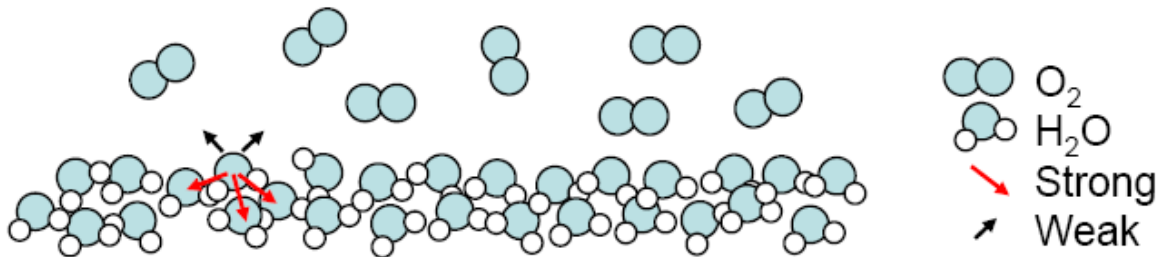


Lecture 4: Properties of Fluids (Continued)

Surface Tension

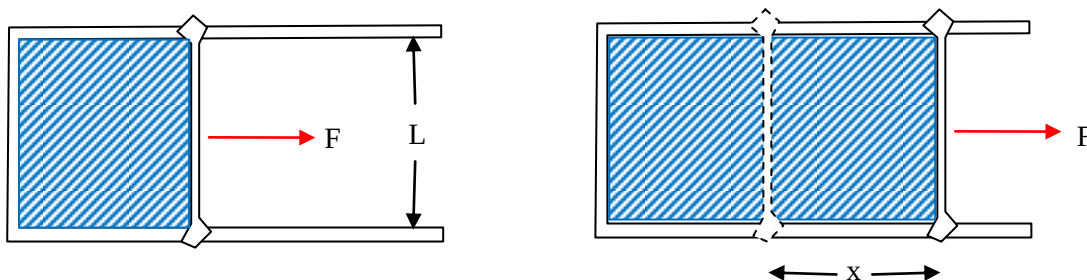
Surface tension arises due to the differing molecular attraction between particles of fluid and the surrounding gas: fluid particles at the surface are bound more strongly to the underlying fluid than to the air:



Imagined water-air surface showing relative cohesion forces at the boundary.

As a result, a fluid body will tend to draw in on itself and minimise its surface area if possible, eg by forming spherical droplets. The “skin” of the droplets provides the tensile force required to do this, and exerts energy as the tensile force produces motion causing the reduction in surface area.

It takes a certain amount of force (and energy) to work against surface tension and increase the surface area. A classic experiment involves arranging a thin film of fluid on a mobile wire frame and applying a force to stretch the film:



It is found that the force F is proportional to the width L of film, and thus the energy $E = F \cdot x$ is proportional to the additional area, $L \cdot x$:

$$F = \sigma L \quad \text{and} \quad E = \sigma L x$$

These are multiplied by 2 where there are two surfaces, as in a film with top and bottom surfaces.

The proportionality constant σ (sigma) is a property of the fluid called the *surface tension*, and has units of force per length or energy per area:

$$\sigma = \frac{F}{L} = \frac{E}{A} \quad \text{Surface Tension (units } \text{Nm}^{-1} \text{ or } \text{Jm}^{-2} \text{)}$$

In some texts surface tension is again referred to as γ , but σ is more common.

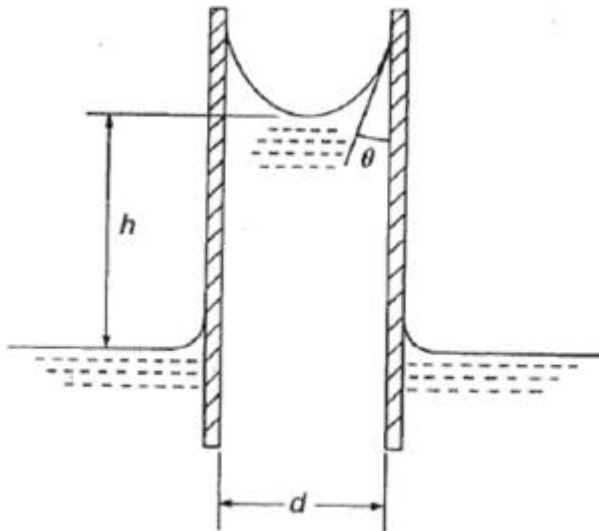
Surface tension is most significant for small-scale systems where fluid surfaces interact with boundaries, creating additional forces that appear as a source of friction. Small scale models are particularly prone to surface tension forces that are not realistic in the full-scale model.

Insect biology and other small scale systems may be strongly influenced by surface tension forces.

Cohesion and Adhesion

As with surface tension, fluids exhibit varying degrees of *boundary tension* due to the varying attraction of fluid molecules to themselves (cohesion) as opposed to their attraction to container walls of other materials (adhesion).

The relative values of adhesive and cohesive forces influences the behaviour of fluids in small conduits where these forces are significant compared to body forces. *Capillary tension* is an important example with applications in plant and animal vascular biology.



Capillary tension arises from a balance of cohesive and adhesive forces

A thin tube (called a *capillary* in both biology and fluid mechanics) provides a natural measure of the width L to which surface tension applies: the circumference of the *meniscus* where the fluid contacts both the air and the tube. The capillary lift force is given by:

$$F = \sigma \pi d \quad \text{where } \pi d \text{ is the inside circumference of the capillary.}$$

This force acts along the plane of the surface at the contact point, ie at the angle θ . The vertical component of force $F_v = F \cos \theta$ balances the weight of fluid:

$$\sigma \pi d \cos \theta = mg = \rho \forall g = \rho \frac{\pi d^2 h}{4} g$$

Thus the fluid will rise by capillary tension to a height h given by:

$$h = \frac{4\sigma \cos \theta}{\rho g d} \quad \text{Capillary Rise Equation : } \theta \text{ is often assumed to be zero.}$$

Wetting and Waterproofing

If the forces of adhesion between the molecules of a particular liquid and a particular solid are greater than the forces of cohesion among the liquid molecules, the liquid molecules tend to crowd towards the solid surface, and the area of contact between liquid and solid tends to increase.

Given the opportunity, the liquid then spreads over the solid surface and 'wets' it. Water will wet clean glass, but mercury will not. Water, however, will not wet wax or a greasy surface.

Surface Tension is a function of Temperature:

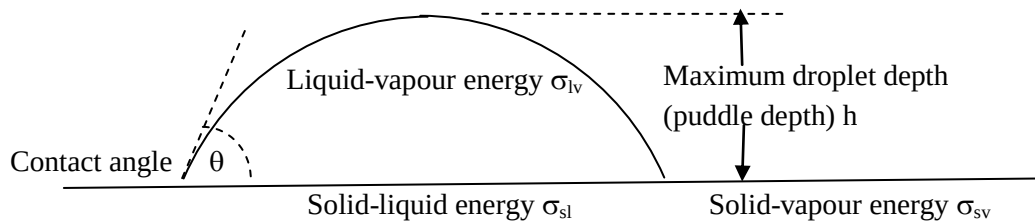
Temperature °C	Water-air Surface Tension $\sigma \text{ Nm}^{-1}$
0	7.62×10^{-2}
10	7.48
20	7.36
30	7.18
40	7.01
50	6.82
60	6.68
70	6.50
80	6.30
90	6.12
100	5.94×10^{-2}

Droplet Formation: Solid, Liquid and Vapour Surface Energies

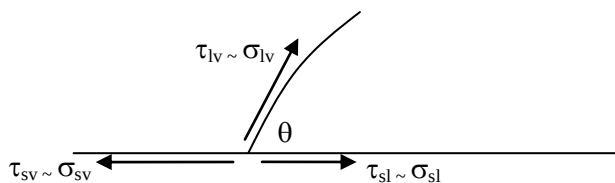
All surfaces of contact between different materials, whether solid, liquid or gas, can be described as having a specific *surface energy*. Liquid-air surface tension is only one such type of surface energy.

The formation of droplets on a non-wetting surface is governed by three distinct surface energies, which interact to produce droplets with the least possible combined surface and potential energy. This *minimum energy* rule is a feature of the stable equilibrium of all types of systems.

Two features of a droplet on a level surface are the *contact angle* θ and the maximum droplet depth h , which is the level depth to which puddles will form for a given liquid-solid-vapour combination:



The contact angle θ can be derived by considering a balance of forces between the three interfaces: Since the respective surface tensions are proportional to their surface energies, we can draw three tensile forces (denoted τ proportional to σ) which are in equilibrium or balance:



Thus
$$\sigma_{sv} = \sigma_{sl} + \sigma_{lv} \cos \theta$$

Or
$$\theta = \cos^{-1} \left(\frac{\sigma_{sv} - \sigma_{sl}}{\sigma_{lv}} \right)$$

Young's (Contact Angle) Equation

Example: Find the contact angle of water on Teflon (a low surface energy material) in air at 20°C:

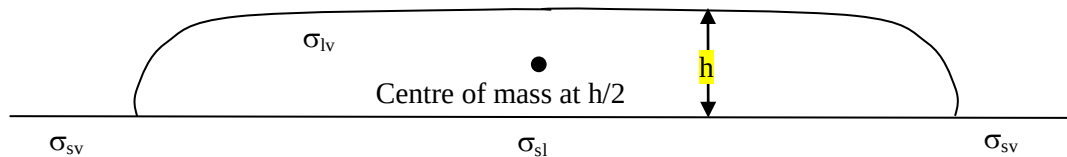
A quick search¹ for the relevant surface energies yields $\sigma_{sv} = 18.5$ mN/m and $\sigma_{sl} = 43.4$ mN/m. The water-air surface tension of $\sigma_{lv} = 73.6$ mN/m is obtained from the table on the previous page. Thus

$$\theta = \cos^{-1}\left(\frac{\sigma_{sv} - \sigma_{sl}}{\sigma_{lv}}\right) = \cos^{-1}\left(\frac{18.5 - 43.4}{73.6}\right) = 109.8^\circ$$

Surfaces that produce contact angles with water greater than 90° are known as *hydrophobic* surfaces. In general this occurs whenever the solid-vapour tension is less than the solid-liquid. Teflon, a well known non-stick material, has one of the lowest known solid-vapour surface energies. It's so slippery even a gecko can't climb it.

Hydrophobic materials can be nano-engineered to produce a surface texture that has microscopic air pockets. Liquids on such a surface are partially supported on trapped air, producing contact angles greater than 150°. Such high contact angle surfaces, which include naturally occurring plant surface such as lotus leaf, are known as *superhydrophobic surfaces*. These materials have a range of useful microtechnological applications.

The maximum droplet depth or *puddle depth* can be determined by a minimum energy analysis. The method is to evaluate the total surface area energy + potential energy of a droplet spreading to any depth, and differentiate to find a minimum:



The puddle has a volume V and depth h . For large volumes this gives approximately equal solid-liquid and liquid-vapour surface areas: $A = V/h$.

The gravitational potential energy of a puddle of mass ρV and depth h is $E_p = \rho Vgh/2$ (centre of mass at $h/2$)

The solid-liquid surface energy of the puddle is $E_s = (\sigma_{sl} - \sigma_{sv})A = (\sigma_{sl} - \sigma_{sv})V/h$ (allows for the decrease in solid-vapour area at the same time as the increase in solid-liquid area).

The liquid-vapour surface energy is $E_v = \sigma_{lv}A = \sigma_{lv}V/h$

The total potential + surface energy is thus $E = \rho Vgh/2 + (\sigma_{sl} - \sigma_{sv})V/h + \sigma_{lv}V/h$

We can differentiate this to find the minimum energy as a function of h :

$$\frac{dE}{dh} = \frac{\rho Vg}{2} - (\sigma_{sl} - \sigma_{sv} + \sigma_{lv})\frac{V}{h^2} = 0 \quad @ \quad \frac{\rho Vg}{2} = (\sigma_{sl} - \sigma_{sv} + \sigma_{lv})\frac{V}{h^2}$$

The volume V cancels from both sides, as you would expect since the puddle depth is the same regardless of the size of the puddle.

$$\text{Thus} \quad h = \sqrt{\frac{2(\sigma_{sl} - \sigma_{sv} + \sigma_{lv})}{\rho g}}$$

For water on Teflon, this gives a depth of about 4.5 mm, easily confirmed by experiment in your own kitchen (you will need a good-quality Teflon pan).

¹ <http://www.surface-tension.de/solid-surface-energy.htm>



Droplets and puddles of water on lotus leaf at 20°C. Note the maximum droplet depth = puddle depth. The waxy superhydrophobic surface produces a glossy submerged surface due to air entrainment. This gives a high contact angle of ~150°.

Droplet technology is highly developed. Bubble jet printers use a matrix of fine capillaries activated by temperature signals to eject micro-droplets of three-colour + black inks. Coupled with digital processing, this basic fluid control system can reproduce colour images to any resolution.

Vapour Pressure

Liquids exist in a state of physical-chemical equilibrium with their surroundings. All liquids have a tendency to evaporate, and all gases have a tendency to condense to liquid. While there are distinct temperatures at which a phase change must occur, the tendency to evaporate or condense is present at all temperatures. The phase change is a reversible process which will move in a particular direction (from liquid to vapour or vice versa) until a saturation concentration develops, at which point the rates of evaporation and condensation are equal.

One way of expressing this relationship is through the concept of *saturation vapour pressure*. A liquid exerts a certain pressure on the atmosphere, representing a balance of forces between evaporation and condensation as a function of temperature. If the absolute pressure on the liquid is reduced to this value, the liquid will boil at that temperature. This is a critical design consideration for liquids at low pressure, such as flows through suction pumping systems or high-speed propellers.

The vapour pressure in water at 100°C is 101.3 kPa, roughly equal to the standard atmospheric pressure. Thus water boils at 100 °C in normal sea-level circumstances, or less at higher altitudes.

The saturation vapour pressure of water as a function of temperature is given by:

$$P_{sat}(T) = 610.7 \times 10^{\frac{7.5T}{237.3+T}} \quad \text{Water vapour pressure (Pa for T in } ^\circ\text{C)}$$

Thus at 20 °C, water vapour pressure is:

$$P_{sat}(20^\circ\text{C}) = 610.7 \times 10^{\frac{7.5 \times 20}{237.3+20}} = 2340 \text{ Pa}$$

To avoid damage caused by boiling in machinery, water at 20°C must not be subjected to a lower pressure than this.

Saturation vapour pressure also represents a humidity of 100% in the atmosphere. Relative humidity is calculated as a ratio of the actual vapour pressure in the air to the saturation vapour pressure:

$$\text{Relative Humidity} = P_{\text{vap}} / P_{\text{sat}} \times 100\%$$



Thought Experiment: Breaking a water column

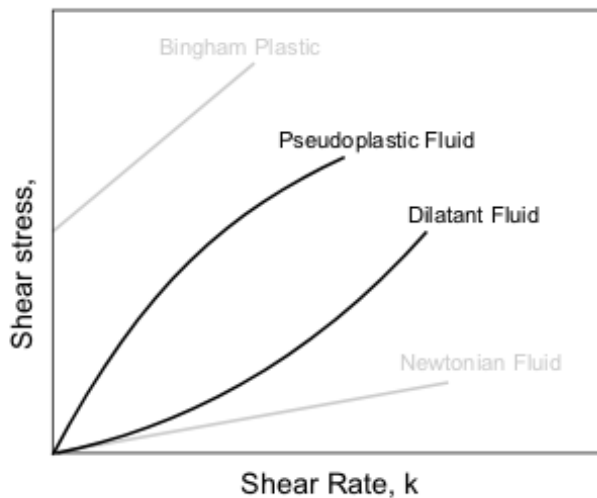
10 metres is the theoretical maximum height of trees that draw water using ordinary stomatal suction pressures. Trees taller than this (in fact all trees) use a combination of suction, capillary action and cellular diffusion to draw water to their leaves. But what is the basis of this theoretical maximum? Can you picture a situation where you fill a 10 metre length of flexible PVC tube with water, then raise one closed end of the tube to a 10 metre height? What would happen to the water, and why?

Stress-Response Categories of Fluids

Fluids can be divided into two broad categories depending on their response to shear stress:

Newtonian Fluids: Fluids that conform to Newton's Law of Viscosity, ie viscosity as a constant of proportionality dependent only on temperature.

Non-Newtonian Fluids: Fluids whose viscosity is also a function of the *rate of application* of shear force:



Many polymer solutions and molten polymers are non-Newtonian fluids, as are many commonly found substances such as tomato sauce, starch suspensions, paint, blood and shampoo.

For example a *dilatant* fluid such as corn starch resists a rapid application of shear more effectively than a slow application – it appears plastic if you hit it hard or run on it, but liquid if you hold it gently or try to stand on it.



Game-show contestants walking fast on corn starch suspension. If they stop, they sink. A video of some unusual properties of corn starch solution is available at <http://www.youtube.com/watch?v=nq3ZjY0Uf-g>



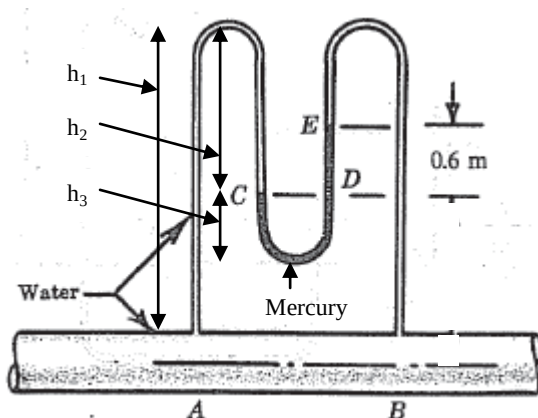
Thought Experiment: A Bouncing Spoonful

If you mixed a sample of corn starch in a bowl, could you scoop out a spoonful, roll it into a ball and bounce it on a table? What happens when it stops bouncing?

Fluid Instruments and Components

One of the key roles of a fluid scientist or engineer is the design and construction of instruments and components that use static and dynamic principles to perform tasks based on the distribution of forces.

In analysing and designing such devices it is usually essential to find some way of identifying pressure differences between points A and B (say):



The usual technique is to trace pressure changes as a result of measureable loads from one end of the system to the other, or from point A to point B, using static or dynamic principles to create algebraic statements of pressure changes.

For example, static pressures vary through the series of depths of water and mercury between A and B. Given densities of ρ_w and ρ_m respectively, we can trace from A to B, *adding* static pressures wherever the depth *increases* from one point to the next, and *subtracting* wherever the depth *decreases*:

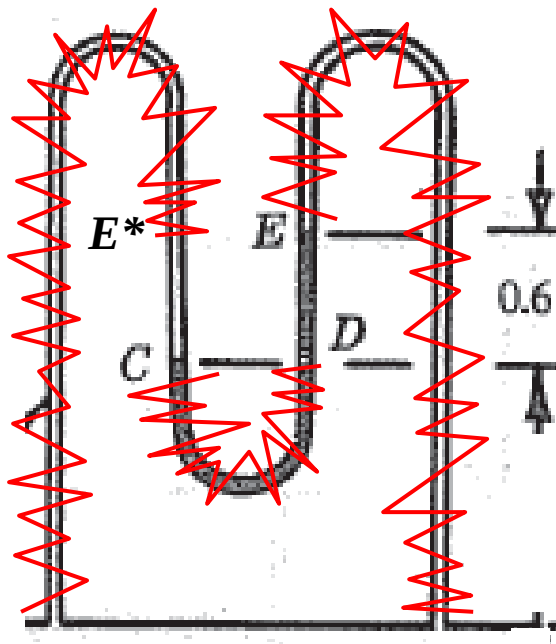
Thus for example $P_A - \rho_w g h_1 + \rho_w g h_2 + \rho_m g h_3 - \rho_m g h_3 - \rho_m g (0.6) - \rho_w g (h_2 - 0.6) + \rho_w g h_1 = P_B$
(Trace the line from A to B as you read this through: it follows the statement of static pressures)

Before evaluating the pressure difference $P_A - P_B$ we can simplify by eliminating repeated terms:

$$P_A - \rho_m g (0.6) + \rho_w g (0.6) = P_B$$

So the Pressure Difference: $P_A - P_B = (\rho_m - \rho_w) g (0.6)$

This much simpler expression could have been found by first eliminating all fluid depths that cancel each other out, that is depths of the same fluid that are equally paired:



The inverted U-tubes A-E* and E-B are identical and create equal-opposite static pressures that cancel out. Similarly the U-tube C-D has two uprights which cancel. The remaining pipe lengths are those which remain in the algebraic statement of the pressure difference. E*-C and D-E do not cancel as they are not the same fluid. The designer has scribbled out the paired elements to simplify the calculation.

This is a useful general rule that works with both static and dynamic pressures as well as friction, adhesion and other fluid forces. *You will use it in your first assignment and the exam.*

What can you expect in PHYS207?

So far we have seen a bewildering range of categories, definitions and ranges of behaviour of fluids. Fluid mechanics is in fact one of the most difficult of the great branches of classical mechanics, mainly because of the sheer diversity of ways in which fluids and fluid behaviours can be classified.

How can we begin to organise this complex field for practical purposes?

- In most applications, the mechanics of fluids are dominated by an equilibrium of forces: the flow is either static or steady, or can be approximated as such.
- The geometry of most fluid flow situations can be simplified as one-dimensional or at most two-dimensional: pressure and velocity changes occur along the length of a conduit (1st dimension) and to some extent across the depth or width of the conduit (2nd dimension). It is relatively rare to perform an analysis across all three dimensions.
- Fluid flows are nearly always non-uniform in practice. This is unavoidable, and it keeps things interesting.
- Gauge pressures are generally sufficient to define the mechanical forces involved in fluids: atmospheric pressure is constant across most hydraulic systems, and hydrostatic pressure is not significant in the small depths of pipes and most channels.
- Surface tension is often small compared to other forces and can be neglected. Precision engineering, miniature components and biological fluids are important exceptions.
- Newtonian fluids are far more common in practice than non-Newtonian ones. Research in fluid mechanics however frequently crosses over with materials science and involves unusual fluid types and behaviours.

It takes a certain amount of experience and judgement to determine which of these approximations is valid, either in practice or in the laboratory. This issue will be revisited in Section 3, Fundamentals of Fluid Flow.

Reading

**Massey 1983 (Reading 1) Section 1.5.6 and 1.6
Review Learning Outcomes for Section 1**

Tutorial

Properties of Fluids