

Lecture 5: Static Pressure

Statics is the branch of mechanics that deals with bodies in equilibrium, where all forces and turning moments (often called *torques*) are equal and opposite, so there is no acceleration.

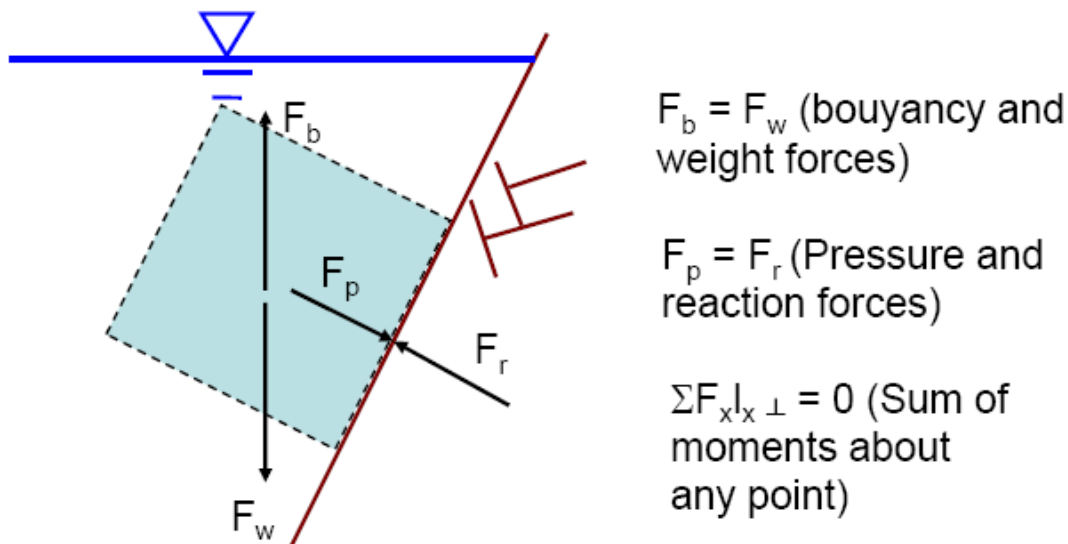
Fluid statics is a special case in that fluid bodies are unable to support themselves, so the forces required to maintain a fluid body in a given shape must be supplied by the walls of a container. Free-body diagrams often assume rigid bodies with supportive forces attached at particular points. In fluid statics the supportive forces must be distributed across every face of a fluid body other than the horizontal fluid surface. Designing a dam wall or pressure vessel requires an understanding of force distributions dominated by the static pressure of a fluid body's weight.

From the definition of a fluid (Lecture 1) there can be no static shear forces on a fluid, so *all static forces exerted on a fluid by its containing structure must act at right angles to the solid-fluid interface*. If the supporting wall is curved, the pressure forces must curve with it so as to remain at right angles.

So long as there is no relative motion in the fluid we can be sure there are no shear forces. This can greatly simplify the analysis.

Free Body Diagrams

In the analysis of a problem it is usual to consider an element of the fluid defined by solid boundaries or imaginary planes around the fluid: a free body diagram can be drawn for this element, showing the forces acting on it due to the solid boundaries or the surrounding fluid.

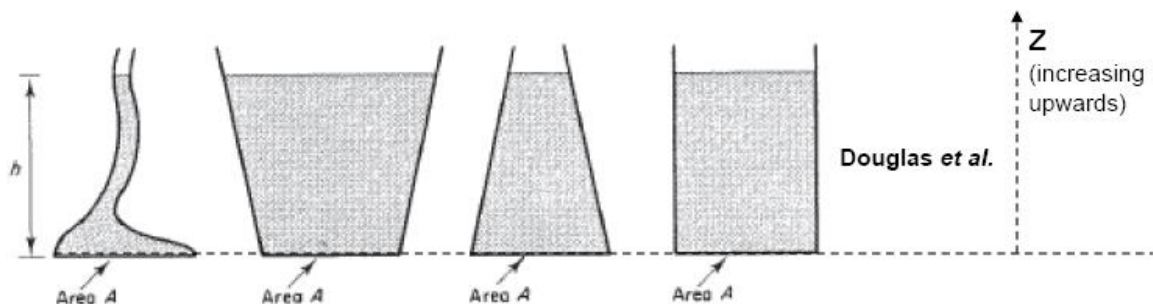


Fluid element resting against an inclined plane eg a weir wall. All forces and moments are in balance.

Hydrostatic Pressure

As we saw in Lecture 2, the static pressure in a body of fluid varies with depth, as the weight of fluid creates increasing pressure at increasing depth. The depth of fluid at any point is often referred to as the Head, denoted h or H , at that point.

Interestingly, but not quite paradoxically, the pressure increase is independent of the shape of container. Static pressure depends only on density and head. The reason for this is that there is a geometric relationship between the container's shape and the area of its base, such that the weight force per area (pressure) is independent of the shape.



In all these odd-shaped containers, the static pressure at any depth is the same.

In each above case, the total weight force divided by the base area is a simple function of fluid height:

Again,

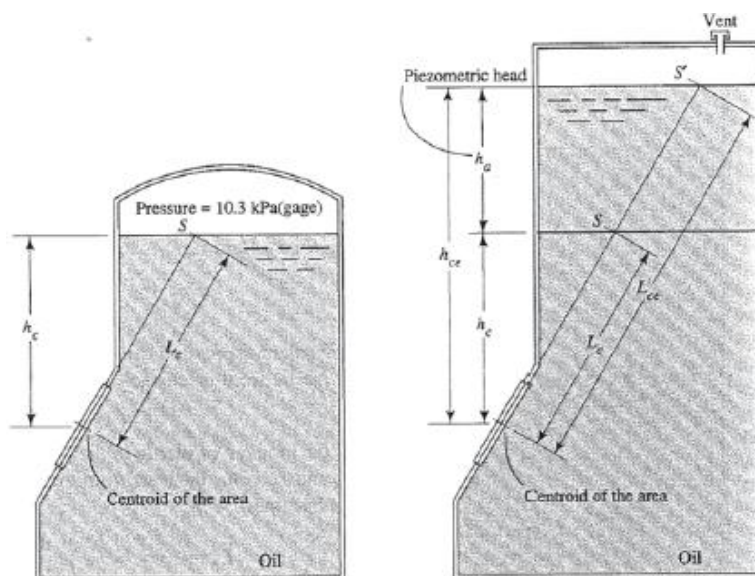
$$P = \rho gh \quad \text{Static Pressure Distribution}$$

Piezometric Head

In cases where an additional pressure is applied to a fluid, the total pressure is the sum of hydrostatic + applied pressure.

It is sometimes convenient to express the applied pressure as an equivalent depth of fluid or *piezometric head*:

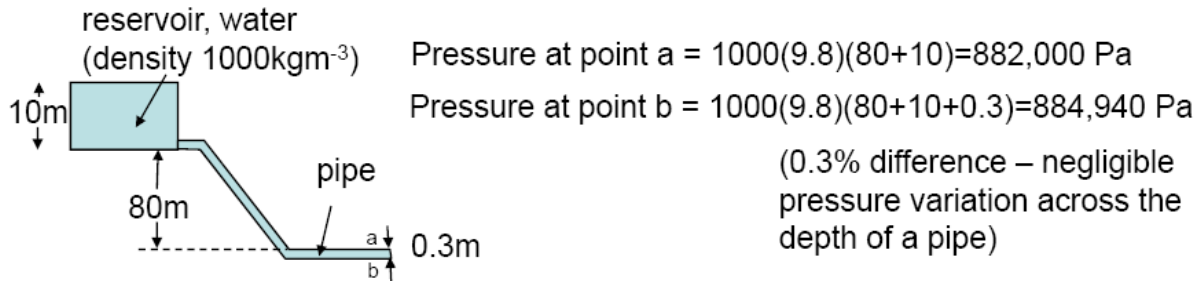
$$h_a = P_a / \rho g \quad \text{Piezometric head } h_a \text{ for an applied pressure } P_a$$



A pressurised tank showing actual configuration (a) and configuration with assumed piezometric head. This simplifies the structural design of the pressure vessel.

Negligible Hydrostatic Pressure

In some cases hydrostatic pressure variation can be neglected or ignored in calculations. For example hydrostatic pressure variation in a pipe may be small compared to the absolute pressure, or hydrostatic pressure in a channel may be small compared to momentum forces:



Negligible difference in hydrostatic pressure between the top and bottom of a pipe.

Gauge Pressures

All calculations in hydrostatic and pressure vessel design use gauge pressure (see Section 1).

Turning Moment

A *turning moment* or torque is a force cross-multiplied by a distance, for example the action of a force on a door which acts around the hinge, or the force of fluid static pressure on a hinged sluice gate:

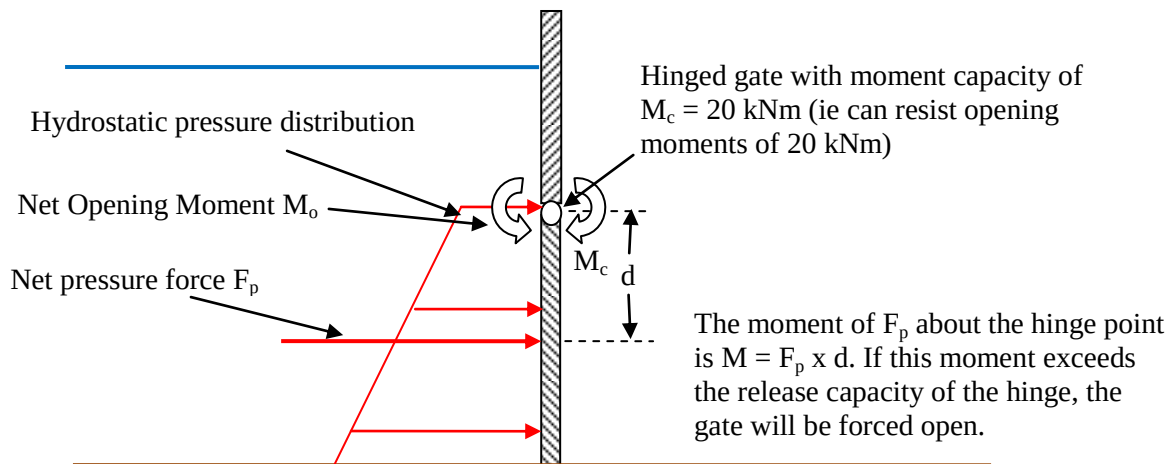


Illustration of turning moment action of force x distance (sluice gate in cross-section)

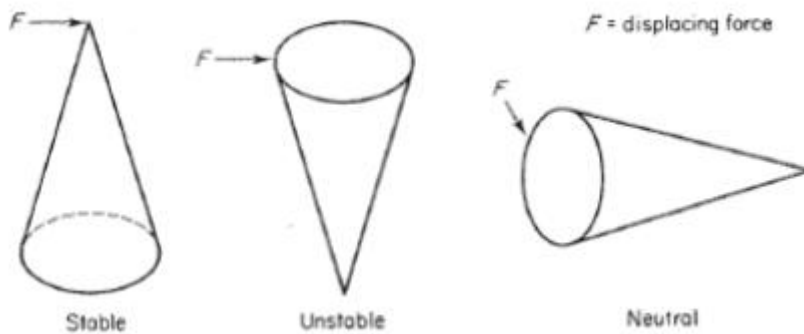
For static equilibrium (ie no accelerations) it is essential that all forces and moments are equal and opposite, ie in balance. A balance of forces ensures no linear acceleration, while a balance of moments ensures no angular acceleration. Both are essential conditions for a static system.

In many of the structural and buoyancy problems of this section *it is more important to consider the balance of moments than of forces*. Design techniques are based on a moment balance, with unresolved forces assumed to be balanced by small deformations in the structure. A full structural design will ensure that the structure is robust enough to supply these forces, however for the moment we can make some loose assumptions.

Equilibrium: Stable, Unstable and Neutral

Although a body or element may be in equilibrium, it can also be of interest to know what will happen if it is displaced from its equilibrium position. There are three possible conditions of equilibrium.

- Stable equilibrium. A small displacement from the equilibrium position generates a force producing a righting moment tending to restore the body to its equilibrium position.
- Unstable equilibrium. A small displacement produces an overturning moment tending to displace the body further from its equilibrium position.
- Neutral equilibrium. The body remains at rest in any position to which it is displaced.

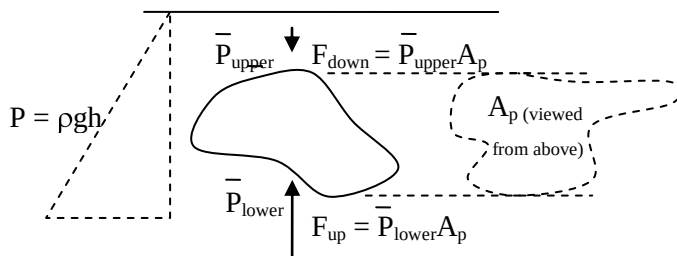


3 possible equilibria.

Buoyancy: Archimedes' Principal

The concept of buoyancy is usually covered in first-year physics courses. It is the one idea of fluid mechanics we hope you already have in place. Just in case however, a quick refresher:

Buoyancy arises because of the variation in static pressure with depth: a submerged object experiences higher static pressures on its lower profile than on its upper profile, so there is a net upthrust. We can simplify this by considering average pressures and a profile area in the vertical direction as per the method derived in Section 1:



Static pressure forces on a submerged object with a profile area A_p in the vertical direction

$\bar{P}_{\text{lower}} > \bar{P}_{\text{upper}}$ so the vertical forces on the (equal) upper and lower profile areas are unequal.

This principle applies to all irregularly-shaped objects. The calculation of average pressure by profile area becomes a *surface integration over depth*, which yields the *volume* of the submerged object:

$$F_{\text{buoyant}} = F_{\text{up}} - F_{\text{down}} = \rho g \nabla$$

Where V is the volume of the submerged object and ρ is the fluid density. This is equal to the weight of fluid *displaced* by the submerged object.

The submerged object's apparent weight is reduced by that amount:

$$F_{\text{submerged weight}} = F_{\text{dry weight}} - F_{\text{buoyant}}$$

But the dry weight force is based on the volume and the density of the solid:

$$F_{\text{dry weight}} = \rho_{\text{solid}} g \nabla$$

Thus $F_{\text{submerged weight}} = (\rho_{\text{solid}} - \rho_{\text{fluid}}) g \nabla$ Buoyancy Law of Archimedes

Archimedes's principle states that the net buoyant force on an immersed object is equal to the weight of liquid displaced by the object. If this is greater than the weight of the object, it will float at the level where the displaced liquid weight equals the object's weight.

Similarly less dense liquids will tend to form a layer on top of a more dense liquid, especially where density differences are due to temperature differences. These *thermoclines*, or temperature-density gradients, are a defining characteristic of natural water bodies and the atmosphere. Thermal buoyancy effects are also important in processes where gases or liquids are heated differentially, creating convective currents whereby warmer fluid rises and cools, falling again to create a possible rotational motion in an air or water column.

Different gases will separate partially into temperature-dependent density strata, but will tend to remain mixed as gas temperature dynamics are the same generally for different gas species.

Buoyancy and Effective Stress

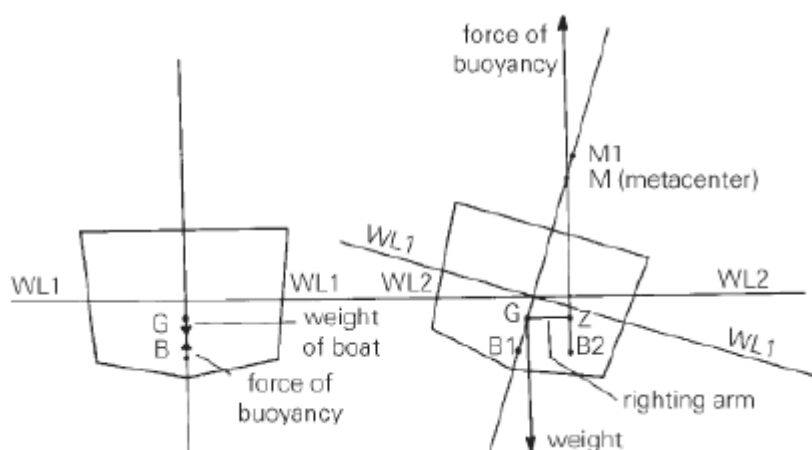
In soil mechanics and geotechnology it is often necessary to evaluate the *effective stress* in soil masses below the water table, ie in soil masses that are saturated with water. In these cases the static pressure between soil particles due to the weight of soil, which relates to the degree of soil compression and possible slip failure, is reduced by the buoyancy imparted by the water table.

The effective stress calculation is typically performed by subtracting the density of water from the dry density of the soil mass before evaluating the self-weight of the soil. This calculation is equivalent to the effect of a varying static liquid pressure exerting a larger force below than above soil particles.

Stable Equilibrium and Vessel Design

A vessel designed to float upright should ideally have only one stable equilibrium, that is hull down with righting moments that will restore the vessel to upright in the event of heeling or capsizing.

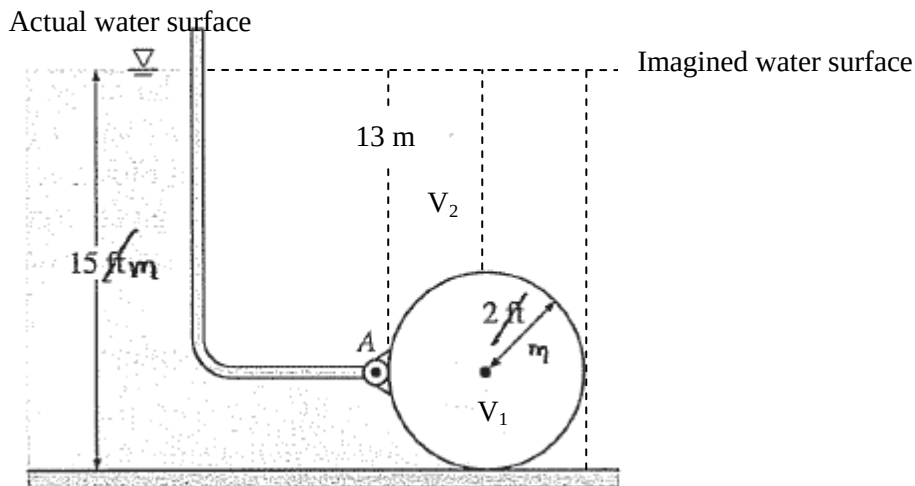
This requires careful design of the *centre of buoyancy* relative to the centre of mass (or gravity), such that the buoyant force always produces a righting moment—a force acting at a distance from the centre of gravity such that a rotational action restores the vessel to upright:



A vessel has variable stable equilibria produced by the changing geometry of the immersed volume as the vessel “heels over”: the centre of buoyancy moves laterally due to the changing 3D geometry of the submerged volume, creating a restoring or “righting” force.

The design of floating vessels such as pontoons or hulls is relatively advanced and is not covered in this course.

Example: The hinged cylindrical structure in the cross-section below is a *gravity gate*, holding down water pressure by its own weight. If the cylinder weighs 40 tonnes per meter of width (into the page) what is the net vertical force on the hinged structure? Will it hold water at this depth?



In this case the static pressure under the cylinder is quite high due to 15 m of water, but there is no corresponding static pressure above the cylinder, it is open to atmosphere. It is as if the cylinder has displaced a depth of between 11 and 13 m, but has not replaced it with a corresponding solid volume. We need to break the forces into components consisting of the “missing” water depth and the available mass of the structure.

The weight of cylinder per metre of width is

$$W = mg = 40 \times 10^3 (9.8) = 392 \text{ kN per m of width}$$

Imagining for a moment that the cylinder is completely submerged on all sides to the full 15 m depth, the weight of water displaced by the cylinder would be:

$$W_1 = -\rho g V_1 = -1000(9.8)\pi R^2 = -9800\pi 4 = -123.2 \text{ kN per metre}$$

Where the – sign indicates upward force. If we mentally remove the entire right hand half of the imagined water depth, we lose exactly half that buoyant force. If we then remove the left hand part that lies over the hinge side of the cylinder, we lose the overlying weight of water, which is

$$W_2 = -\rho g V_2 = -1000(9.8)[(13 \times 2) - \pi R^2/4] = -9800(26 - \pi) = -224.0 \text{ kN per metre}$$

The buoyant force is thus $B = W_1/2 + W_2 = -123.2/2 - 224.0 = -285.6 \text{ kN/m}$

The buoyant force is subtracted from the weight of the structure to determine the net force holding the gate closed:

$$\text{Gravity Gate Force } G = W - B = 392.0 - 285.6 = 106.4 \text{ kN per m of width.}$$

The moment of this force about A is $M = Fd = 106.4(2) = 212.4 \text{ kN.m per metre width}$

This vertical force-moment calculation is not the end of the story however. There is, in addition, a *horizontal* hydrostatic pressure moment which must be less than the vertical “holding down” moment for the structure to retain water.

That’s a job for lecture 6.