

Lecture 2: Categories and Conditions of Fluid Mechanics

Fluid Mechanics can be divided into several branches with different methods of analysis:

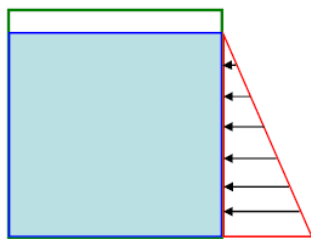
Fluid Statics – Motionless Fluid Masses

Fluid bodies are seldom truly static, due to the long lag time involved in the decay of motion by friction. Medium-sized reservoirs require from days to weeks to become even approximately stationary, and large reservoirs effectively never settle due to the dynamics of changing seasonal temperatures and flows.

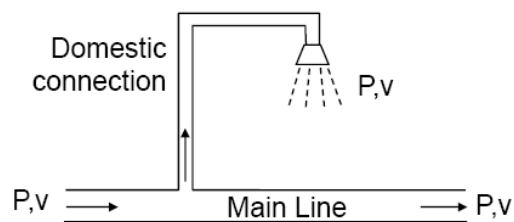
Nevertheless a static fluid is a good approximation for calculating the weight and resulting structural loads of fluid bodies. Designing a dam wall or pressure vessel depends almost entirely on static body forces (weight) and applied pressure forces (surcharge). Active forces due to fluid momentum and kinetic energy can in these cases be ignored or greatly simplified.

Fluid Dynamics – Fluid Masses in Motion

In many other cases however the motion of the fluid is a critical condition on its behaviour and the resulting forces and energies. In particular, the geometry (size, shape and elevation) of a channel or conduit can change from place to place along a flow line, inducing effects which may require one-, two- or three-dimensional descriptions.



Statics: dominated by hydrostatic pressure and its induced forces



Dynamics: Energy and geometry induce varying pressure and velocity

Fluid Dynamics: Steady, Unsteady, Uniform and Non-Uniform

Fluid Dynamics can be further classified into steady, unsteady, uniform and non-uniform flow:

Steady Flow

In steady flow, fluid velocities may differ from place to place, but do not vary with time. The fluid is said to be in steady state or *dynamic equilibrium*, ie pressure forces are in balance with viscous shear and momentum forces, so there is no acceleration.

Unsteady Flow

In unsteady flow, fluid velocities vary in both space and time: this usually arises in situations where dynamic forces are not yet in equilibrium, ie at the moment of a dam breaking, or where flowrates are changing as in a flood wave:



Steady flow: flow is *established* with all forces in balance on average



Unsteady flow: accelerations must be accounted for to balance forces

Steady and unsteady flow are categories of fluid variability in *time*. Similarly we can classify variability in *space* by the descriptions of uniform and non-uniform flow:

Uniform Flow

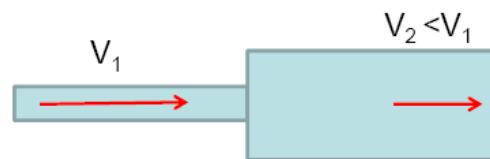
In uniform flow, fluid velocities are the same at all points under consideration, even though they may vary with time. This is very rare in practice, for example a long straight pipe with no end effects of interest.

Non-Uniform Flow

In non-uniform flow, fluid velocities vary in space and may also vary in time: this is the most general case, as systems of interest “do things” such as change direction, speed up or slow down as they move along.



Uniform flow: no change in velocity from point to point



Non-uniform flow: velocities change, creating local accelerations

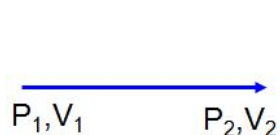
In reality all flows are both unsteady and non-uniform, but in practice an assumption of steady flow is often sufficiently accurate and provides useful simplifications.

One, Two and Three-Dimensional Flow

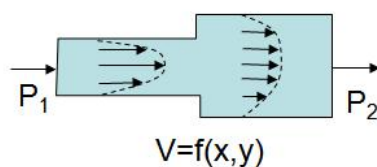
All fluids flow in three dimensions, along the length, width and depth of a pipe or channel.

However in practice we can often approximate a situation as being one-or two-dimensional, since the key changes in pressure, velocity etc occur along the length of a conduit, and to a lesser degree across the width or depth of the flow.

High powered computing has made it possible to routinely calculate fully three-dimensional flows, however the uncertainties associated with many physical details mean these complex models may be no more accurate than a simple one-dimensional model.



one-dimensional



two-dimensional



three-dimensional

While this unit introduces the concepts of some 2- and 3-dimensional methods, as well as some useful software, all methods presented in detail are 1- or 2-dimensional. Fully developed 3-dimensional methods are beyond the scope of the unit.

Conditions of Fluids

Climate, Humidity and Temperature

Climate, in particular humidity and temperature, are key determinants of the *hydrological* condition of landscapes and water-bearing systems. Hydrology is generally regarded as a separate science from *hydraulics*, and as such is covered in courses other than fluid mechanics.

However, these climate variables at a local scale do affect fluid flow properties in important ways. Temperatures are a significant controlling variable on fluid viscosity, density and thermal energy. Humidity is a factor in the density of air and the availability of moisture on free surfaces. An awareness of climate factors is generally a first essential of air and water resource engineering.

Pressure

Pressure is the single most important environmental or technical condition on fluid behaviour, dominating both the static and dynamic branches of fluid mechanics. A glance at the reading (see end of this lecture) is strongly recommended for an adequate understanding of pressure.

Pressure is induced by forces, including gravitational, dynamic and thermal forces, some of which may be inherited from the process of gas or liquid formation. As per Newton's second law, fluid forces arise in reaction to applied forces by the action of huge numbers of particle collisions against a container's surface, and as such on average always take the direction of a *normal* to the surface ($\perp$).

Since all *shear* forces are associated with fluid motion (see Lecture 1) we know that *any static force-inducing pressure has no shear stress component and so must be at right angles to the surface on which it acts*. In consequence, as many textbooks show by direct derivation, *the pressure at any point is the same in all directions from that point*.

Pressure is not a vector quantity, since it has no single direction. However it is also not a scalar or ordinary number quantity, which has no direction. Pressure is properly called a *tensor*, which is a type of poly-vector quantity which points in many directions.

Pressure produces reaction forces in all directions, but which can be resolved into a dominant or *resultant* force which is at right angles (normal) to a supporting surface. The vector description of the reaction force must include a unit normal vector at right angles to the surface. If A is the area over which the pressure P is applied, the reaction force as a vector is:

$$\underline{F} = PA\underline{i} \quad \text{Force-Pressure Vector Relationship}$$

Where $\underline{i}$ is a unit vector (of length 1 unit) *normal* to the surface. This relation can be simplified by mentally orienting the surface to a horizontal or vertical plane, at which point the vectors simplify to:

$$F = PA \quad \text{or} \quad P = \frac{F}{A} \quad \text{Pressure-Normal Force Relation. Units } \text{Nm}^{-2} \text{ or Pascals (Pa)}$$

Older units such as PSI (pounds per square inch), bar and mmHg are still used in some texts or manuals. Check that you know how these are defined / converted from Pascals.

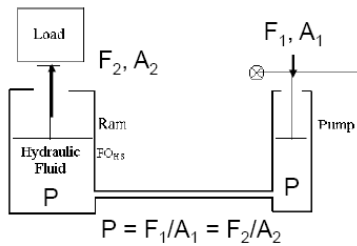
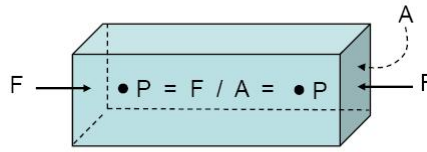
Applied or Externally-Induced Pressure

Applied Pressure is any pressure created by forces applied externally to a fluid. A pressurised fluid must be in an enclosed container or the induced shear forces and flow reactions will result in the fluid simply moving away from the applied force.

Pascal's Law states that *pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel*:



Blaise Pascal, 1623-1662



In a hydraulic jack, a small force F_1 over a small area A_1 induces a pressure P which is constant throughout the fluid body. This pressure acts on the larger area A_2 with a larger force F_2 , lifting a heavy load.

The classic application of Pascal's Law is the Hydraulic Jack.

For example, atmospheric pressure applied to the surface of a body of fluid is an applied pressure and as such is constant throughout the body.

Gauge and Absolute Pressure

Pressures are frequently measured relative to atmospheric pressure, particularly in situations where the same atmospheric pressure applies at all points in a system. This “relative” form of pressure is known as *gauge pressure*. Gauge pressure is calculated by subtracting the atmospheric contribution of pressure (about 101,300 Pa at sea level, or 90,000 Pa at 1000 m altitude) from the absolute or total pressure in a system.

$$P(\text{gauge}) = P(\text{absolute}) - 101,300 \text{ Pa}$$

or
$$P(\text{absolute}) = P(\text{gauge}) + 101,300 \text{ Pa}$$

(for 1000 m altitude such as UNE, estimate mean atmospheric pressure as 90,000 Pa)

Gauge pressure is a standard description of many types of fluid systems. Atmospheric pressure, being the same at all points, creates no differential effect and can be ignored. *Most importantly a gauge pressure of zero indicates the point at which compression turns to suction, producing a change in the direction of induced forces.*

In contrast, absolute pressure is important for calculations involving the thermodynamic state of gases, for example the true density of a gas is related to the absolute temperature and pressure of the gas by the ideal gas law. For an amount n (in moles) of gas:

$$PV = nRT$$

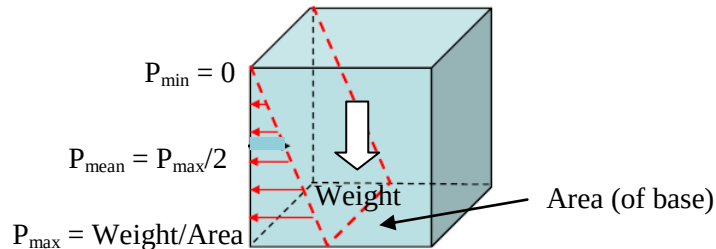
Ideal Gas Law

where P and T are absolute pressure (Pascals) and temperature (Kelvin). R is the universal gas constant $8.31 \text{ JK}^{-1}\text{mol}^{-1}$.

Solving fluid mechanics problems often involves a choice between gauge and absolute pressures. If in doubt, or until better judgement develops with practice, it is recommended that all pressure calculations be done in absolute terms, ie inclusive of atmospheric pressure. Conversions to gauge pressure can be provided at the end of a problem or as a consideration within a problem by subtracting a standard atmosphere, say 90.0 or 101.3 kPa.

Static or Weight-Induced Pressure

Static pressure (called *Hydrostatic* in the common case of water) is the pressure induced by the static weight of a fluid body acting upon itself. This varies with depth, since the weight above a horizontal plane surface increases with increasing depth. Static pressure is a *body force*, in that it is induced by the fluid body itself. Gravity combines with the mass to produce force and hence pressure:



Static pressure, a gauge pressure, produces a triangular distribution of induced forces in a tank.

A static or hydrostatic pressure distribution is a linear variation of pressure with depth forming a triangular pressure profile. Since this starts at a zero pressure (the apex of the triangle) regardless of the overlying atmospheric pressure, the *static pressure distribution is a gauge pressure*. Because it varies with depth we often use the *average or mean pressure*, which is half the maximum pressure.

The absolute pressure of a fluid body under atmospheric pressure is the static pressure varying with depth plus the atmospheric pressure constant throughout the depth. An important result is that *the pressure is the same at all points of the same depth*:

$$P = P_{atm} + \rho gh \quad \text{Pressure is the sum of applied (absolute) and static (gauge) pressures.}$$

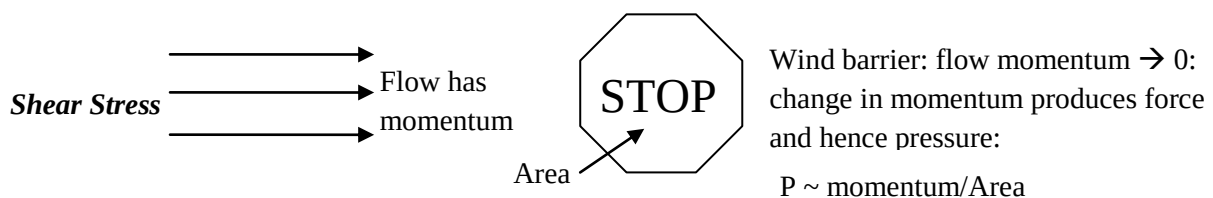
Thought Experiment: Hydrostatic pressure

What would happen if you taped a 3 m length of 5-mm diameter PVC tubing to the side of a house and injected water into the bottom end until the tubing was full? A 10 ml syringe blank would be useful. Would there be any difference in the feeling of pressure or force in the syringe if you used 10-mm diameter tubing instead? Is it a gauge pressure or an absolute pressure you are feeling (ie is atmospheric pressure pushing on both ends of the system)?

Dynamic or Momentum-Induced Pressure

Dynamic pressure is a result of fluid motions impacting on the walls of conduits and on surfaces and bodies in the flow. Any change in the direction or magnitude of flow velocity results in reaction forces and pressures that must be contained by structural vessels, and which can be exploited in the design of turbines and aircraft.

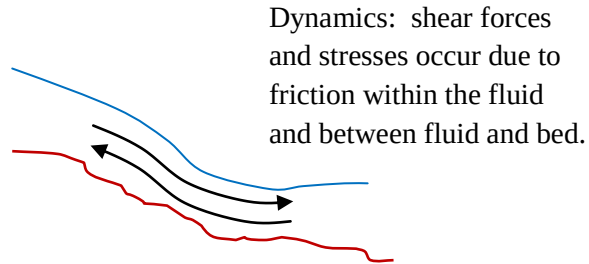
Dynamic pressure is another type of weight-induced pressure, induced in this case by the acceleration of the fluid due to forces other than gravity eg wind pressure on a sign post:



As we saw with the lava flow example in lecture 1, the weight of a mass of fluid on a sloping surface induces shear stresses in the fluid and its supporting surface. Shear stresses depend on the state of flow:

—
$P = \rho gh$ $\tau = 0 :$ (No Shear)

Statics: all shear forces are supported by the walls of the container so there is zero shear stress

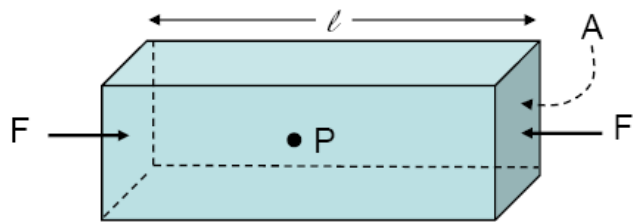


As with normal pressure, shear stress has units of N/m^2 or Pascals. Shear stress can be induced by dynamic weight forces, however in a static fluid there can be no shear stress, only normal pressure.

Pressure as Stored Energy

Pressure can be regarded as a measure of the elastic energy per unit volume stored in a fluid as a result of induced and applied forces:

$$P = \frac{F}{A} = \frac{F \cdot l}{V} = \frac{E}{V}$$



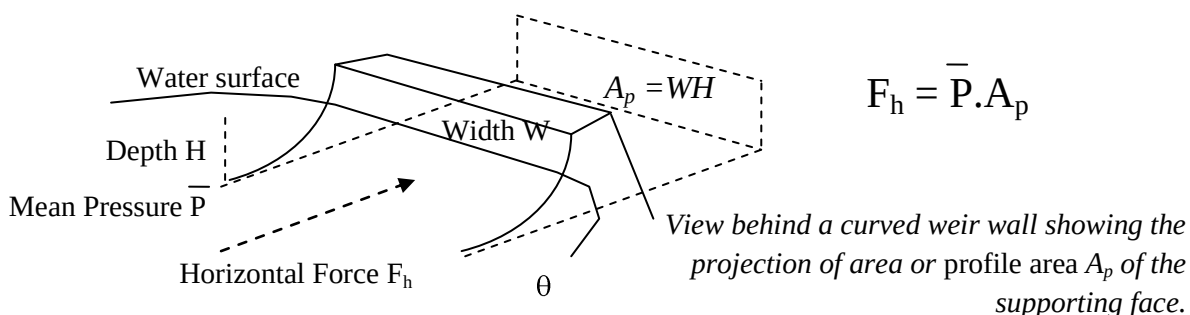
F is a force applied to a fluid body to pressurise it. The fluid compresses very slightly due to its elastic modulus. The action of the force over that distance requires a certain amount of mechanical energy. This is stored in the fluid as elastic energy. The stored energy per volume equals the pressure.

As with pressure, energy has no particular direction, or rather the same value in every direction.

3-Dimensional Pressures as 2-Dimensional Components

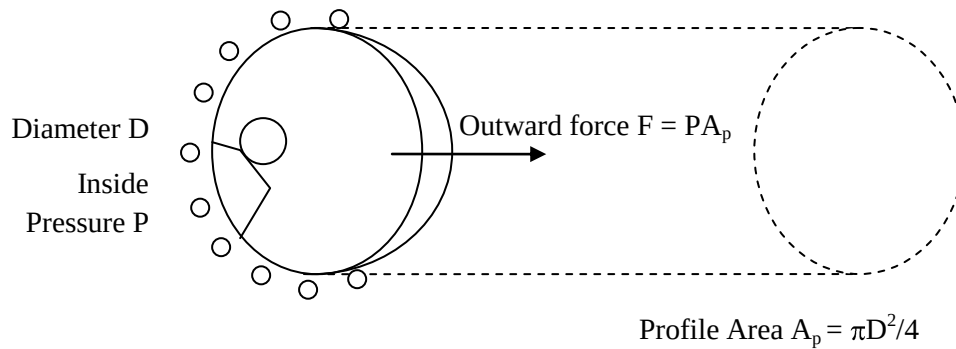
The total force in any given direction (say vertical or horizontal) due to pressure on a curved surface is equal to the pressure multiplied by the *projection of area*, or *profile area*, in that direction:

For example the horizontal force on a curved dam wall:



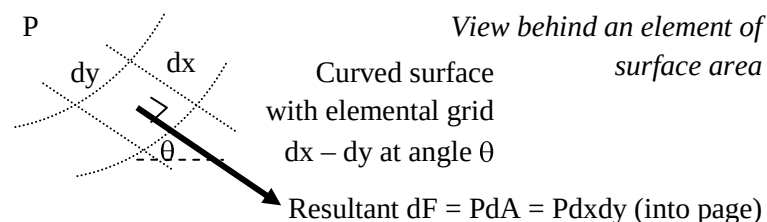
The full surface area of the sloping surface is not required to work out the horizontal force: The projected area multiplied by the average static pressure is all you need.

For another example, the outward force on a bubble window in a space station:

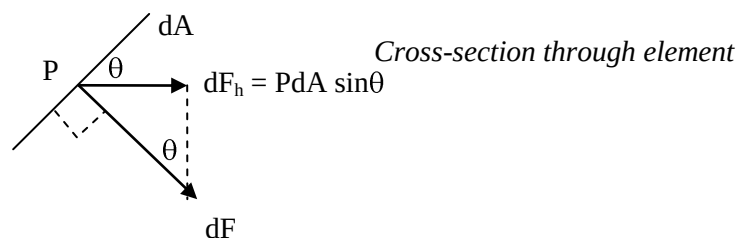


$$F = \bar{P}A_p \quad \text{Mean Pressure – Profile Area Relationship}$$

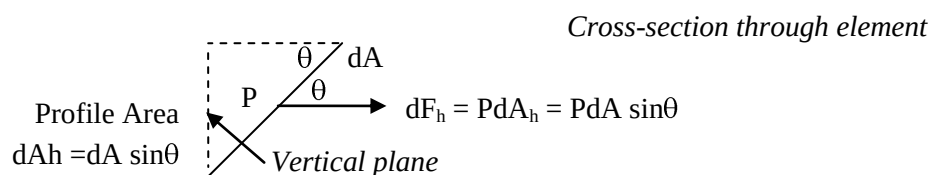
This very useful result arises because of the tensor form of pressure, in that we can select any direction, such as the principle horizontal or outward directions shown above, and calculate pressure forces on a plane in that direction. We start with an element (as in surface integration) of the dominant or resultant force:



The resultant is projected in the horizontal by using the sine of the angle to horizontal:



This is equivalent to applying the pressure against the profile area normal to that direction:



This simplifies the integration of elements of force over the surface: The total force is projected onto a 2-D plane which integrates as the overall area (as projected rectangles or circles) of the profile.

If that seems too easy, use calculus to integrate the horizontal resultant of a constant pressure over a hemispherical surface. Alternatively, try this mind-bending thought experiment:

Thought Experiment: The Impossible Self-Propelled Hemisphere

A hemispherical globe with a flat bottom surface is filled with compressed air and placed on a table. If the reaction forces on the plane and curved faces are not exactly the same, why doesn't it fly away?

Reading

Massey 1983 (Reading 1) Section 1.2

Douglas *et al.* 1979 (Reading 2) Sections 2.1 – 2.7

