

Lecture 6: Static Pressure and Structural Design

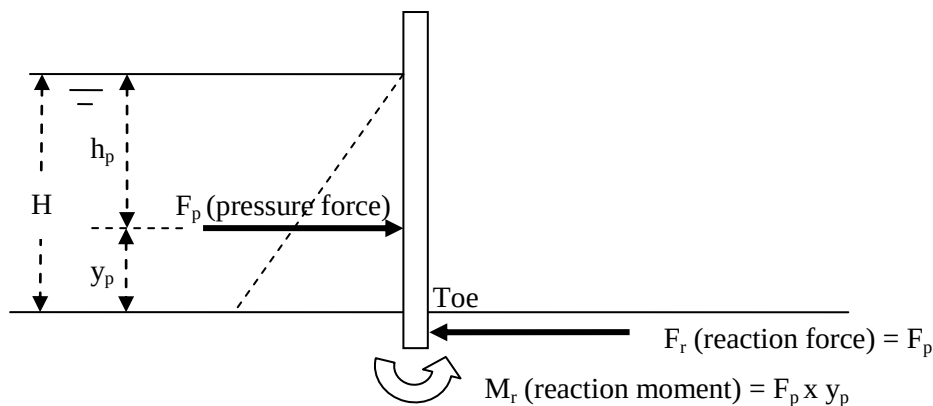
Hydrostatic Pressure, Reaction Forces and Reaction Moments

To determine the stability of a fluid-retaining structure it is necessary to calculate the resultant or *net force* due to pressure. This can then be multiplied by the leverage of worst case failure points to determine the net *overturning moment*. Overturning often bears a more significant risk of failure than modes that are due to force alone.

This complex subject can be simplified by breaking the problem into horizontal and vertical forces, which are recombined by rectilinear force vectors.

Vertical Walls and Horizontal Pressure Forces

The simplest type of water-retaining structure is a vertical wall: we consider forces at the base or *toe* of the wall, which is an *overturning failure point*:



The wall supports a hydrostatic pressure force:

$$F_p = \bar{P}A_p$$

The average pressure is half the maximum pressure:

$$\bar{P} = \rho g \frac{H}{2}$$

It is usual to work in terms of quantities per metre width of wall: Thus the wall area per metre width is the height of wall:

$$A_p = H \quad \text{Pressure Face Area per metre width of wall}$$

The net horizontal pressure force is thus proportional to the *square of the full depth*:

$$F_p = \bar{P}A_p = \rho g \frac{H^2}{2} \quad \text{Horizontal Static Pressure Force}$$

The reaction force F_r supplied by the foundations must easily exceed this amount. Foundations must also provide a reaction to the *overturning moment* around the base of the wall:

$$M_o = F_p y_p$$

Where y_p is the pressure centre elevation, ie the position of the resultant force relative to the base of the wall.

The position of the pressure centre can be determined by considering the effect of a triangular pressure distribution: As with the centre of mass of a triangular shape, the centre of pressure occurs at an offset $1/3$ of the distance from the base of the triangle.

Thus the centre of pressure is:

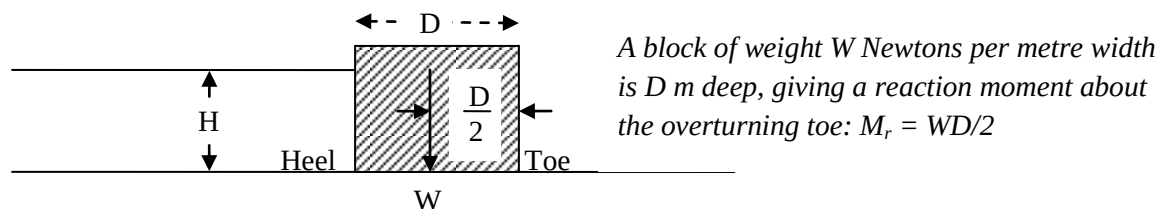
$$y_p = \frac{H}{3} \text{ or } h_p = \frac{2H}{3} \quad \text{Pressure Centre as depth } h_p \text{ or elevation } y_p$$

With this result, the overturning moment is proportional to the *cube of the full depth*:

$$M_o = F_p y_p = \rho g \frac{H^2}{2} \frac{H}{3} = \rho g \frac{H^3}{6} \quad \text{Horizontal Static Pressure Moment}$$

This is one reason why dam operators are extremely cautious when dams are above safety levels: the risk of catastrophic failure can increase very rapidly with only small increases in depth.

The structural reaction force must be provided by material strength or weight: for instance a *gravity wall* uses its own self-weight to balance the overturning moment:

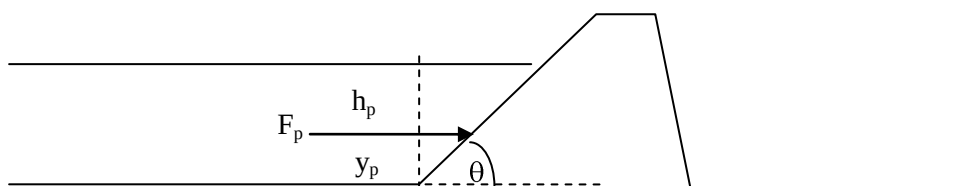


$$M_r = W \frac{D}{2} \gg \rho g \frac{H^3}{6}$$

Sloping Walls and Vertical Pressure Forces

For the more common problem of sloping dam walls, we can divide forces into horizontal and vertical, and recombine them as vectors.

The horizontal force is a projection of the mean pressure against the vertical plane as per the method set out in lecture 2.



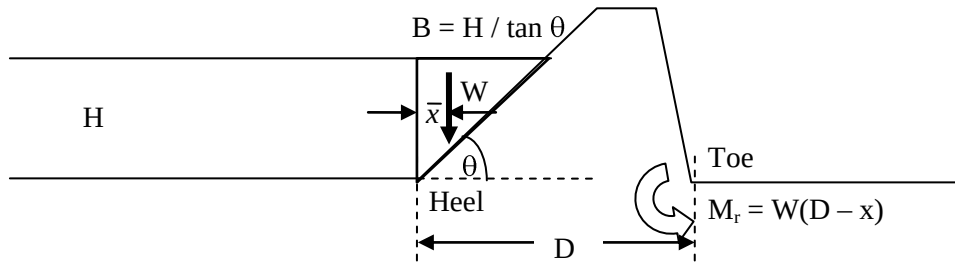
The result is exactly the same as for a vertical wall:

$$F_p = \bar{P} A_p = \rho g \frac{H^2}{2} \quad \text{Horizontal Static Pressure Force}$$

$$y_p = \frac{H}{3} \text{ or } h_p = \frac{2H}{3} \quad \text{Pressure Centre as depth } h_p \text{ or elevation } y_p$$

And
$$M_o = F_p y_p = \rho g \frac{H^2}{2} \frac{H}{3} = \rho g \frac{H^3}{6} \quad \text{Horizontal Static Pressure Moment}$$

The vertical force can be resolved by considering the weight of the “wedge” of water above the sloping face. This is supported by the wall and so bears down on the back face, creating a force which holds the wall down and a moment which counters the horizontal overturning moment:



The weight of this wedge per metre width of wall is

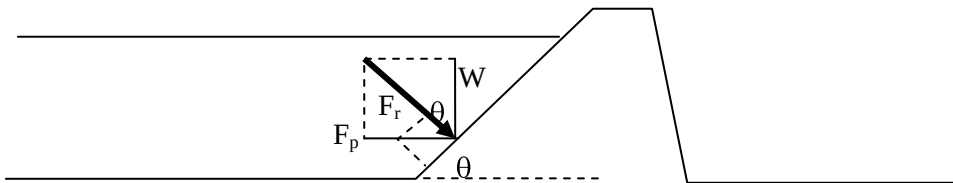
$$W = \rho g A = \rho g \frac{BH}{2} = \rho g \frac{H^2}{2 \tan \theta} = \frac{F_p}{\tan \theta} \quad \text{“Wedge” Weight Force}$$

The angle θ to the horizontal is a convention that we will adhere to.

The line of the weight force is as per a triangular pressure distribution, ie it occurs at 1/3 of the base width: This horizontal ordinate is usually given the symbol $\bar{x}$ from the *heel* of the wall:

$$\bar{x} = \frac{B}{3} = \frac{H}{3 \tan \theta} \quad \text{Wedge Weight Position}$$

Conveniently this locates the horizontal and vertical forces at the same point on the wall: we can thus add the two vectors by a simple rectilinear pair of vectors. The resultant must be at right angles to the wall:



With some elementary geometry we arrive at:

$$F_r = \sqrt{F_p^2 + W^2} \quad \text{Force Resultant on Sloping Wall}$$

$$\theta = \tan^{-1} \left(\frac{F_r}{W} \right) \quad \text{Angle to vertical of Force Resultant = Angle to horizontal of wall}$$

The angle θ can be calculated and compared to the wall angle, as a check.

The resisting moment provided by the wedge can be evaluated as:

$$M_r = W(D - \bar{x}) \quad \text{Wedge Resisting Moment}$$

The net overturning moment is evaluated by calculating the horizontal overturning moment and subtracting the wedge resisting moment. This can then be compared to the resisting moment provided by structural configurations and strengths.

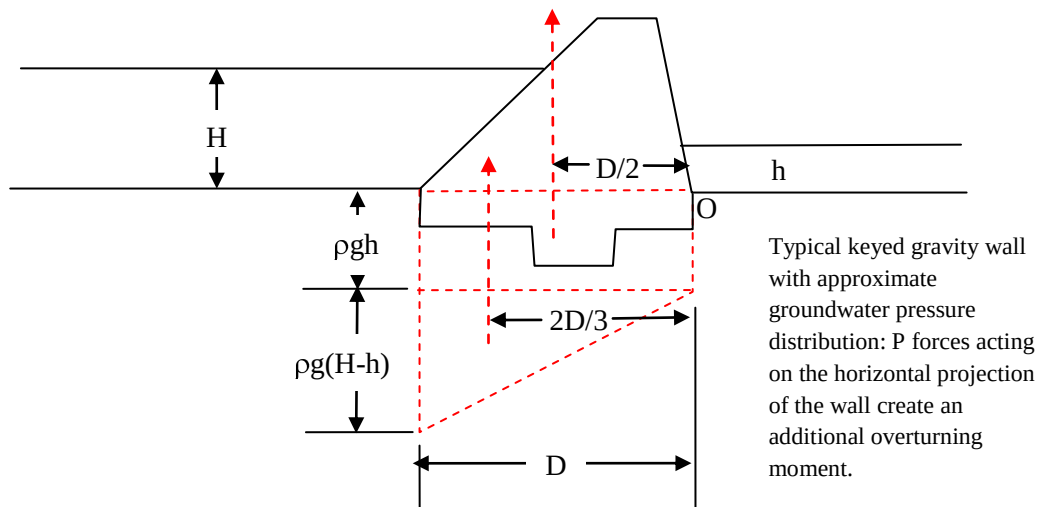
In practice failure modes of sloping walls are due to undermining by water seepage under the heel and toe, which can loosen the soil or bedrock foundations causing structural failure. Failure by overturning is not without historical precedent, for example the St Francis Dam in California, which

suffered progressive structural failure and failed by overturning on March 12 1928, causing a flood wave 38 m high which killed more than 450 people.

Uplift: Groundwater Pressure Acting Upwards from Underneath a Dam

As well as the horizontal overturning moment due to varying static pressure, in almost all real hydraulic structures there is a large pressure excess under the base of the wall due to groundwater flow. It is almost impossible to eliminate this baseflow or seepage, and even when the actual flowrate is small the pressure excess can be high.

Generally an analysis of groundwater movement is required (this is covered in a 3rd year Geotechnology unit), however a conservative (safe) estimate can be based on a linear pressure distribution from the maximum $P = \rho gH$ (upstream) to $P = \rho gh$ (downstream) can be added into the overturning moment:



The trapezoidal pressure across the base D can be analysed as superposed triangular and rectangular sections with their own force and action around the overturning toe at O: The uplift force F_u and moment M_u per unit length of wall can be evaluated:

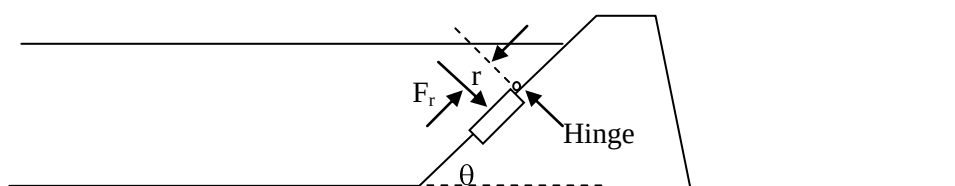
$$F_u = \frac{1}{2}\rho g(H - h).D + \rho gh.D \quad (\text{triangle} + \text{rectangle areas} \rightarrow \text{pressure} \cdot \text{width} = \text{force/length})$$

$$M_u = \frac{1}{2}\rho g(H - h).D.2D/3 + \rho gh.D.D/2 = \rho g(H - h).D^2/3 + \rho gh.D^2/2 \quad (\text{force} \cdot \text{leverage} = M)$$

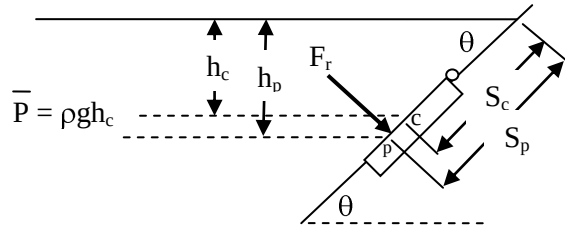
Forces on Apertures and Sluice Gates at Depth

Frequently we need to determine force resultants on apertures designed to open and close under pressure, for example to operate the low level offtakes on a dam for hydropower or irrigation releases.

For these problems it makes sense to work with the true orientation of the aperture, since this gives the true area and area-related properties of the opening. Again we adhere to the convention of setting angles to the horizontal:



The force resultant is determined by an average pressure on the true area. The average pressure on the aperture face occurs at the depth of the centroid of the aperture: For circular or square apertures (by far the most common) the centroid is the aperture's geometric centre at depth h_c or projected depth y_c . As before, the pressure centre occurs at a greater depth h_p than the centroid depth h_c :



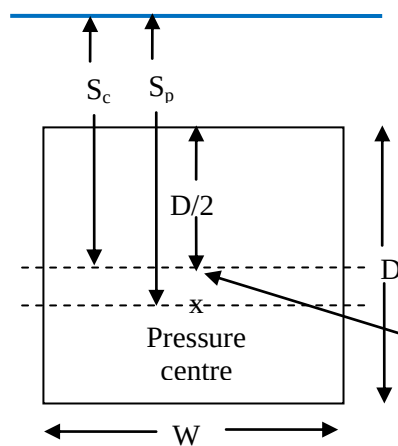
c: Centroid of square or circular aperture

p: Pressure centre

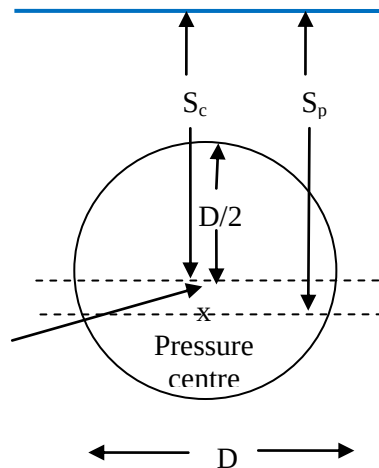
$$\bar{P} = \rho g h_c \quad \text{mean pressure on aperture at centroid depth } h_c$$

And $F_r = \rho g h_c A$ Pressure resultant force on aperture of true area A

Projected views (normal to face of aperture: $h_c = S_c \sin \theta$):



Rectangular gate:
Mean $P = \rho g h_c$
Resultant $F_r = \rho g h_c D W$



Circular gate:
Mean $P = \rho g h_c$
Resultant $F_r = \rho g h_c \pi D^2 / 4$

Conventional square or round apertures: use the average pressure at half the depth of the structure.

The line of the resultant must again be determined, because it does not coincide with the centroid. This is derived from the unequal distribution of pressure over the face of the aperture, modified by the effect of the distribution of area or shape of the aperture, which differs for square, rectangular and circular faces. The centre of pressure occurs at a projected depth S_p .

This is given by:

$$S_p = S_c + \frac{I_c}{S_c A} \quad \text{Centre of Pressure}$$

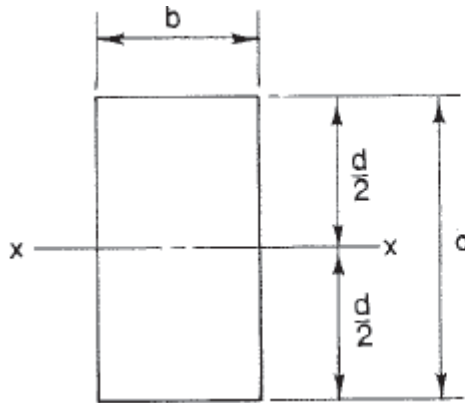
Where

S_p = projected depth to the pressure centre
 S_c = projected depth to the centroid of the aperture
 I_c = the *Moment of Inertia* of the aperture
 A = the true area of the aperture

The depth to the pressure centre can be found by $h_p = S_p \sin \theta$

The *moment of inertia* is a modification of the centroid calculation to include the rotational inertia of the aperture, for example its ability to generate turning moments from an applied pressure distribution. The moment of inertia is calculated by determining the contribution of elements of area to a moment about the centroidal axis (see your statics courses or first-year physics).

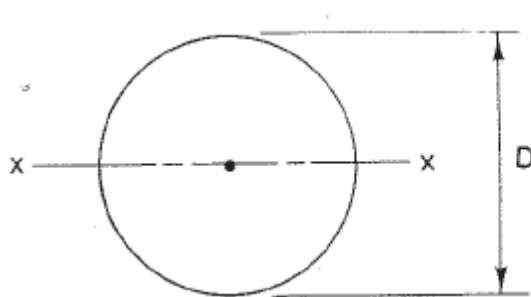
Moments of inertia have been tabulated for a number of shapes (see Reading 13 Mullins 1983) but the most common values in structural applications are for rectangular and circular shapes:



$$I_{xx} = \frac{bd^3}{12}$$

$$(I_{xx} = \frac{b^4}{12} \text{ for a square})$$

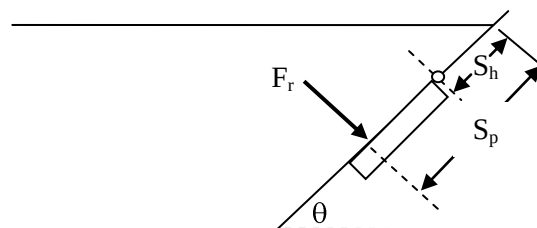
Moment of Inertia of a rectangular shape about the centroid: $I_c = bd^3/12$



$$I_{xx} = \frac{\pi D^4}{64}$$

Moment of Inertia of a circular shape about the centroid: $I_c = \pi D^4/64$

Once the projected pressure depth S_p is found, the resultant pressure force can be multiplied by offset distances to key rotational failure points, such as the hydraulic hinge mechanism of a dam aperture, to calculate the *opening moment* which the structure or mechanism must be capable of resisting: Assuming the failure point or hinge has a projected depth S_h :



$$M_o = F_r(S_p - S_h) \quad \text{Opening moment by pressure force and offset}$$

Thought Experiment: Balancing a Sluice Gate.

If you had a hinged sluice gate (even a model sluice gate) how would you *physically* locate the position of the centre of pressure so as to design an appropriate hinge?

Reading

Mott 2006 (Reading 3) Sections 4.5 to 4.8