

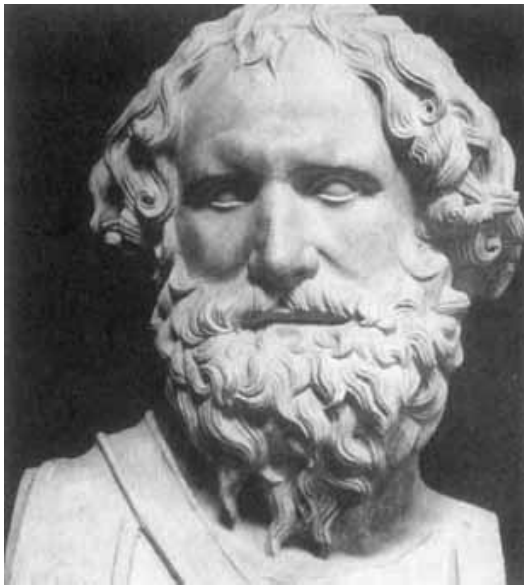
Lecture 3: Properties of Fluids

Density

The density of a fluid, as measured from any representative sample of the substance, is the mass of the sample divided by the volume. Care must be taken to measure the true volume of the sample, excluding all air space and contaminants.

$$\rho = \frac{M}{\forall} \quad \text{Density equation (units kgm}^{-3}\text{) where } \forall \text{ is volume.}$$

A number of classical experiments have been developed to measure density, notably that devised by Archimedes, who first described the concept of density and developed a simple method of confirming the gold content of a suspect crown made for the king of Syracuse:



Archimedes (c. 287 – 212 BC)

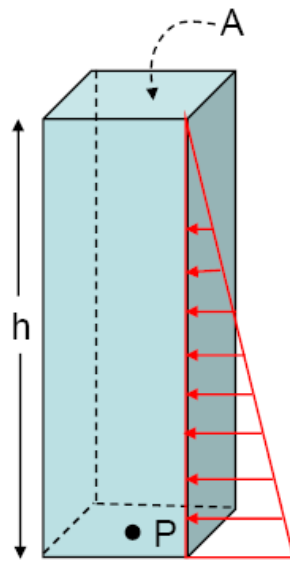
Density is often stated relative to some standard material at a given temperature. *Relative density* (sometimes referred to as *specific gravity*) uses water at 4.4 C° having an absolute density of 1000 kg/m³ and Rel.D 1.000.

Density varies with temperature, since all materials undergo thermal expansion (or contraction) over a range of temperatures:

Temp (C°)	Water Rel.D or S.G.
4.4	1.000
10.0	1.000
15.6	0.999
21.1	0.998
26.7	0.997
32.2	0.995
37.8	0.993
43.3	0.991
48.9	0.990
65.6	0.98

Density and Static Pressure

Static pressure is induced by the weight of a fluid body above a horizontal plane at any depth: The weight is a function of density, which we assume initially to be constant:



$$\text{Mass} = \rho.V = \rho.h.A$$

$$\text{Weight} = M.g = \rho.h.A.g$$

$$\text{Pressure} = \frac{W}{A} = \rho.h.g$$

Thus $P = \rho gh$ Static Pressure equation for constant density

This is the most common equation you will use in fluid mechanics. Understand/Remember it!

For hydrostatic (water) pressure, $\rho g \sim 9,800 \text{ kg m}^{-2}\text{s}^{-2}$ is a useful figure to remember.

The average pressure in a layer of water is the pressure at half the depth: $P_{\text{mean}} = \rho g H/2$

The *specific weight* $\gamma = \rho g$ is a useful quantity simply due to its frequent use. Tables of fluid properties often list γ rather than ρ . Note possible confusion with γ the ratio of specific heats.

A depth such as h is often referred to as *Head*, which simply means the depth of fluid in suitable units ie metres. Head is a method of quantifying all forms of pressure, including static, dynamic and applied forms, in a single measure that is scaled to the vertical scale of the system.

Gas Density

Gases have density, and as such exert measurable weight forces on their surroundings. To evaluate this, the ideal gas law (possibly familiar to you from chemistry) can be written in terms of density:

$$P\forall = nRT$$

So $P = \frac{nRT}{\forall} = \rho RT$ Ideal Gas Law for Density

Where $R = \frac{R}{M}$ M the molar mass of the gas

R is the *specific gas constant* for the gas under consideration, with units $\text{J kg}^{-1}\text{K}^{-1}$. Note that it is essential to use the Kelvin or absolute temperature scale in these calculations ($\text{K} = ^\circ\text{C} + 273.3$).

Specific gas constants for some common gases:

Gas	Specific Gas Constant $R \text{ J kg}^{-1}\text{K}^{-1}$	Molar Mass g/mol
Air	287.1	28.97
CO ₂	189	44.01
Water Vapour	462	18.02
Hydrogen	4124	2.016
Helium	2077	4.003
Methane	518	16.01

Density in gases is thus a function of temperature *and* pressure. Gases can be compressed in a number of different ways depending on how the gas temperature and thermal energy are controlled, a condition known as *process*. We can use the *gas constitutive law*, which relates pressure to density, according to the type of process:

$$\frac{P}{\rho^n} = \text{constant} \quad \text{Gas Constitutive Law}$$

Where n depends on the process:

Isothermal Process (Constant Temperature):	$n = 1: P/\rho = k \text{ constant}$
Isobaric Process (Constant Pressure):	$n = 0: P = k \text{ constant}$
Isentropic or Adiabatic Process*:	$n = \gamma: P/\rho^\gamma = k \text{ constant}$
Polytropic (Natural Processes):	$n = \text{other values determined by experiment.}$

* The parameter γ (gamma) is the *ratio of specific heats*:

$$\gamma = \frac{C_p}{C_v} \quad \text{Ratio of specific heats } (\gamma \text{ is referred to as } k \text{ in some texts})$$

where C_p and C_v are the specific heats at constant pressure and constant volume.

γ , C_p and C_v are constants with numerous applications that cross over between thermodynamics and fluid mechanics. These values are empirical (can only be determined by experiment) and are tabulated for a range of different gases. For most diatomic gases (eg all the main constituents of air) $\gamma = 1.4$.



Thought Experiment: Air Scales

Air, like all gases, expands and contracts as you apply pressure to it. Can you picture an alternative use for a 50 ml syringe blank, where you attach a hook to the plunger and use it to weigh fish? How would you calibrate the scale to give an accurate reading? Could you design it to weigh fish over a fairly wide weight range, say from 100 grams (tiddlers) to 10 kg (whoppers)?

The Effect of Humidity on Air Density

Humid air is less dense than dry air at the same atmospheric pressure. This is because water vapour is less dense than any of the principal components of air.

Dalton's law of partial pressures states that the mixed components of a gas contribute individually (ie they sum) to the total pressure of the gas. This means that in an air mass which increases in humidity

while its pressure is held constant, the heavier gas components (mainly O₂ and N₂) are displaced by a lighter one (H₂O).

If the water vapour pressure P_v and atmospheric pressure P_a are known then the net density can be calculated as:

$$\rho = \frac{P_v}{R_v T} + \frac{P_a - P_v}{R_a T}$$

where R_v and R_a are the gas constants for water vapour and air (see table previous page)

Atmospheric Variation of Density and Pressure with depth

The density of the atmosphere is not constant but decreases with height as the overlying fluid weight decreases and the gas accordingly decompresses. In this case we need to integrate through the depth of a fluid body to determine its total weight and hence pressure at some height or depth:

$$\Delta P = - \int_0^h \rho g dz \quad \text{Static Pressure equation for variable density}$$

Here both ρ and g may vary with height z . In practice the variation of g is negligible in the ~100 km depth of the Earth's atmosphere (3% of the Earth's radius) and is usually treated as constant.

The variation of density and pressure depends on *how* the pressure is applied, in particular how the temperature is controlled, and as such is a thermodynamic problem. A common solution used in conventional meteorology and weather reporting is the *barometric approximation*:

$$P(h) = P_o \left[\frac{T_o}{T_o - \lambda h} \right]^{\left(\frac{-g}{R\lambda} \right)}$$

where P_o is the pressure at sea level, $T_o = 288.15$ is a standard reference temperature (15 °C = 288.15 K), λ is the environmental lapse rate (the rate at which temperature decreases with altitude, -0.0065 °C/m), $g = 9.81 \text{ m/s}^2$ and $R = 287.1 \text{ J/kgK}$ is the specific gas constant for air.

Hence on a day of average pressure, the pressure at 1000 m altitude is:

$$P(1000\text{m}) = 101325 \left[\frac{288.15}{288.15 - 0.0065(1000)} \right]^{\left(\frac{-9.81}{287.1(0.0065)} \right)} = 89873 \text{ Pa}$$

Density and Compressibility: the Bulk Modulus

The density of any bulk material, whether solid, liquid or gas, varies with applied pressure. The variation depends on a material property known as the *Bulk Modulus K*:

$$K = - \frac{\Delta P}{\Delta \forall / \forall_0} \quad \text{Bulk Modulus (Units N/m}^2 \text{ or Pa)}$$

ΔP and $\Delta \forall$ refer to changes in pressure and volume respectively, while $\forall_0$ is the original (uncompressed) volume of the sample.

Gases have a small bulk modulus that varies depending on how the pressure is applied, while liquids typically have a very large bulk modulus reflecting the approximate incompressibility (see Lecture 2) of liquid and solid materials. As with density, the bulk modulus varies with temperature:

Temp (C°)	Water Modulus N/m ²	Bulk
0	1.98 x 10 ⁹	
4.4	2.04	
10.0	2.10	
15.6	2.16	
21.1	2.20	
26.7	2.24	
32.2	2.27	
37.8	2.28	
48.9	2.29 x 10 ⁹	

A gas bulk modulus can be determined from the Universal Gas Law and a description of the process:

Example: if the temperature of a fixed amount of gas is held constant as pressure is applied (Isothermal process), then $PV = \text{Constant} = C$:

Given a pressure change from P_1 to P_2 , $\Delta P = P_2 - P_1$ and $V_1 = C/P_1$, $V_2 = C/P_2$, thus

$\Delta V = C(1/P_2 - 1/P_1) = C(P_1 - P_2)/P_1P_2$ and the Bulk Modulus is given by

$$K = -\frac{(P_2 - P_1)P_1P_2}{(P_2 - P_1)/(C/P_1)} = P_2$$

This gives the surprising result that the gas bulk modulus, supposedly a property of the fluid, is in fact solely a function of the final pressure. The bulk modulus in general depends on conditions such as temperature and pressure, most obviously in the case of gases.

Thermal Expansion and Contraction

All materials react to temperature changes by changing in volume: as the molecular energy increases so the space between particles increases. In most cases an increase in temperature produces an increase in volume, with some notable exceptions such as the contraction of water as it warms between zero and about 4° C. If the volume is constrained so it cannot expand or contract, pressure changes will result (as in burst water pipes in winter).

Since there is no mass change in a fixed sample, thermal expansion involves density changes. In the case of gases, thermal expansion and density are related by the ideal gas law or its fluid analogue as described above. For liquids, volume expansion is governed by a *Thermal Volume Expansion Coefficient* β :

$$\Delta V = \beta V_o \Delta T \quad \text{Thermal Volume Expansion (Liquids)}$$

Where V_o is the initial volume and Δ represents difference from the initial value ie $\Delta T = T - T_o$.

β is usually a variable with temperature, for example water's expansion factor varies from a negative to a positive value at around 4°C.

Ethanol (methylated spirits is 95% ethanol) is a common liquid with a high and nearly constant thermal expansion. 100 ml of ethanol will expand or contract by 0.1 ml for every 1°C temperature change. Ethanol is used in thermometers because (as with mercury) its thermal expansion is steady and provides a linear thermometer scale.

Density and Dynamic Pressure

Since static pressure is driven by a fluid's self-weight and hence density, we would expect dynamic or momentum-induced pressure to have a similar dependence on density. It will be shown in Sections 4 and 5 that dynamic pressure in fact relates to the kinetic energy (due to velocity V) of the flow:

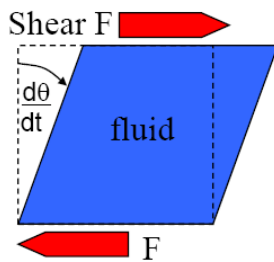
$$P \sim \rho \frac{V^2}{2} \quad \text{Dynamic Pressure (Nm}^{-2} \text{ or Pa)}$$

A casual examination of the static and dynamic forms of pressure reveals a pattern that should be familiar from first-year physics: they are *fluid analogues of potential and kinetic energy*. The mass m of a solid body having kinetic and/or potential energy is replaced by the mass per unit volume (or density ρ) of a fluid, producing a type of distributed energy which can be applied to elements of a flow or the entire body of flow or fluid mass.

This relationship between different forms of pressure and energy in a flow is one of the most important concepts in fluid mechanics. We will return to take a closer look at the dynamic form of fluid pressure in later sections, but now is a good moment to take this idea on board.

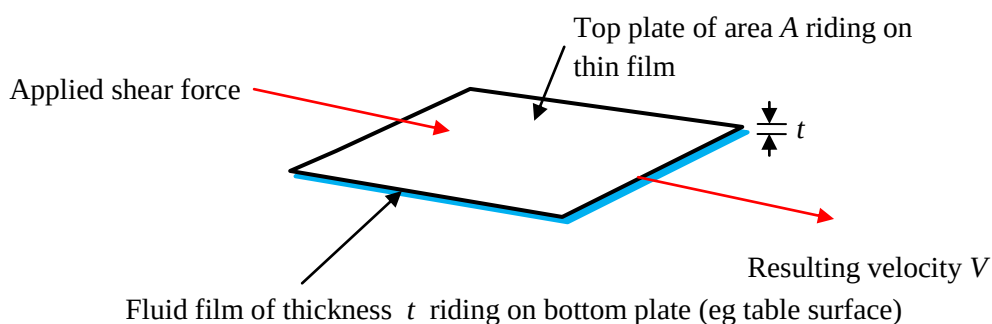
Viscosity

Viscosity is the “stickiness” of a fluid, expressed as a function of the shear force required to induce a given rate of rotation (see Lecture 1):



Element of fluid rotating under shear stress

A classic experiment attributed to Newton places a thin film of fluid between two plates. A shear force is applied to the top plate, and the resulting sliding motion is measured. Newton found that the velocity is proportional to the applied shear force, inversely proportional to the thickness of the fluid film, and also inversely proportional to the contact area of the plates:



Newton also observed that the shear force differs for different fluids, and that a larger force is required for fluids observed to be more “sticky”, ie with slower pouring rates.

Combining the values for force and contact area, Newton derived a shear stress equivalent to a shear pressure:

$$\tau = \frac{\text{Shear Force}}{\text{Contact Area}} \quad \text{Shear Stress – a form of lateral pressure (Units Nm}^{-2} \text{ or Pa)}$$

Considering the proportionalities observed in the data, Newton concluded:

$$\tau = \mu \frac{V}{t} \quad \text{where } \mu \text{ is a proportionality constant identified with viscosity or “stickiness”}$$

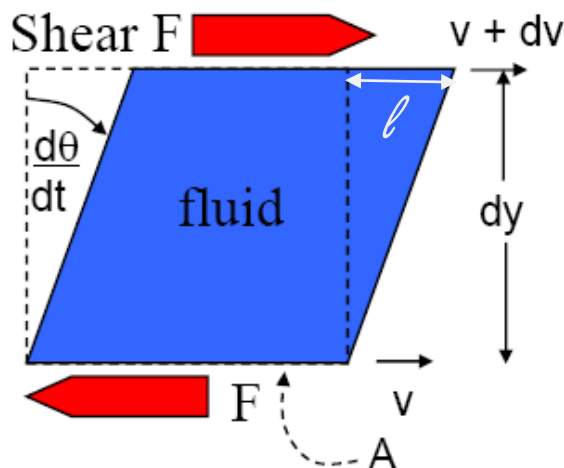
Newton reasoned that the fluid is “stuck” to the two plates and has the same velocity as the plates. The velocity of the fluid thus varies from zero at the bottom plate to V at the top plate, and varies as

$$dV/dy = V/t$$

where y is the vertical dimension.

Thus
$$\tau = \mu \frac{dV}{dy} \quad \text{Newton’s Law of Viscosity}$$

In Lecture 1 we saw that the rate of rotation of a fluid element is proportional to the shear force. Newton’s law provides an explanation for this and introduces the concept of a *velocity distribution*, dV/dy , the variation of velocity with depth: The entire element has a varying velocity which is unevenly distributed across the element, with the relative velocity increasing in the lateral direction.



The short edge length ℓ is the distance that the top edge will travel relative to the bottom edge at the relative velocity dV in a short time dt . This can be related to the angular displacement $d\theta$ in that time:

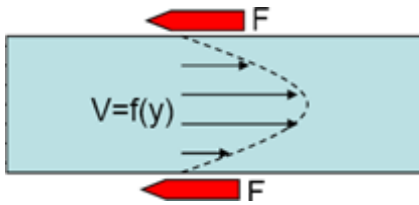
$$\ell = dV \cdot dt = d\theta \cdot dy$$

$$\text{thus } d\theta/dt = dV/dy$$

That is the rate of angular displacement is equal to the slope of the velocity distribution. From Newton’s Law we see the shear force is proportional to the rate of rotation, as observed in Lecture 1.

Viscosity is thus one of the key properties of any fluid, since it controls the rate of motion in response to applied forces, and also provides a link between applied forces and the distribution of velocity within the fluid. Both points are of fundamental importance to fluid mechanics.

A third important point is Newton’s reasoning in the above experiment that the fluid is “stuck” to the plates, ie there is no relative motion between a fluid and a boundary surface at the point of contact. A fluid assumes the same velocity as the boundary, ie if the boundary is a fixed wall the velocity at the wall is zero:



Pipe flow with velocity distribution. $V = 0$ at the pipe wall

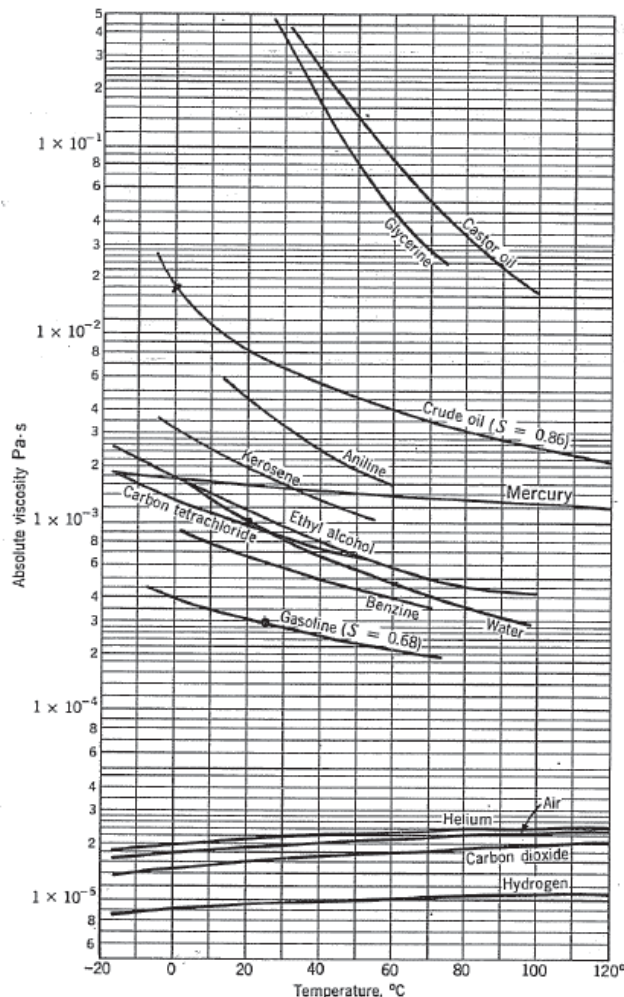
This “no slip” condition controls the initial values of the velocity distribution and also determines the intensity of friction forces wherever a fluid is in contact with a boundary. Both points will be returned to later in this course.

Viscosity varies with temperature, and is often published with different units. The standard SI units for viscosity are $\text{N}\cdot\text{m}^{-2}$ or Pascals-seconds:

$$\mu = \tau \frac{dy}{dV} = \text{Pa} \left[\frac{\text{m}}{\text{m/s}} \right] = \text{Pa}\cdot\text{s}$$

It is always a good idea to check your units by a unit formula such as this.

The chart below shows viscosities for a range of fluids over a -20 to 120°C temperature range.



Viscosities for certain gases and liquids (Streeter and Wylie, 1983:536)



Thought Experiment: The Lubricated Sliding Block

In first-year physics you may have tried the classic sliding block experiment, in which you slide a block down an inclined plane to determine the coefficients of static and active friction between the two surfaces. But what would happen if you poured a lubricating fluid such as oil (or honey even)

onto the inclined plane and *then* tried to slide a block on it? Would you still see a static friction coefficient, ie is there a shallow incline below which a well-lubricated block will not slide at all? Would the active friction coefficient depend only on the surface types and the normal force, or would there be an additional dependency on the sliding velocity?

Kinematic Viscosity

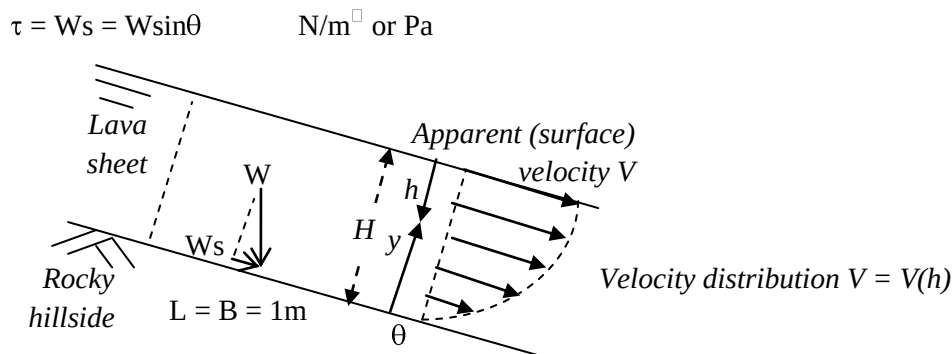
In most cases both the viscosity and the density of a fluid determine the motion resulting from a given applied force. A common parameter that arises is the ratio of viscosity to density, usually referred to as the *kinematic viscosity*:

$$\nu = \frac{\mu}{\rho} \quad \text{Kinematic Viscosity: units m}^2\text{s}^{-1}$$

Kinematic viscosity is important in the development of *turbulence*, a type of complex random flow that is the dominant feature of high flow rates and rough surfaces. This will also be returned to later in this course.

Example: The lava sheet in lecture 1 is now 50 cm thick and flows downhill at an apparent surface velocity V . If its density is $2,500 \text{ kg/m}^3$ (a very heavy fluid) and viscosity is $1,500 \text{ Pa.s}$ (very sticky), determine the distribution of velocity within the fluid. The hillside slope is 20° to the horizontal.

In the previous example we worked out the shear stress on the quasi-fluid:



Again we can use a free body analysis of a $1 \times 1 \text{ m}^2$ block perched, but now in uniform motion, at an angle θ on the slope. The weight force W is still the weight per square metre of fluid. We know the density, and the volume is of a $50 \text{ cm} \times 1 \text{ m}^2$ block, so we can determine the mass m and hence weight force mg of the block:

$$W = mg = \rho g \nabla = \rho g(1 \times H) = 2500(9.8)0.5 = 12.25 \times 10^3 \text{ N/m}^2 = 12.25 \text{ kPa}$$

So the maximum shear stress is:

$$\tau_o = W\sin\theta = 12.25 \times 10^3 \sin(20) = 4.16 \text{ kPa}$$

This is the maximum shear stress at the base (depth H) of the flow. At any depth h , the shear stress varies as the weight above that depth:

$$\tau(h) = \rho gh\sin\theta \quad (\text{Depth variable shear stress})$$

The stress varies from a maximum at the base (depth $h = H$) to zero at the surface ($h = 0$). The velocity distribution and shear stress at depth h are related by Newton's Law:

$$\tau(h) = \mu \frac{dV}{dy} = \rho gh\sin\theta \quad \text{or} \quad \frac{dV}{dy} = \frac{\rho}{\mu} gh\sin\theta$$

The velocity profile or slope, dV/dy thus varies with depth. The velocity must also vary with depth, from $V = 0$ at the base of the flow (due to Newton's non-slip condition) to $V = V_{\text{apparent}}$ at the surface.

To analyse the velocity as a function of depth h , note that h (depth) and y (height above base) are complementary: as h increases, y decreases. Thus *any* rate as a function of y is equal and opposite to the rate as a function of h :

$$\text{So } \frac{dV}{dh} = -\frac{dV}{dy} = -\frac{\rho}{\mu} g h \sin \theta$$

We can rearrange the derivative and integrate to solve for V : take a definite integral and use the boundary condition of $V = 0$ at $h = H$ to determine the integration constant:

$$dV = -\frac{\rho}{\mu} g h \sin \theta dh \quad \text{so} \quad V = \frac{\rho}{\mu} g \left[\frac{H^2}{2} - \frac{h^2}{2} \right] \sin \theta = 0.7 \text{ m/s at } h = 0 \text{ (apparent } V)$$

The velocity profile is thus parabolic. In fact this is an approximation that only works for very smooth surfaces and very viscous true fluids. The hillside would need to be very smooth, and the lava very hot, to match this solution.

Reading **Massey 1983 (Reading 1) Section 1.4 and 1.5 to 1.5.5**