

$$\text{ie } V = \frac{b}{\mu} \frac{r^2}{4} + d = 0 \text{ at } r = \pm D/2 = R \text{ (R the pipe radius, not hydraulic radius)}$$

$$\text{hence } d = -\frac{bR^2}{4\mu}$$

$$\text{and } V = -\frac{b}{4\mu}(R^2 - r^2)$$

but $b = \frac{d\tau}{dy} = \rho g \frac{dH}{dx}$

so $V = -\frac{\rho g}{4\mu} \frac{dH}{dx} (R^2 - r^2) = -\frac{\rho g}{16\mu} \frac{dH}{dx} (D^2 - 4r^2)$ Hagen-Poiseuille Equation

Note that $-\rho g \frac{dH}{dx} = -\frac{dP}{dx}$ is the rate of pressure loss.

The negative sign in this equation indicates that the velocity is in the direction of *falling* pressure.

This equation relating flow velocity to pressure loss was derived from experimental data in 1839 (Raleigh) and “confirmed” with these theoretical considerations in 1856 (Hagen, Poiseuille).

We can use this to determine maximum and average velocities:

$$V_{max} = -\rho g \frac{dH}{dx} \frac{D^2}{16\mu} \quad \text{at } r = 0 \quad \text{and} \quad V_{mean} = \frac{V_{max}}{2} = -\rho g \frac{dH}{dx} \frac{D^2}{32\mu}$$

The average velocity V is the most easily measured (by flowrate and area) and provides a measurable relationship for the friction slope in laminar pipes:

$$\frac{-dH}{dx} = S_f = \frac{32\mu V}{\rho g D^2} \quad \text{Friction slope, laminar pipe flow } Re < 2000$$

Laminar and Turbulent Friction in Pipes

Further development of the friction slope equation was influenced by three ideas:

- The head loss should be proportional to the velocity head $V^2/2g$.
- The friction slope should be inversely proportional to diameter, not diameter squared.
- There should be an independent coefficient or *friction factor* f , expected to be a function of Reynolds' number, the key dimensionless variable relating viscous phenomena.

ie $h_f = f \frac{1}{D} \frac{V^2}{2g}$

where $f = f(Re) = f\left(\frac{VD\rho}{\mu}\right)$ f = Dimensionless Friction factor (function of Re)

(some texts use λ for f - check that they use the same units ie there is no conversion factor)

Darcy and Weisbach (1850) modified the laminar friction slope equation to isolate Reynolds number and the velocity head:

$$S_f = \frac{32\mu V}{\rho g D^2} = \frac{64\mu V^2}{2g\rho V D^2} = \frac{64}{Re} \frac{V^2}{2gD} = f \frac{V^2}{2gD}$$

where $f = f(Re) = \frac{64}{Re}$ Laminar pipe friction factor $Re < 2000$

Laminar pipe friction slope is thus proportional to velocity head and inversely proportional to Reynolds' number and diameter. The proportionality in the Hagen-Poiseuille equation is nominally to V/D^2 ; however, the friction factor f is more complex over its whole range and any simple proportionality tends to be masked.

The Darcy-Weisbach equation is now understood to apply over the whole range of laminar and turbulent flow, in both gases and liquids, so long as an appropriate value of f can be found: We are thus able to determine total friction headloss h_f in m for a given length L of pipe as:

$$h_f = S_f L = f \frac{LV^2}{2gD} \quad f = f(Re) \quad \text{Darcy-Weisbach Headloss Equations}$$

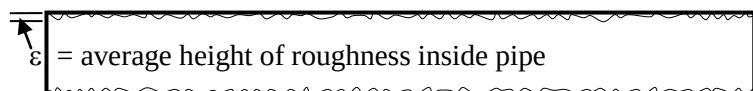
Three principal flow *regimes* have been experimentally identified for the determination of f :

Laminar:	$Re < 2000$	$f = \frac{64}{Re}$	(Darcy-Weisbach, 1850)
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Transitional:	$2,000 < Re < 4,000$	$f = \frac{0.316}{Re^{1/4}}$	(Blasius, 1913)
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Turbulent:	$Re > 4,000$	$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon}{3.7D} + \frac{2.51}{Re \sqrt{f}} \right)$	(Colebrook-White, 1937)
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Where ε is the roughness height, the average dimension (in metres) of roughness elements on the surface of the pipe (Nikuradse, 1932). (Some texts use k for roughness height)



Pipe with somewhat exaggerated roughness.

This three-regime model encompasses more than a century of experimental-theoretical work. Other models (such as the Hazen-Williams equation) are sometimes seen but these are now regarded as textbook or historical models and are seldom used in practice.

Reynolds' original laboratory work suggested a laminar-turbulent transition at about $Re = 10,000$. More recent evidence in practice suggests this occurs at around $Re = 4,000$. Care should be taken in the transitional regime between $Re = 2,000$ and $4,000$.

Non-circular cross-sections

Ducts and culverts frequently have rectangular rather than circular cross-sections. For these types of pipes it is common practice to use the hydraulic radius of the cross section, with a conversion to equivalent diameter:

Hydraulic radius = ratio of cross-sectional area to wetted perimeter:
 $= A/P$

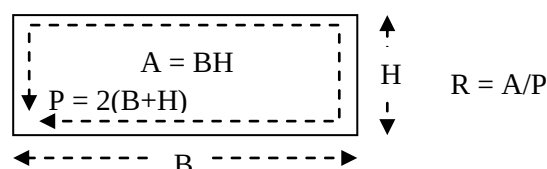
For a box section, $A = \text{width} \times \text{depth}$, $P = 2(\text{width} + \text{depth}) \Rightarrow R = wd/(2(w+d))$

For a circular section $A = \pi D^2/4$, $P = \pi D \Rightarrow R = D/4$

An equivalent *hydraulic diameter* for a given hydraulic radius is thus $D_e = 4R$. The Darcy-Weisbach formula is then written as:

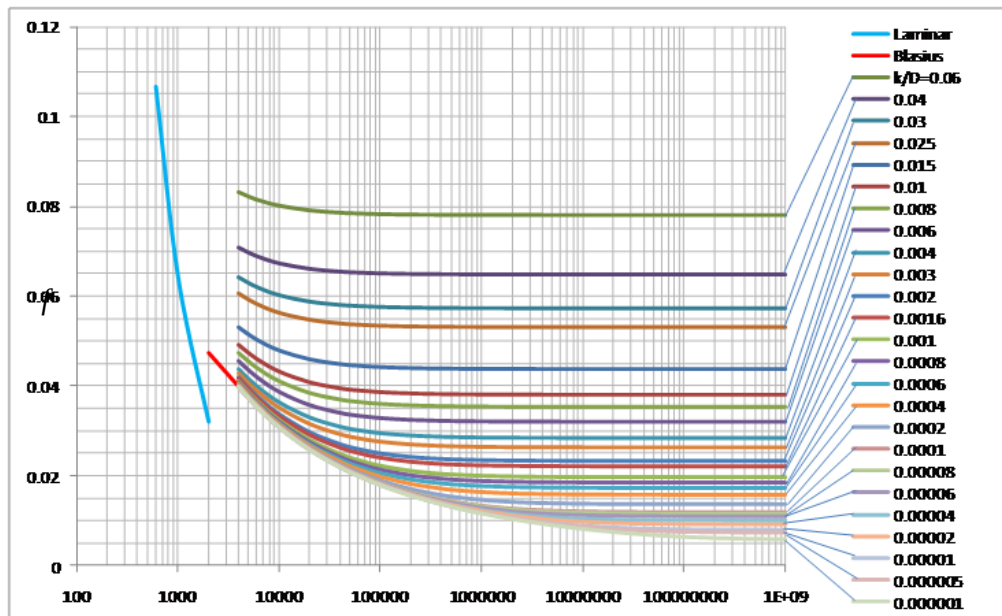
$$h_f = \frac{fLV^2}{2gD_e} = \frac{fLV^2}{8gR}$$

Headloss in non-circular sections



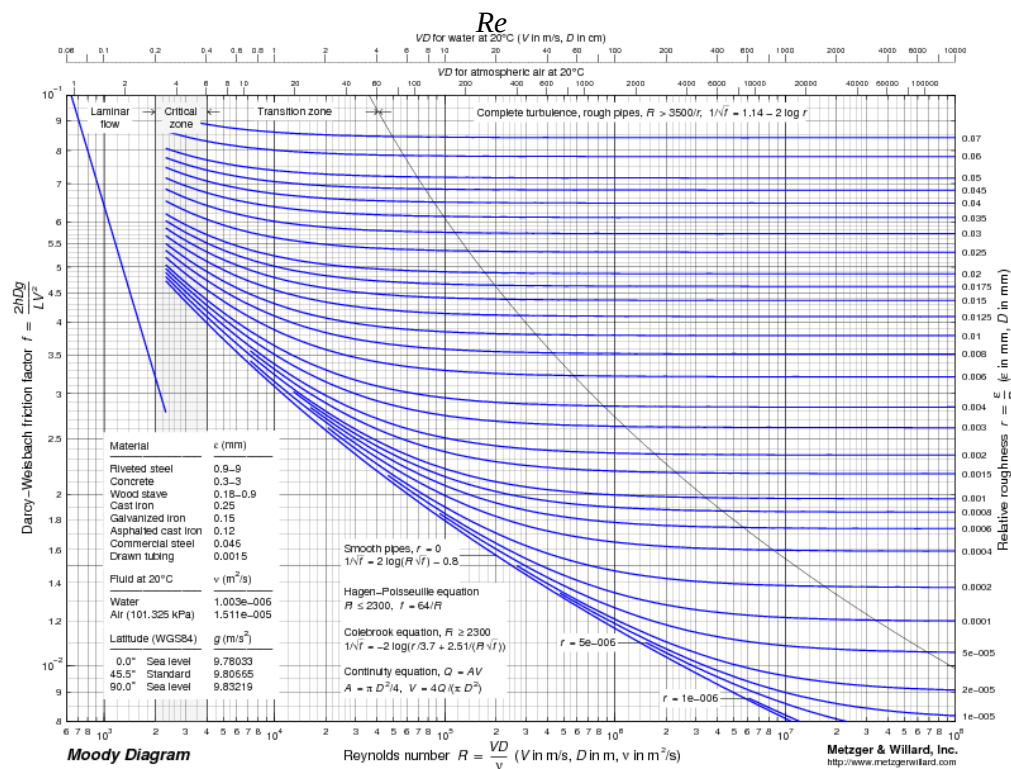
Graphical Method

An alternative graphical representation of f , known as a *Moody diagram* (Moody, 1944) (Giles 1982) provides the same level of information. Moody diagrams are today generally used to calibrate computational models based on the three-regime model:



Three-regime

model (Excel spreadsheet).



Moody diagram (Giles, 1982) : $h_f = f \frac{LV^2}{2gD}$ (see larger version in Tables and Graphs online)

Note that the friction factor varies at most over the range 0.01 to 0.1, usually much less, ie there is a single order of magnitude of potential variation in headloss due to the friction factor. A much larger potential for variation in a pipe of given diameter is in the velocity head. Further potential is available at the design stage by varying the diameter. Pipe length is usually a fixed constraint.

Reading

Streeter and Wylie 1981 (Reading 10) Section 5.2 and 5.10