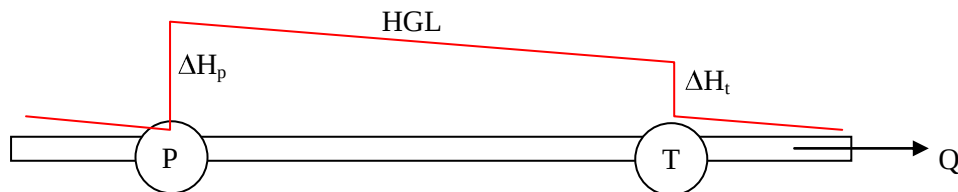


Lecture 16: Turbomachinery

Theory of Turbomachinery

Turbomachines transmit power to or from a flowing fluid by adding or extracting energy:

Fluid Power = $\rho g Q \Delta H$ where ΔH may be positive or negative (Section 4.3).



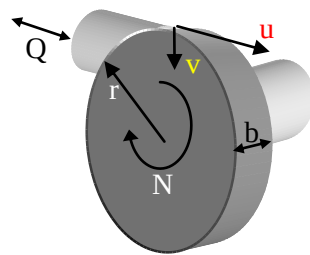
There is a specific relationship between head and discharge for any given pump or turbine. Finding the right size and speed for a required head and flowrate is the basic design task.

Turbines can in many respects be regarded as pumps operating in reverse, except for the irreversibility of losses. A pump produces less head than predicted by linear theory, while a turbine extracts more head, due to friction losses.

“Black Box” Theory of Turbomachines: We don’t need to know all the details of momentum transfer in pumps and turbines (as per Supplementary Lecture 1) to do accurate design or modelling. We can measure the performance of test units, construct charts of head, discharge and efficiency, and use this data to accurately scale up to any size pump or turbine we need.

Only two parameters regarding flow in a turbomachine are needed:

- Lateral velocity u : the flow velocity around the circumference of the machine
 - Radial velocity v : the outward velocity of flow at the circumference
- $u = rN$ r = pump or turbine impeller radius
 N = speed in rads/second
 $v = Q/2\pi r b$ Q = flowrate
 b = pump or turbine depth



These measurements are for any geometry of radial or axial pump or turbine, not limited to the dimensions shown in the diagram as an example. Machines with similar geometry over a range of different sizes are called a *homologous turbomachine series*, which can be compared and modeled by a single set of performance curves.

In a homologous series, both r and b are proportional to Diameter (same geometry):

$$\begin{array}{ll}
 u = kDN & (kD = r) \\
 v = kQ/D^2 & (k/D^2 = 1/2\pi r b)
 \end{array}$$

A comparison at kinematically similar flowrates (smaller flow for smaller diameter) between units of a homologous series should show the same pattern of streamlines, thus the ratio of v/u at any point is constant for all units in a series:

$$v/u = (kQ/D^2)/(kDN) = kQ/ND^3 = \text{constant}$$

Without specifying the value of any of these constants, we have

$Q/ND^3 = \text{constant}$ for all units of a homologous series at kinematically similar flowrates.

$$\frac{Q}{ND^3} = \text{constant} \quad \text{Discharge relationship for a homologous series of pumps or turbines}$$

A simplified Bernoulli equation provides a relationship between pump or turbine head and discharge:

$$Q = kA\sqrt{2gH}$$

Where area A is proportional to D^2 :

$$\text{Thus } Q = kD^2\sqrt{H}$$

But from the discharge relationship, we have $Q = kND^3$

$$\text{So } kND^3 = kD^2\sqrt{H}$$

Squaring both sides we obtain

$$kN^2D^6 = kD^4H$$

Or $H/N^2D^2 = \text{constant}$ for all units of a homologous series at kinematically similar flowrates

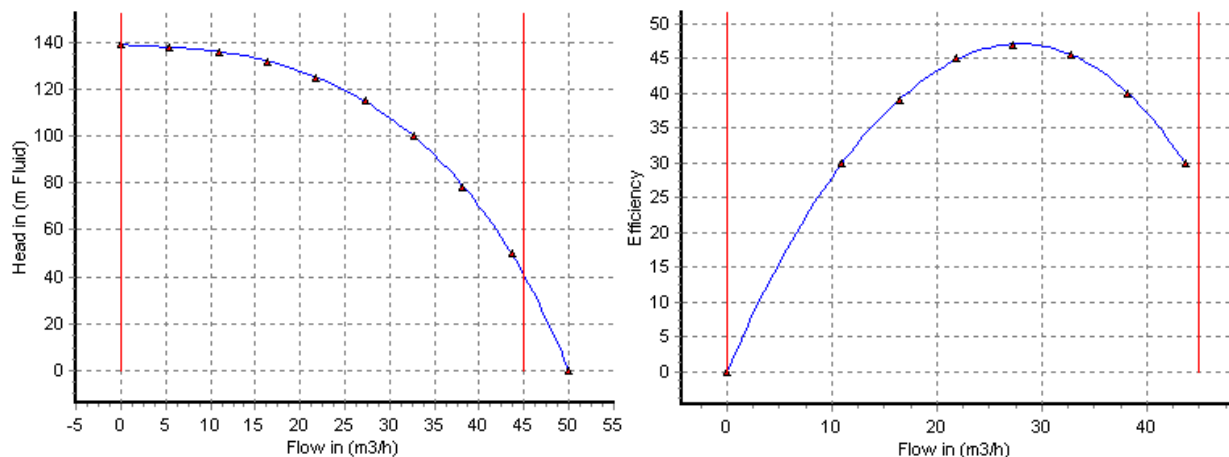
$$\frac{H}{N^2D^2} = \text{constant} \quad \text{Head relationship for a homologous series of pumps or turbines}$$

These two relationships model the variation of head H , discharge Q and diameter D for kinematically similar speeds N in homologous units of different sizes. The formulae are not dimensionless and depend on the units used. The most common units are RPM, m and m^3/s . Low discharge performance tests may sometimes be reported in m^3/hour .

This two-equation model is not entirely accurate across the full range of sizes. The constant k in the simplified Bernoulli's equation is not constant, but varies with Q , larger for smaller Q , due to losses. This means that smaller units are less efficient than larger ones. This scale effect is ignored in the general turbomachinery theory but is included in more precise calculations and in performance measurements.

Losses are reflected in particular in the efficiency of pumps and turbines at different speeds.

Example: Performance data on a 220mm radial pump at 3000 rpm are shown in the two graphs. What size and speed are required to produce 10 m³/h for 40 m head at best efficiency?



Capacity and efficiency curves for a 220mm centrifugal pump at 3000 RPM (image from FluidFlow © Flite Software Ltd)

This is a fairly typical question, given that the design parameters of any pipeline will include a target headloss and flow rate. *Take care with the units:* You could convert the speed to radians per second, but in fact this isn't necessary. It is essential however to have the diameter in the same units as for head and discharge, ie m, m³/h etc.

From the efficiency graph, the best efficiency at 47% is around 27 m³/h. On the capacity curve this equates to a head of about 115 m. Best efficiencies can be higher or lower than this for different designs. Pump efficiencies vary around 60%, turbines around 90%.

The available performance data include the chart speed of 3000 RPM, the prototype diameter of 220mm, and the flowrate and head at best efficiency of 27 m³/h and 115 metres. The design targets of Q = 10 m³/hour and H = 40 metres head lead to a unique pump diameter and speed:

Applying the performance data to the Head and Discharge relationships we can calculate constant values for the homologous series:

- (1): $Q/ND^3 = 27/((3000 \times 0.22^3) = 0.845$
- (2): $H/N^2D^2 = 115/(3000^2 \times 0.22^2) = 2.64 \times 10^{-4}$

We need to solve for a design N and D given a target Q and H: Begin by eliminating N:

- (3) = (1)²: $Q^2/N^2D^6 = 0.845^2 = 0.714$
- (4) = (2)/(3): $HD^4/Q^2 = 2.64 \times 10^{-4} / 0.714 = 3.7 \times 10^{-4}$

Use equation 4 to solve for D given the target flowrate and head:

Diameter: $D = (3.7 \times 10^{-4} \times 10^2 / 40)^{0.25} = 0.174 = 174 \text{ mm}$

Lastly use either equation 1 or 2 to solve for N given the design diameter, head and flowrate:

Speed: $N = 10 / (0.845 \times 0.174^3) = 2246 \text{ RPM}$

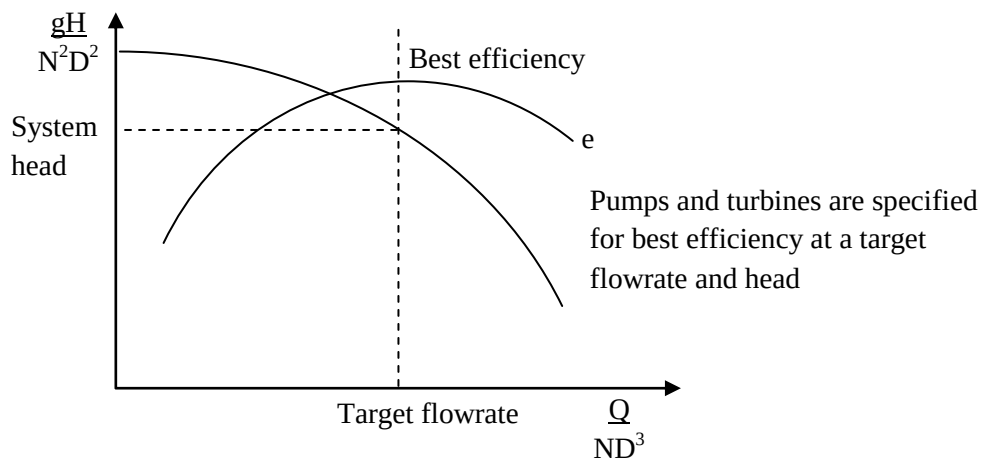
A 170 mm pump from this series operating at 2250 RPM should do the job.

We can generalise that solution as a useful design formula:

Let $Q_r = Q/ND^3$
 And $H_r = H/N^2D^2$ (Discharge and Head relationships based on performance data)
 Then $D = (H_r/Q_r^2 \times Q^2/H)^{0.25}$
 And $N = Q/(Q_r D^3)$

$$D = \left(\frac{H_r}{Q_r^2} \frac{Q^2}{H} \right)^{0.25} \quad \text{and} \quad N = \frac{Q}{Q_r D^3} \quad \text{Diameter and speed } N \text{ for Head } H \text{ and Discharge } Q$$

Performance data can be presented in a dimensionless form as a function of head and discharge relationships. This *characteristic curve* can be used to determine the actual head-discharge curve for any unit in a homologous series:



Specific Speed

The *specific speed* of a homologous series of pumps or turbines is the speed at which $H = Q = 1$. Pumps and turbines are designed for particular situations based on their specific speed:

- Large discharge at low head requires a large specific speed $N_s \sim 750-1750$ (Axial flow)
- Small discharge at large head requires a small specific speed $N_s \sim 75-1000$ (Radial flow).
- Intermediate situations: use mixed flow system.

The specific speed for a series can be derived from a single unit's performance data at best efficiency:

$$Q_r = \frac{Q_d}{N_d D_d^3}$$

$$H_r = \frac{H_d}{N_d^2 D_d^2}$$

Solve for D:

$$D = \sqrt[3]{\frac{Q_d}{Q_r N_d}} = \sqrt[2]{\frac{H_d}{N_d^2 H_r}}$$

Q_r and H_r are constants for a homologous series: we can rearrange this to have all the constants on the RHS, and all the variables on the LHS:

$$\frac{Q_d^{1/3}}{N_d^{1/3}} \frac{N_d}{H_d^{1/2}} = K \text{ which leads to } \frac{\sqrt{Q_d} N_d}{H_d^{3/4}} = K^{3/2} = K$$

K is a constant for all units of a series: This constant is known as the specific speed N_s :

ie
$$N_s = \frac{N \sqrt{Q}}{H^{3/4}} \quad \text{Pump Specific Speed} \quad \text{Units RPM, m, m}^3/\text{s}$$

This is often used as an alternative method for calculating a target speed given a target head H and discharge Q. Given the specific speed N_s as a design specification for the homologous series, the target speed is obtained by:

$$N = \frac{N_s H^{3/4}}{\sqrt{Q}}$$

The target diameter can then be determined by either the Head or Discharge relationship.

Turbines use a slightly different formula for specific speed based on their Power:

For a turbine,
$$\frac{Power}{QH} = constant$$

Thus
$$N_s = \frac{N \sqrt{Q}}{H^{3/4}} = \frac{N \sqrt{Power / H}}{H^{3/4}} = \frac{N \sqrt{Power}}{H^{5/4}} \quad \text{Turbine Specific Speed}$$

Reading

Fox and McDonald 1992 (Reading 8) Section 11.4