

Lecture 14: Applications Continued: Flow Measurement Technology

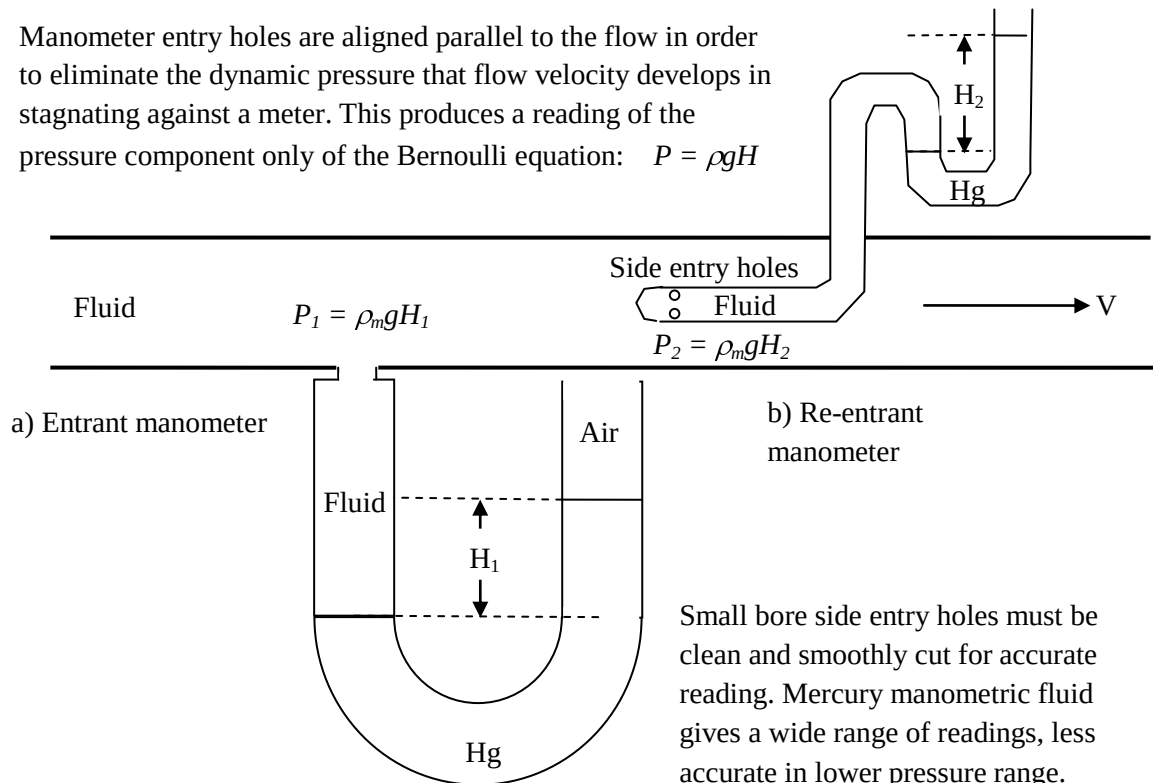
Flow instruments typically use energy principles to measure three parameters of interest:

- Pressure
- Velocity
- Flowrate

Pressure Measurement

Mercury Manometers: The principle of pressure measurement in a flow is the same as for static pressure, taking care to eliminate dynamic pressures due to the measurement process itself:

Manometer entry holes are aligned parallel to the flow in order to eliminate the dynamic pressure that flow velocity develops in stagnating against a meter. This produces a reading of the pressure component only of the Bernoulli equation: $P = \rho g H$



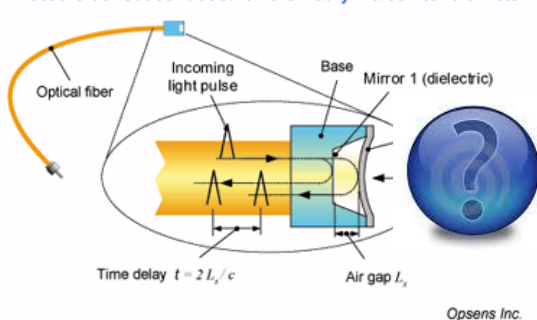
Transducers

Pressure transducers are devices designed to flex with pressure, the degree of movement is read electronically and processed as a pressure reading. Recent technology uses optical fibre and interferometry to measure the displacement of a diaphragm:

<http://www.sensorland.com/Images/HowPicAccurate02.gif>

Applications:

Pressure transducer based on the Fabry-Perot interferometer

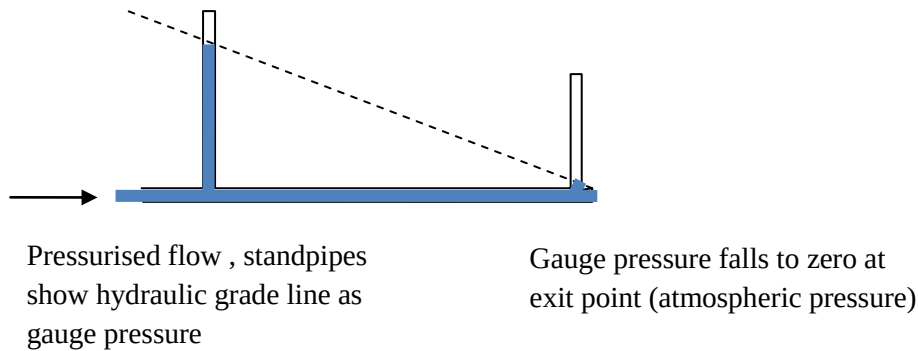


- Pressure gauges in major pipelines
- Fuel line pressure in aircraft, vehicles

Thought Experiment: Standpipe Hydraulics

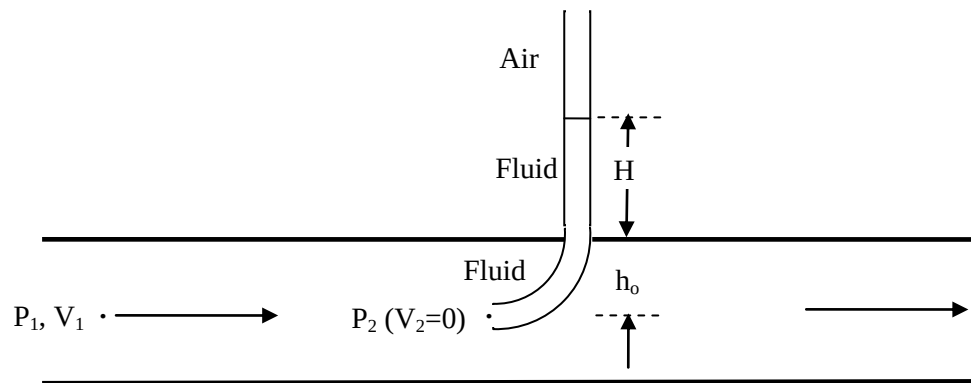
Your garden hose is a typical pressurised water supply system. A remarkable thing about such systems is that the pressure always falls to zero gauge pressure (or absolute atmospheric pressure) at

the mouth, where it flows out into the air. Try fitting vertical PVC tubes to your garden hose to see this in action.



Velocity Measurement: Anemometers

Pitot Tubes: These devices use the stagnation pressure of a flow velocity stalling against a meter:



The flow velocity stagnates against the pitot entry hole, with static and dynamic pressures combining as $P_2 = \rho g(H + h_0)$ (the pitot manometric fluid is the same as the working fluid in the pipe)

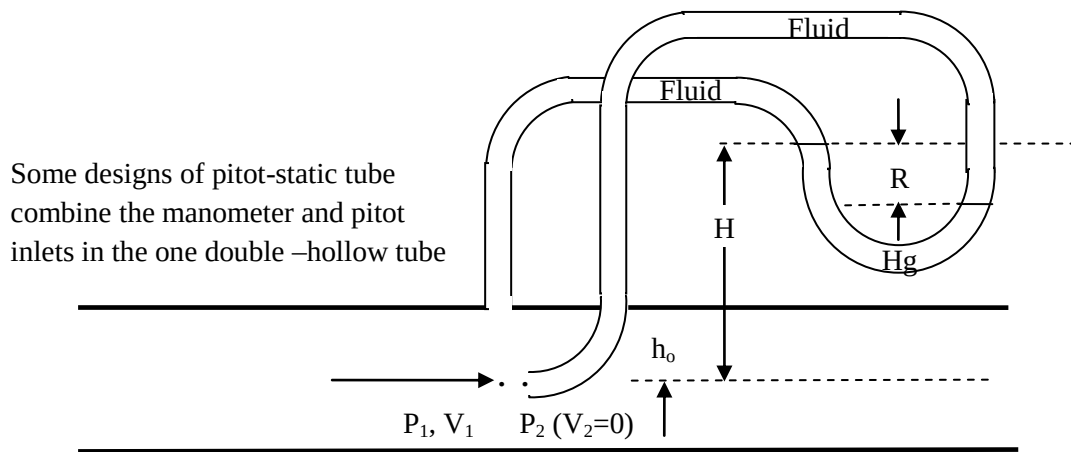
Applying Bernoulli from 1 to 2:

$$\frac{V_1^2}{2g} + \frac{P_1}{\rho g} = 0 + \frac{P_2}{\rho g} = H + h_0$$

Thus $V_1 = \sqrt{2g(H + h_0) - 2P_1/\rho}$

The simple pitot tube thus requires a manometer and a calculation step for determination of velocity.

A direct measurement can be obtained by the difference of two manometer measurements using a pitot-static or *Prandtl* tube:



Balancing pressures from 1 to 2 via the pitot-static line we have

$$P_2 = P_1 - \rho g H + \rho_m g R + \rho g (H - R) = P_1 + \rho_m g R - \rho g R$$

$$= P_1 + \rho g R \left(\frac{\rho_m}{\rho} - 1 \right) \text{ where } \frac{\rho_m}{\rho} \text{ is the ratio of manometer to working fluid density}$$

Bernoulli's equation from 1 to 2 gives us

$$\frac{V_1^2}{2g} + \frac{P_1}{\rho g} = \frac{P_2}{\rho g}$$

Thus
$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = R \left(\frac{\rho_m}{\rho} - 1 \right)$$

and

$$V_1 = \sqrt{2gR \left(\frac{\rho_m}{\rho} - 1 \right)}$$

Pitot-static Equation

Instruments are typically calibrated with a scale that can be read directly from a difference in fluid heights, or are processed via transducer.

Applications: Aircraft airspeed instruments

Gas pitot tubes

Pitot tubes for compressible gases, for example in aircraft airspeed indicators, make additional use of the temperature changes (increases) that occur in compression. While this requires a thermodynamic derivation beyond the scope of this course, it is noted here:

A gas entering a pitot-static stagnation inlet undergoes rapid compression with no significant heat transfer or friction and is thus treated as isentropic, warming on compression. Thermometric devices in the free flow (manometer inlet T_1) and stalled air (pitot inlet T_2) would read:

$$\frac{V^2}{2} = C_p T_1 \left(\frac{T_2}{T_1} - 1 \right) \quad C_p = 1004 \text{ J/kg.K (Air)} \quad \text{Thermometric pitot-static tube.}$$

Temperatures in Kelvin

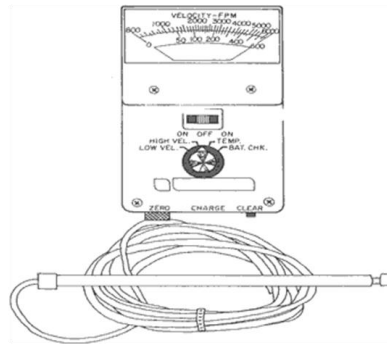
Ideally this would be cross-read with pressure measurements at the same inlets.

It is now known that the crash of Air France Flight 446 in 2009 was caused in part by a faulty thermal pitot tube airspeed reading.

Other types of Velocity gauges:



Cup and gauge anemometer



Hot-wire anemometer

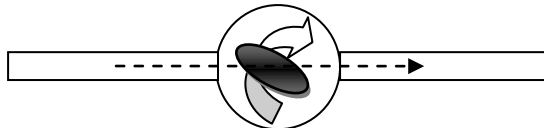


Ventimeter: for airflow strengths from light breeze to gale force.

<http://weblogs.newsday.com/news/local/longisland/politics/blog/anemometer.jpg>

Flowrate (Discharge) Measurement

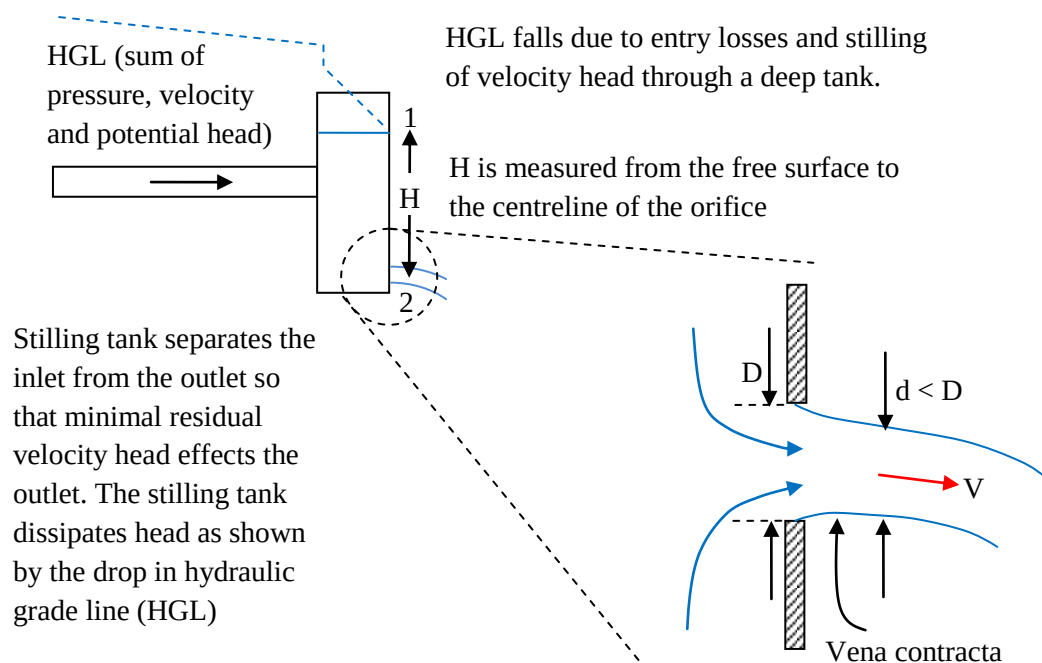
Wobble Meters:



Bead fitted inside spherical chamber, flowrate causes precession, motion is connected to a gearing system and read/processed electronically. Often used in fuel pumps.

Orifices:

A pipeline discharging via an orifice with a deep stilling tank allows a free head to develop, providing a useful indication of the flowrate. The head H is proportional to the velocity head in the orifice, which can be converted to a discharge by the orifice area and a contraction factor:



Streamlines converge on the outlet, continuing to contract after the orifice. This region is known as the *vena contracta*, which becomes approximately parallel (after allowing for fall) with a diameter less than the orifice diameter. At this point the average velocity V is proportional to the basin head and discharge is proportional to the contracted area:

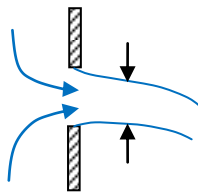
Again applying Bernoulli from 1 to 2:

$$0 + 0 + H = 0 + \frac{V^2}{2g} + 0 \quad (\text{pressure} + \text{velocity} + \text{head terms}) \quad \text{thus} \quad V = \sqrt{2gH}$$

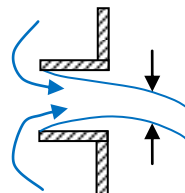
The flow area corresponding to this velocity depends on contraction and loss factors that produce the vena contracta: These are collected together in a discharge coefficient C_d :

$$Q = C_d \frac{\pi D^2}{4} \sqrt{2gH} \quad C_d \sim 0.5 - 0.6 \quad \text{Orifice Discharge Equation}$$

Orifice discharge coefficients depend on the specific type of orifice:

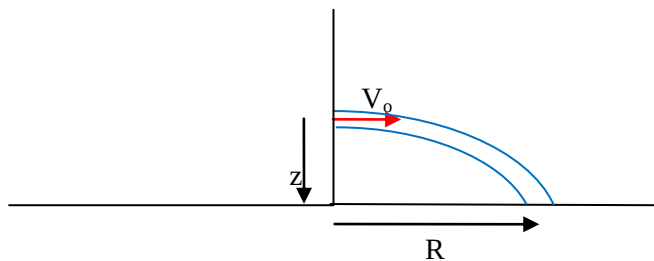


Square: $C_d \sim 0.6$



Re-entrant: $C_d \sim 0.5$

The exit velocity can be estimated by the *range* of the outflow jet, ie using the projectile equation:



$$R = V_o t \quad \text{and} \quad z = \frac{1}{2} g t^2$$

Eliminating t ,

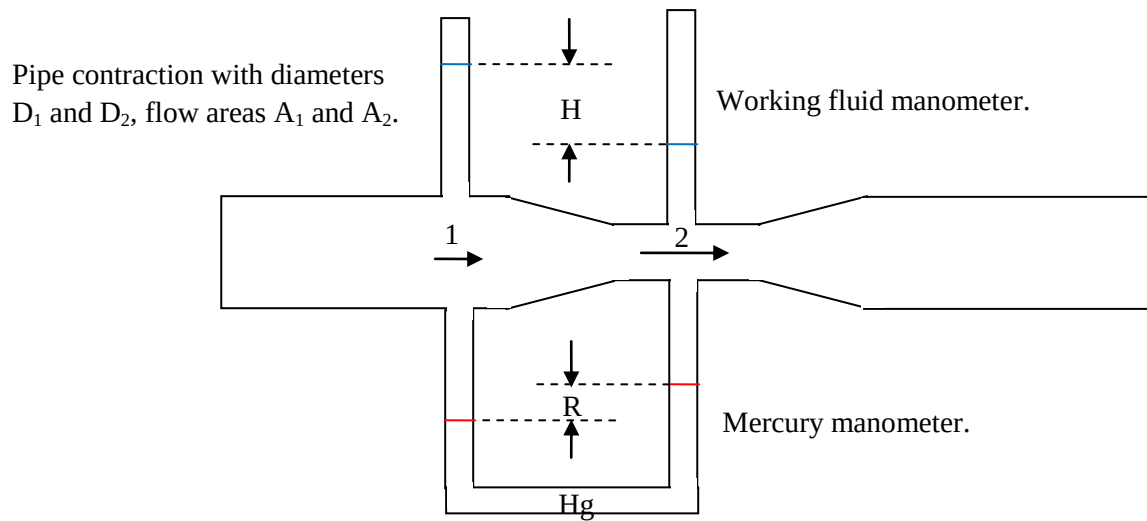
$$\frac{R^2}{z} = \frac{2V_o^2}{g}$$

$$\text{Or} \quad V_o = R \sqrt{\frac{g}{2z}}$$

Again, a discharge coefficient C_d can be applied to the area of the outlet for a good estimate of discharge.

Venturi Meters

Venturi meters compare static pressures at two different flow velocities:



For the working fluid manometer we have:

$$\frac{V_1^2}{2g} + H_1 = \frac{V_2^2}{2g} + H_2 \quad \text{or} \quad \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = H_1 - H_2 = H$$

But $V_1 A_1 = V_2 A_2$

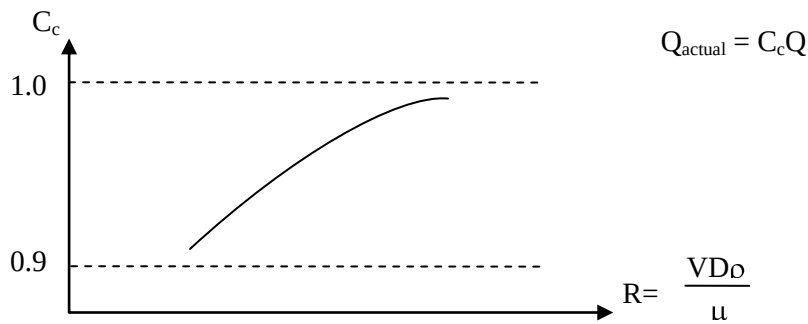
or $V_2 = \frac{V_1 A_1}{A_2}$

So $\frac{V_1^2}{2g} \left(\frac{A_1^2}{A_2^2} - 1 \right) = H$ or $V_1 = \sqrt{\frac{2gH}{\frac{A_1^2}{A_2^2} - 1}} = A_2 \sqrt{\frac{2gH}{A_1^2 - A_2^2}}$ and $Q = A_1 A_2 \sqrt{\frac{2gH}{A_1^2 - A_2^2}}$

The mercury manometer uses a construction similar to the pitot-static tube to arrive at:

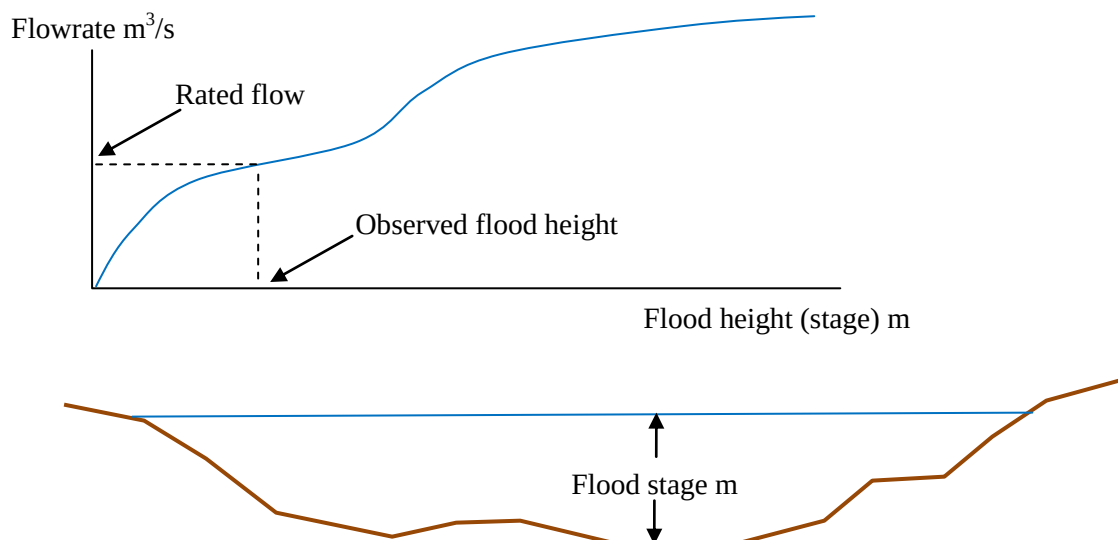
$$V_1 = \sqrt{\frac{2gR \left(\frac{\rho_m}{\rho} - 1 \right)}{\frac{A_1^2}{A_2^2} - 1}} = A_2 \sqrt{\frac{2gR \left(\frac{\rho_m}{\rho} - 1 \right)}{A_1^2 - A_2^2}} \quad \text{and} \quad Q = A_1 A_2 \sqrt{\frac{2gR \left(\frac{\rho_m}{\rho} - 1 \right)}{A_1^2 - A_2^2}}$$

Venturi meters are normally provided with a contraction coefficient that depends on flowrate: a partly iterative process. To a good approximation at reasonable flowrates $C_c = 1$



Measurement of open channel flow

Open channel flows are typically metered by gauging stations that measure flood height or *stage* at a control point, typically a natural or man-made weir or overflow. Control points are not significantly affected by upstream or downstream flow conditions and provide a consistent correlation of flow height with flowrate, known as a *rating curve*:



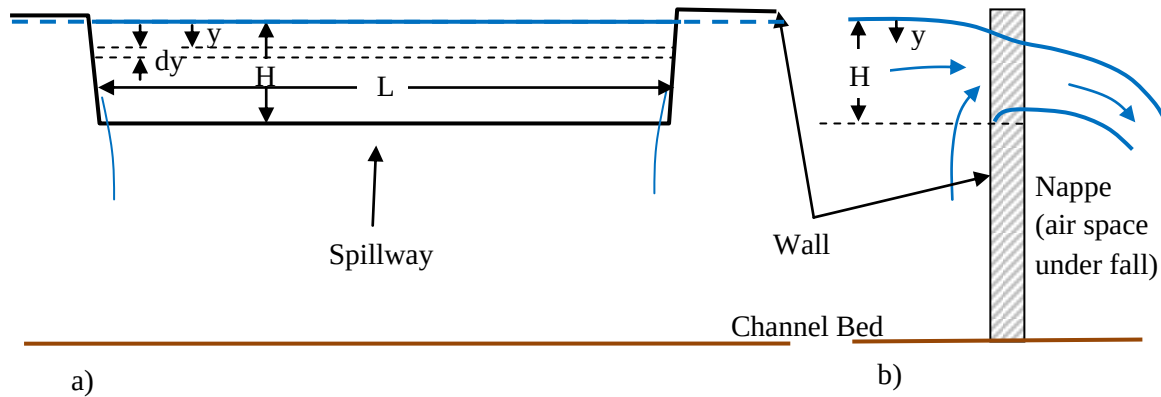
Typical rating curve for a channel. Cross-section view shows flood stage at a given flow.

Rating curves are constructed by multiple observations of flood stage and flowrate measured by other means eg flow velocimetry across the section or flow information provided by a water operator.

The shape of a rating curve can give useful information about the shape of the cross section, for example steeper regions of the rating curve indicate that a channel spills into a broader channel or floodplain with a large increase in flowrate for only a small increase in stage.

Weirs

Artificial weirs provide the most consistent correlation between flowrate and flood stage. The rating curve for a weir can usually be developed theoretically by simple Bernoulli methods, with at most an experimentally determined contraction factor:



Simple sharp-crested weir; a) front view, b) side view.

To calculate the flow rate across the weir we integrate elements of area $\times$ flow velocity across the width and depth of the flow. The flow velocity at a given depth y is assumed to vary as per the familiar velocity head equation:

$$V_y = \sqrt{2gy} \quad ; \quad dQ = V_y dA = \sqrt{2gy} L dy$$

Assuming a contraction coefficient C and integrating we have:

$$Q = C \int_0^H \sqrt{2gy} L dy = CL \sqrt{2g} \int_0^H \sqrt{y} dy = CL \sqrt{2g} \left[\frac{2}{3} y^{3/2} \right]_0^H = CL \sqrt{2g} \frac{2}{3} H^{3/2}$$

With $C \sim 0.62$ for a free nappe, sharp-crested weir, we have

$$Q = 1.83 L H^{3/2} \quad \text{Sharp-crested weir equation}$$

If the channel flow velocity is significant (shallow headwater upstream of weir) then the velocity head may need to be taken into account as part of the total head:

$$H_t = H_1 + \frac{\alpha V_1^2}{2g} \quad \text{and} \quad Q = 1.83 L H_t^{3/2} \quad \text{Shallow channel weir equations}$$

$\alpha = 1.4$ Energy correction factor

The shallow weir equations can be solved by iteration: guess a value of flow velocity V_1 (say 1m/s) and calculate H_t and Q , then recalculate V_1 based on Q and the flow area of the channel. Repeat and improve your guess until the solution converges with V_1 . Best practice is to have relatively still water behind the weir.

For narrow spillways a correction for end effects (an additional contraction factor) should be applied:

Calculate an *effective spillway length* L' :

$$L' = L - 0.2H \quad Q = 1.83 L' H^{3/2} \quad \text{Narrow weir equation}$$

The spreadsheet solution below can be written from the formulas shown: Note that the Excel Options Formulas must be set to iterative calculations.

Spillway Length	L	m		5
Weir Head	H	m		0.5
Effective Length	L'	m	=L-0.2H	4.9
Channel Depth at spillway	D	m		2
Channel Width	W	m		4
Channel Velocity	V	m/s	=Q/(W(D+H))	0.324191
Total Head	Ht	m	=H+1.4V ² /2g	0.507499
Flowrate	Q	m ³ /s	=1.83L'Ht ^{3/2}	3.241907
(Channel Depth at spillway: the depth of channel below the spillway level)				

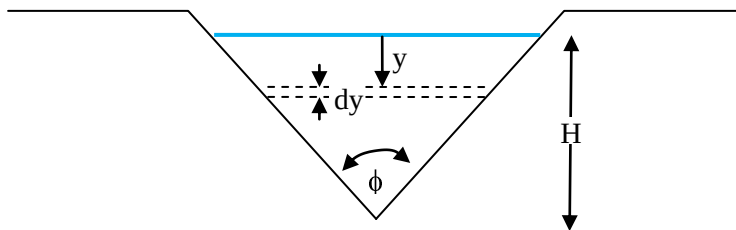


Thought Experiment: Weir-Orifice Transitions

Your sink is an ideal fluid mechanics laboratory. For example, at low flow rates the plughole acts like a weir, ie the hole is open to the air and the water forms a critical depth as calculated by the weir equations. At higher flow rates the water level increases until the plughole suddenly drowns, ie it fills with water and behaves more like an orifice. We say the behaviour *transitions* from weir flow to orifice flow. The trouble is there's no law to say that weir and orifice flows must overlap or neatly baton-change at some agreed water level. So what actually happens?

V-notch weirs

Commonly used in laboratory applications for accurate measurement of small flow rates. V-notch weirs have a precise rating curve, derived with flow width L varying with depth y:



$$Q = C \int_0^H \sqrt{2gy} L dy = C \sqrt{2g} \int_0^H \sqrt{y} \cdot 2(H-y) \tan\left(\frac{\phi}{2}\right) dy = C \sqrt{2g} \tan\left(\frac{\phi}{2}\right) \frac{8}{15} H^{5/2}$$

Note that H is now raised to the power of 5/2, so the shape of the V-notch rating curve is systematically different to that of a crested weir.

With $C \sim 0.58$ (higher contraction effect due to narrow notch) we have

$$Q = 1.37 \tan\left(\frac{\phi}{2}\right) H^{5/2} \quad \text{V-notch weir equation} \quad \text{For a typical } 90^\circ \text{ notch, } \tan(\phi/2) = 1.$$

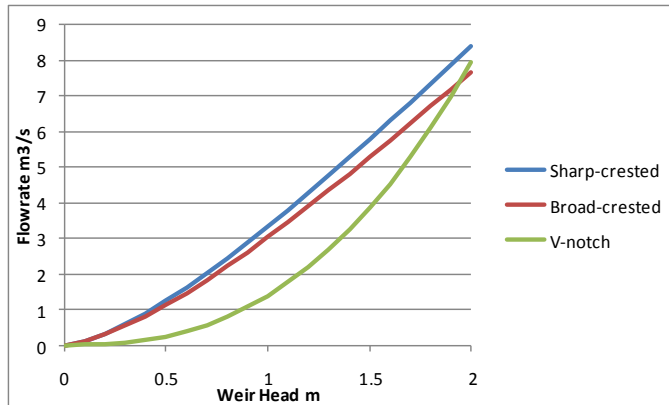
Broad-crested weirs

Weirs are often massive constructions or even roadway overflows that cannot be designed with a sharp edge or free nappe. Often the overflow “spillway” is not a horizontal section but includes the shape of the bank in its cross-section.

The flow over a broad-crested weir or road culvert is a varying open channel flow with relatively still water above critical depth and a rapid flow region downstream.

For a limited range of simple broad-crested weirs, a first-order model is similar to the sharp-crested weir:

$$Q = 1.67LH^{3/2} \quad \text{Broad-crested weir equation}$$



Comparison of rating curves for 3 weir types: 2-m spillway sharp-crested and broad-crested weirs, and a V-notch weir. The V-notch weir is unrealistically large (as lab instruments they are rarely manufactured to a head capacity of 2 m) but gives an indication of the different form of rating curve.

Reading

Giles 1982 (Reading 7)

Tutorial

Mass and Energy Conservation and Applications