

PHYS207
Fluid Mechanics

Home Laboratory Manual



Vortex motion in a cropduster's wake (image courtesy DANTEC measurement Technology Inc.)

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INTRODUCTION

This home laboratory manual is provided as an opportunity for you to gain experience with the handling and measurement of fluids in experimental situations. Fluid mechanics is primarily an experimental science, so this is an important component of your training.

Given the need to strike a balance between engaging your interest and committing your time and financial resources, we have selected a range of activities that efficiently cover the main topics of the subject using materials that are affordable and commonplace. Items such as plastic tubs, PVC tubing and syringe blanks are usually available in budget supermarkets, hardware stores and chemists.

The exercises in this manual are designed to be fun and interesting, much like a “Dangerous Book for Boys” except with second year university students in mind and not as dangerous as might be expected of boys.

With the word “dangerous” in mind, ALWAYS GIVE CONSIDERATION TO SAFETY ISSUES AND MAKE SAFETY YOUR FIRST PRIORITY. See the safety instructions in the following section.

The exercises include some inductive or purely observational work, where you try things out and think about what happens; some deductive work where you make measurements and perform analytic calculations; and some instrumental work, where you construct devices and instruments to make measurements with. Some are easy, some are hard. We won’t tell you which is which, because we don’t want to put you off the hard ones.

We have included, as an appendix, notes on uncertainties, the use of graphs, good experimental procedure and technical report writing. Please ensure that your results are written up as a properly formatted (not handwritten) technical lab report.

SAFETY INSTRUCTIONS FOR HOME LABORATORY WORK

All activities in this manual are designed with safety in mind. The following safety instructions must be read and followed exactly as you would in a supervised laboratory.

If you are doing university work off-campus you are required to complete a general fieldwork risk assessment form and submit it to your supervisor or unit coordinator for approval. This will ensure you are covered by the University's insurance policies. (www.une.edu.au/hrs/handbook/04/04.13b-fieldworkrisk.doc)

General

1. Do not conduct experimental work of any kind unless another person is present.
2. All injuries in particular eye injuries must be considered as serious and be given immediate first aid.
3. Closed and preferably non-conductive shoes must be worn. For outdoor work wear a hat, long sleeves and sunscreen.
4. Smoking, eating and drinking are not recommended while engaging in experimental work.
5. Extreme care must be taken when working with open watercourses, high pressure water supplies and electrical equipment in conjunction with water. Consideration must be given to safety measures against falling into water or electrical contamination or other risks in your own home, college or surrounds.

Open channels in flood are a particular hazard for hydraulic engineers and scientists. Trunk drains are often constructed within grass swales which may be steep and slippery when wet. *EXERCISE EXTREME CARE WHEN WORKING WITH OR OBSERVING MAJOR OPEN CHANNEL FLOWS.*

General Safety Issues

1. Handle electrical equipment carefully and according to instructions. Do not accept any risk of electrical contamination of water or conductive conduits.
2. Personal protective equipment such as closed, non-conductive shoes and sun protective gear must be worn as directed.

Section 1: Properties of Fluids

Pressure and Density

1.1 Hydrostatic pressure is a function of depth only

This exercise is designed to give you a direct hands-on feel (literally) for hydrostatic pressure and resultant forces.

You will need:

- 1 x 50 ml syringe
- 3 m length of 5 mm diameter clear PVC tubing
- 3 m length of 10 mm diameter clear PVC tubing
- A ladder and pole or high wall
- Gaffer or clear tape
- A foldback (stationery) clip

As a child you would have discovered the ‘water-squirtin’ fun of plastic syringes. Here’s a way to measure (and experience again) the raw power behind the plunger!

1. Use the ladder to tape both lengths of the PVC tubing 3 m up the pole
2. Fill the syringe to 50 ml and insert it into the bottom end of the 5 mm tubing. You may need to experiment with taping up the nozzle of the syringe to get a good fit inside the tubing
3. Inject the 50 ml into the tube. It should rise about 2.5 m inside the tubing.
4. Note the increasing pressure required to fill the tubing as the water level rises. Mark the position of the water surface.

The pressure in the syringe at a height (H) of 2.5 m is $\rho gH = 9800(2.5) = 24.9 \text{ kPa}$, about 25% of an atmosphere. This amounts to a force of about 15 N or 1.5 kg depending on the model of syringe, specifically the diameter of the plunger.

5. Pinch the tubing to close it, remove the syringe and then put your thumb over the end. Can you resist the water pressure with your thumb? How else could you measure or sense the pressure directly?
6. Empty the tubing and repeat the exercise using the 10 mm tubing. This time the fluid level will rise only 0.6 m. You should be able to fill it to 2.5 m with another 3 refills.
7. Can you detect any difference in fluid pressure with your thumb? Don’t mistake the flow rate for the pressure; the flow rate and pressure force will be larger, but the pressure the same as before.
8. Fill the syringe again and squirt it straight up into the air. Can you reach a height of 2.5 m? What about 3 m? Why can’t you squirt as high as you want to? Sorry to gross you out, but can you spit anywhere near that high?

Conclusions

You can create large fairly hydrostatic pressures with only a small amount of water. The pressure is the same regardless of the diameter of the vertical tubing.

1.2 Boyle's law: Negative pressure (suction) fishing scales

Some (not many) of the exercises in this manual require a reasonably accurate measurement of weight. If you don't have a set of scales, here's a way you can build and calibrate your own, though these are probably most ideal as portable fishing scales rather than lab scales.

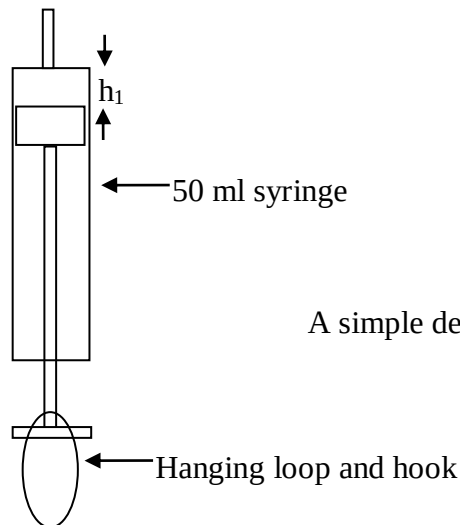
The accuracy of this design improves with the weight, so you can be more scrupulously honest the more impressive your catch. Possibly the most remarkable thing about this design is the load range: while a conventional spring-loaded fishing scale is accurate over a fairly limited range, typically 3 – 5 kg, a suction scale can (with variable accuracy) weigh between 100g and 10kg. Just what you need for weighing fish!

You will need

- 1 x 50 ml syringes
- A short length of 1 mm wire
- A simple engraving tool such as a heavy needle
- A candle
- A ruler with 1 mm markings

The syringe must be absolutely clean and dry to work with minimum friction. If you've previously filled it with water or any other liquid, clean it with detergent if necessary (eg to remove oil) and dry it in a 110°C oven for a few minutes.

1. Use the candle to heat the needle. Pierce the thumb end of the plunger with the hot needle to make a hole.
2. Thread a loop of wire through the hole, and twist-tie it to make a loop with a hook-shaped end.
3. If the barrel does not have accurate 1 mm graduations use the (cool) needle to engrave 2 mm graduations on the barrel of the syringe. Label these at 10, 20, etc mm, with an extra tick mark for 5, 15, etc.



A simple decompressed air fishing scale

4. Find a reference weight to hang off the scale. A 2.4 litre Berri juice bottle has a handy carrying loop and weighs exactly 2555 g (precision chem. lab scale) when filled to the brim, capped and dried off.

5. Extend the plunger to an initial length h_1 , say 10 mm, then seal off the nozzle with your thumb.
6. Hook the scale onto the reference weight and lift it. Observe the plunger in extension until all movement stops (friction can slow it down so keep watching)
7. Read off the new extension of the scale. For the 2.4 litre Berri bottle, this should be about 16 mm
8. Calculate a scale factor $K_s = M/(1 - h_1/h_2)$ where M is your reference mass and h_1 , h_2 are the extensions before and after load.
9. Use the scale to measure and calculate other masses as $M = K_s(1 - h_1/h_2)$

Air is a compressible fluid that closely obeys the ideal gas law at everyday temperatures. The density form of the gas law can be written as:

$$P = \rho RT$$

Where P is absolute pressure, T is absolute temperature in Kelvin and R is the specific gas constant for air, $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$

We can calculate the density of air at room temperature and pressure:

$$T = 23^\circ\text{C (say)} = 23 + 273 = 296 \text{ K}$$

$$P = 101300 \text{ Pa (1 standard atmosphere)}$$

$$\text{So } \rho = P/RT = 101300/(287 \times 296) = 1.19 \text{ kg/m}^3$$

If we start with a syringe at normal pressure and then decompress it, by blocking the nozzle with a thumb and pulling on the plunger, the mass of air in the barrel is constant and the air density decreases in inverse proportion to the volume, which itself is in direct proportion to the length of the barrel:

$$M = \rho_1 V_1 = \rho_2 V_2$$

$$\text{Thus } \rho_2 = \rho_1 V_1 / V_2$$

So long as the temperature is unchanged, the pressure is directly proportional to the density:

$$P = \rho RT = k\rho$$

$$\text{Or equally } P_2 = P_1 V_1 / V_2$$

This is just Boyle's law, $PV = \text{constant}$ at constant temperature.

In a syringe, volume is proportional to plunger movement over a distance:

$$V = \pi D^2 h / 4 \quad \text{So } V_1 / V_2 = h_1 / h_2$$

Let's say atmospheric pressure on a given day is 101,000 Pa

Then $P_1 = 101,000$ is the pressure inside the syringe before you apply the load, with the plunger set at some initial extension, say 10 mm. After you load the scale, the plunger moves to h_2 , say 15 mm.

$$\text{Then } P_2 = P_1 V_1 / V_2 = P_1 h_1 / h_2 = 101000(10/15) = 67333 \text{ Pa}$$

As we'd expect, with the air column inside the plunger stretched by a suspended load, the internal pressure has dropped. How can we relate this to the load on the scale?

The pressure difference between the inside and outside of the syringe creates a force on the plunger:

$$F = \Delta P A = (101000 - P_2) \pi D^2 / 4 = (101000 - 101000(h_1/h_2)) \pi D^2 / 4$$

$$\text{Which relates to the Mass: } M = F/g = 101000(1 - h_1/h_2) \pi D^2 / 4g$$

For a 27.5 mm diameter plunger, $h_1/h_2 = 10/15$ indicates a mass of 2.04 kg, accurate to within about 10 – 20 grams.

So by reading the plunger movement from h_1 to h_2 and applying the current atmospheric pressure we can calculate the mass suspended on the scale. Ideally we would simply calibrate the scale to a known mass and derive a composite term that includes atmospheric pressure, gravity and the area of the plunger:

$$M = K_s(1 - h_1/h_2)$$

Where K_s is a scale constant. For a typical 50 mm syringe, $K_s \sim 6.0$

There are two main uncertainties in this measurement:

- Atmospheric pressure varies by as much as 10%. When you go fishing, you should always check the weather, particularly for low pressure (bad weather) conditions, so that your scale does not overestimate the weight of your catch, and so you don't get washed off the rock.
- Friction inside the syringe may cause the plunger to stick. Lubrication may be ineffective as it tends to leave a residue which worsens the scale's performance in the long term. Be sure to keep your scale clean and dry. Jiggle the loaded scale to make sure it is moving freely, and note the range of measurement.

An estimate of the static friction error in the scale can be added in as a correction:

10. Open the syringe to about 30 mm and block it with your thumb.
11. Pull the syringe open to about 40 mm and release.
12. The plunger returns partially to the first position, with friction bringing it to stop at a final extension of 31 mm, say.

At that pressure the active friction force exceeds the pressure force and the plunger stops. Note the final extension h_2 of the scale.

Say $P_1 = 101300 = \text{atmospheric pressure}$

Then $P_2 = P_1 h_1/h_2$

Pressure equivalent to active friction

$F_a = (P_2 - P_1)A = (P_1 h_1/h_2 - P_1)\pi D^2/4$

Force equal to active friction force

$M_a = F_a/g$

Mass error of scale due to active friction

The calibration equation is now

$K_s = (M - M_a)(1 - h_1/h_2)$ for reference mass, initial and final extensions as before.

And $M = K_s(1 - h_1/h_2) + M_a$

Mass scaling equation

See Viscosity for an experiment that relates dry friction to wet viscous friction.

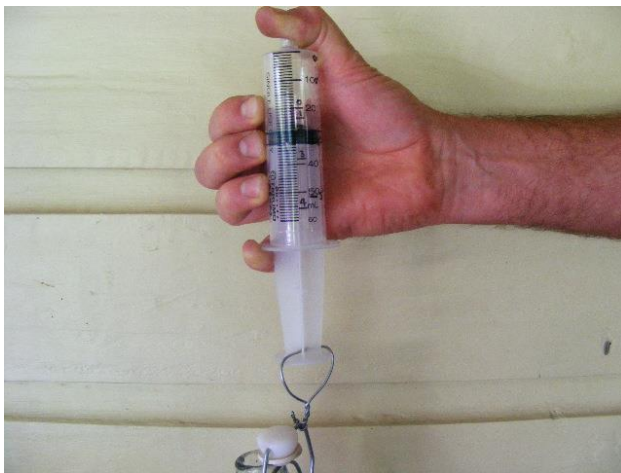
Note that the length scale does not have to be in mm or any other standard unit, since the scaling is purely by the ratio of h_1/h_2 . An extension of from 10 to 15 mm indicates the same weight as an extension from 20 to 30 mm.

You could mark a scale in kg on the barrel, however that would assume you always use the same starting h_1 . The beauty of this device is that for larger masses you can start with a smaller h_1 , giving a wider range of measurement.

If anyone can come up with a scale that can be read for mass with any initial starting point, I will seriously consider what sort of prize might be deserving.

Conclusions

Boyles' law provides a simple and reasonably accurate determination of weight based on the changing volume of a compressible fluid column. A suction scale is very simple to construct and a lot of fun to work with if you're not concerned about gram-kilogram accuracy. Impress your fishing buddies, if that's at all possible.



1.3 Boyles law: Negative pressure (suction) barometer

The fishing scales in the previous exercise depend in part on the atmospheric pressure on the day you make a measurement. You can easily look up current atmospheric conditions for your location via the Bureau of Meteorology website, OR you can construct your own barometer.

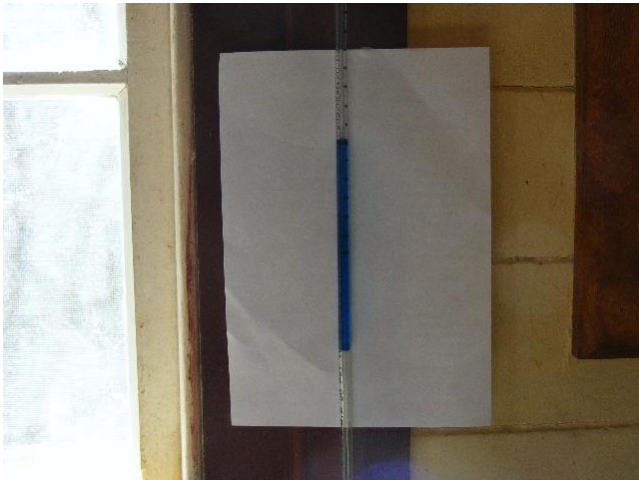
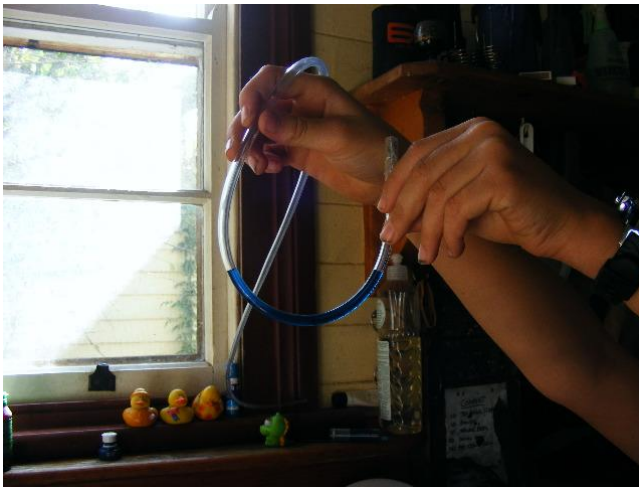
You will need:

- A 1.2 m length of 5 mm clear PVC tubing
- A 10 ml syringe
- A marker pen
- Some blu-tack
- Some pins or thumbtacks

Barometers are generally constructed from a sensitive membrane attached to a guide coil spring and clock-face gauge, or from expensive quantities of high grade mercury. Assuming you're neither rich nor of Swiss artisan stock, you can still make a pretty fair barometer with better than 90% accuracy.

1. Make two marks on the PVC tubing, one at 10 cm from one end and the second at 90 cm from the same end. That end will be the top of the barometer.
2. Use a pin to hang the barometer on a wall from the other (bottom) end.
3. Fill the syringe with water and fit it to the top end of the tube (you may need to add some tape to the nozzle to fit the tubing). Inject a 10 cm length of water into the tube, using the first mark as a guide.
4. Bend the tubing into a U tube with the 10 cm of water in the bottom of the U. Carefully remove the syringe. The water will sit at the bottom of the U.
5. Gently move the 10 cm slug of water along the tube by manipulating the U, keeping the slug contained in the U. If the column of water breaks or gets bubbles in it, re-form it by tapping the tube or start again.
6. When the whole slug has moved past the second 90 cm mark, ie where there is a 90 cm length of air at the top of the barometer, plug the top of the barometer with a small (pea-sized or smaller) pellet of blu-tack. Stick a pin through the blu-tack plug to pin the top end to the wall. You might need to stand carefully on a chair.
7. The 10 cm slug of water is now hanging suspended by negative pressure inside the tubing. The air column will stretch slightly to a little more than 90 cm. External and internal pressures now balance the mass of water. This simple device is a barometer.

See the photos overleaf



Calibrating the barometer

Lets' start by again assuming the atmospheric pressure on a given day (the day we make this barometer) is $P_o = 101000 \text{ Pa}$.

Then the initial pressure inside the syringe is $P_o = 101000 \text{ Pa}$

Once the barometer is mounted the slug of water is supported by a pressure difference:

$$(P_o - P_1)\pi D^2/4 = \rho gh\pi D^2/4$$

Or as usual $(P_o - P_1) = \rho gh$

So $P_1 = P_o - \rho gh$

And $P_1 = P_o h_o/h_1$: $h_1 = P_o h_o/P_1$

Subsequently the external atmospheric pressure rises or falls to a new value, P_e . To provide the same lifting force the pressure inside the barometer changes to P_2 as the plunger moves to a new extension h_2 :

$$(P_e - P_2) = \rho gh$$

$$P_e = P_2 + \rho gh = P_1 h_1/h_2 + \rho gh$$

We can now calibrate a scale on the barrel of the plunger by picking some reference pressures and calculating the corresponding h_2 :

$$h_2 = P_1 h_1/(P_e - \rho gh) = P_o h_o/(P_e - \rho gh) = 101000(0.9)/(P_e - 9800(.1))$$

This is a simple tabular problem:

P_e	h_2
98000	0.93692
99000	0.927362
100000	0.917996
101000	0.908818 (note the small initial extension > 0.9 m due to the weight of water)
102000	0.899822
103000	0.891002

Mark these positions on the barrel. So for example a pressure of 98000 Pa (980 hectapascals in weather reporting units) corresponds to an extension of 93.7 mm.

Conclusions

This barometer is sensitive to sudden pressure fluctuations such as occur inside a house on a windy day. You may find the instrument fails or even breaks in bad weather.

It makes a lot of sense to observe the barometer over a season or two and compare its readings with atmospheric data from the Bureau of Meteorology. After a season or two of regular observations you will find the barometer has leaked and is no longer accurate, but your own sensitivity to pressure and weather will be highly developed.

Strictly speaking this theoretical calibration model neglects air temperature as a component of the ideal gas law, which would mean we need a separate scale for high and low temperatures. This is not a problem so long as you hang it indoors where the temperature range is more controlled than the external seasonal temperature.

1.4 Pascal's Law and the Hydraulic Jack

Pascal's law states that the pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.

A practical consequence of Pascal's law is the hydraulic jack, in which load-bearing pistons of unequal area create unequal lifting forces for the same shared internal pressure.

The principle is analogous to a lever with unequal arms producing unequal forces for the same applied moment or torque around the fulcrum. In either case a large and useful amplification of effort is produced.

You will need:

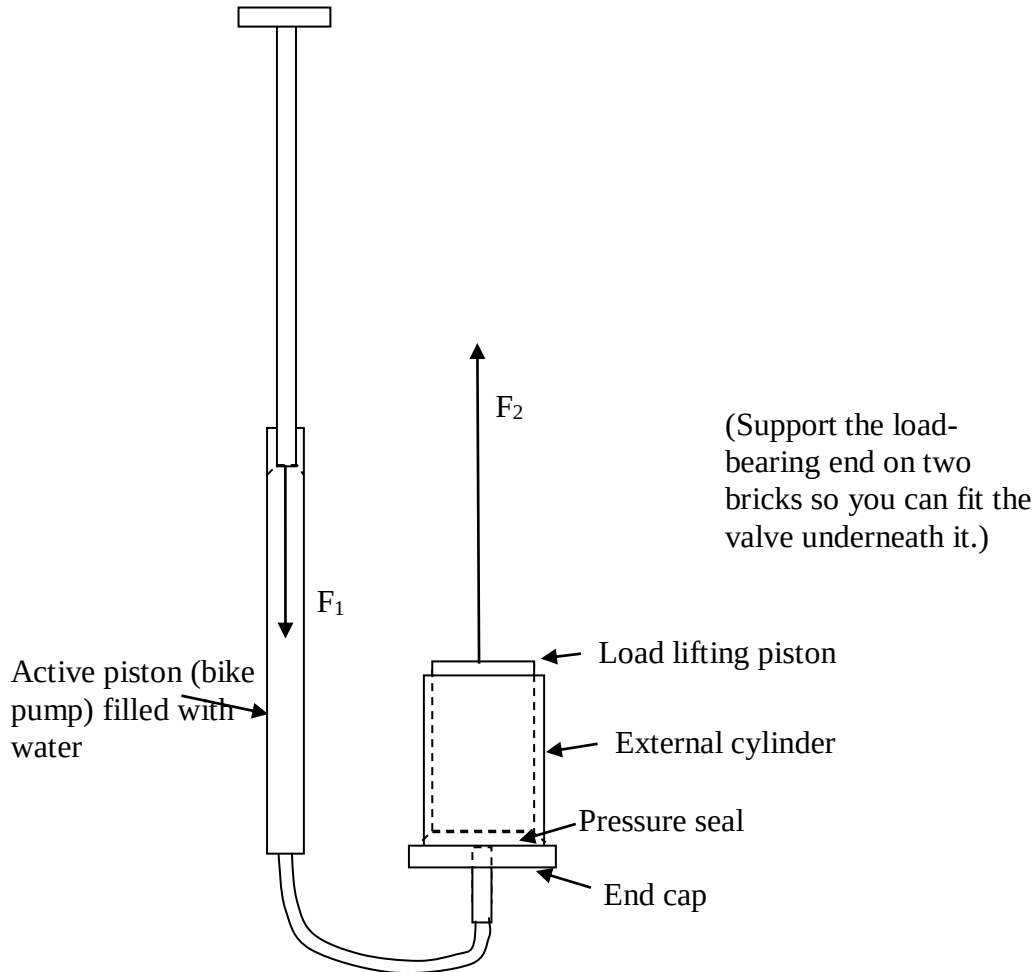
- A bicycle pump with valve connector.
- An old bike tube with valve to fit the connector.
- A 15 cm length of 40 mm SH PVC sewer pipe
- A 15 cm length of 32 mm " " "
- An end cap to fit the 40 mm pipe
- A small piece of tarp or flexible plastic sheet (rubber sheet is even better).
- 1 packet of blu-tac
- Fast-setting glue such as 5-minute araldite.
- 3 bricks
- 1 half-brick

The task is to use these materials to construct a hydraulic jack with a 2:1 lifting ratio. The idea is that the two lengths of PVC pipe fit one inside the other to create a load-lifting piston and cylinder. The bike pump and valve are connected to this as the acting piston. Since a typical bike pump is 25 mm in diameter, while the outside diameter of the lifting piston is about 36 mm, we have an area ratio of $36^2/25^2 = 2.1 =$ the ratio of forces acting on each piston for the same applied pressure.

1. Cut a 40 cm diameter circle from the tarp and place it flat on the bench.
2. Centre the 32 mm pipe on the tarp circle and squash as much blutac as you need inside the bottom of the pipe to create a plug seal with the tarp stuck on the outside end of the pipe. Use the end of the bike pump to squash the blutac inside.
3. Insert this assembly into the 40 mm PVC pipe, open end first and ensuring the tarp creates a good internal seal between the two pipes. The two pipes are of similar internal and external diameter, so they will fit together with about 1 mm clearance all round. The tarp and blutac create a pressure seal.
4. Cut the valve off the old tyre with a few cm of rubber to spare around it.
5. Drill a hole in the centre of the end cap and glue the bike valve into the hole, with the valve's screw end to the outside.
6. Glue the end cap to the 40 mm pipe, still with the valve's screw end outside.

7. Once the glue is set, fill the bike pump with water by inflating it with the connector end under water. Keeping the water in the pump, attach the connector to the valve.

You now have a simple 2:1 hydraulic jack (illustrated not to scale):



8. Test the seals by pumping and drawing water gently back and forth between the lifting and active pistons. The lifting piston should rise and fall at about half the velocity you apply to the active piston.
9. Now place the half brick on the load lifting piston and try that action again. It should seem somewhat lighter than half a brick.
10. Lastly place the whole brick on the load lifting piston and use the half brick to assist your hand on the active piston. You should only need to apply the small additional force required to overcome friction on the pressure seal inside the pump. It should feel as though the jack is lifting the load for free.

Check your theory: Since $P = F/A$ and this is constant (according to Pascal's law) throughout the jack, we have:

$$P = F_1/A_1 = F_2/A_2$$

Thus $F_1/F_2 = A_1/A_2$

Conclusions

With the active area A_1 roughly half the lifting area A_2 we see the active force F_1 is half the lifting force F_2 .

Very convenient, but notice the limit imposed by continuity on the lifting range: the load-bearing piston moves only half the distance of the active piston. Also note that as you pump hydraulic fluid into the lift cylinder you must lift the weight of the fluid as well as the payload.

The main benefit of this exercise, if you try it, is to achieve an effective pressure seal while recognising the inefficiency that the resulting friction forces produce. Hydraulic engines and machinery use light oil to lubricate the pistons and seals, and also because the low fluid density provides a better lift range, both of which increase lift efficiency.

1.5 Breaking the water column

Sometime in the mid 1980's the mystery of how a tree draws water up its trunk to its leaves was solved; biologists measured a combination of capillary and cellular action that sucks water from the ground to sometimes enormous heights, against gravity and indeed against the physical capacity of water to sustain such large suction pressures.

Why is it that water cannot sustain these large negative pressures? What exactly was the problem the biologists solved?

You will need:

- An 11 m length of 5 mm diameter clear PVC tubing
- A bucket
- A 10 m rope
- A 10 m high balcony
- A fold-back (stationery) clip

HEIGHT HAZARD WARNING: exercise extreme care when working with heights. Do not attempt this exercise on unsecured lift equipment, farm equipment or an unprotected roof. Use a safety harness if no railing or fall arrest device is present at your site. You must comply with the universities policy for working at height: (www.une.edu.au/policies/pdf/fieldwork.pdf).

Water exists in vapour and liquid forms, with the vapour dissolved in the liquid and in chemical equilibrium with vapour in the air. The partial pressure of the vapour is a function of temperature, such that at approximately 100°C the vapour pressure exceeds the atmospheric pressure and the water will boil.

At lower pressures, such as at high altitude, water will boil at lower temperatures. If you lower the pressure on the liquid to the vapour pressure at room temperature, it will boil at that temperature.

The purpose of this exercise is to create a low pressure environment by induced suction in an elevated water column. This causes the water to boil, producing vapour that breaks the column.

1. Coil the 11 m of tubing and put it in the bucket
2. Fill the bucket at a tap.
3. Connect the PVC tubing to the tap (you may need to experiment with sticky tape to get the end to fit inside the tap) and fill the 11 m with water.
4. Fold the end of the tubing over and close it off with the foldback clip.
5. Throw one end of the rope off the balcony. Secure the other end to a railing.
6. Tie the closed end of the tubing to the bottom end of the rope.
7. Pull the rope and PVC tubing slowly up to 10 m. Make sure the open bottom end of the tubing stays submerged in the bucket. The line should weigh no more than a few hundred grams at full height.
8. As the top end of the tubing comes in view, try to observe the condition of the liquid. You may see a film of bubbles forming inside the tubing.

9. At around 10 m height, note the formation of a vapour column in the line. The water level will fall and oscillate.

It is not possible (other than by capillary or cellular action) to raise water by suction beyond about 10 m where the pressure falls to vapour pressure:

Vapour pressure ~ 2500 Pa

Pressure is reduced to P_{vap} in a suspended water column of height H_{vap} :

$$P_{\text{vap}} = 101300 - \rho g H_{\text{vap}} = 2500$$

$$H_{\text{vap}} = (101300 - 2500)/\rho g = 10.08 \text{ m}$$

Conclusions

This classic experiment is well worth performing in person. You get a feel for the weight of a water column as well as a direct view of the effect that the self-weight has on the chemical equilibrium in the water.

The boiling effect, known as *cavitation*, can occur whenever water pressures are lowered below vapour pressure. Pumps, turbines, boat propellers and pipeline flow control devices may in some circumstances create strong negative gauge pressures, causing violent cavitation that can erode metal surfaces and destroy impeller bearings.

Cavitation is a thing to be avoided under all circumstances, unless you happen to be studying it to know what to watch out for.

1.6 Viscosity

Viscosity is a material property effected by temperature that determines a fluid's resistance to shear forces. A fluid will flow, due to gravity or any pressure gradient, at a speed which is partly determined by its viscosity.

Viscosity is often described in terms of frictional resistance, ie the effect that it has on frictional forces between surfaces. A viscous flow can be regarded as having *internal friction*, as layers of fluid slide over each other. This provides an insight into the forces at work within fluid masses.

This experiment examines the link between viscosity and friction via a modification of the classic *sliding block experiment*, possibly familiar to you from 1st year physics.

You will need:

- A smooth inclined surface such as a laminated coffee table or small desk propped against a wall. Try the coffee tables in your college common room, with your peers' permission and due care for the walls. Make it fun, and clean up the mess!
- A small, flat moisture-proof object with at least one smooth surface, such as a smooth ceramic tile or block of sanded and varnished wood. You can find this kind of thing in builder's yards or neighbours' sheds.
- A selection of household liquids such as water, honey and cooking oil.
- Some kitchen scales
- A stopwatch and ruler

Dry Friction - Static and Active

First establish the “dry” frictional parameters of your surface:

1. Start with your smooth surface in a horizontal position. Place the smooth block on the surface and gradually increase the slope of the incline until the block begins to slide, accelerating at a rate approaching the in-plane component of gravity. Note the angle of the incline by the difference in height ΔH of the two ends of the table. For the table's length L , the angle $\theta = \sin^{-1}(\Delta H/L)$.
2. Repeat this at least three times and work out the average and standard deviation of angle measurements. This is the angle and uncertainty at which the *static friction force* is overcome by the *shear weight force* of the block's weight component in the plane of the sloping surface.
3. Go back to a level surface again, and this time start your block in motion by giving it a gentle push to overcome the static friction. As you'd expect, it will come to a stop under active friction forces. Gradually increase the slope of the surface and repeat the starting motion until the block no longer comes to a stop. You may see some interesting start-stop behaviours as the forces come into balance. Beyond the balance point the block will not only continue to slide, it will accelerate. To avoid these additional motions, don't go past the balance point.

4. Repeat this at least three times and again work out the average and standard deviation of angle measurements. This is the angle and uncertainty at which the *active friction force* is overcome by shear weight forces.
5. Lastly weigh your block and work out its weight force as $W = mg$.

You can now use your measurements to establish the static and active friction parameters of the block and surface. The shearing (in plane) component of the block's weight is $W\sin\theta$. The plane-normal component is $W\cos\theta$. At the experimentally determined balance points, the shearing weight force is equal to a friction force derived from an unknown static or active friction factor times the normal force:

$$f \cdot W\cos\theta = W\sin\theta$$

Use this equation to work out the static and active friction factors, f_s and f_a .

Fluid Friction - Viscosity

Now try the experiment again, this time using fluid friction, which differs in an important way from either static or active friction:

A fluid resists shear forces by motion-generated viscous forces. As a result, a smooth block riding on a layer of fluid will slide downhill at a constant speed. The sliding motion is only limited by surface tension forces, which may be significant only at very shallow slopes.

A frictional parameter known as the viscosity coefficient can be measured as the velocity and thickness of a fluid layer for any given shear stress. Measuring the fluid thickness can be tricky, since fluid layers, often called *films*, can be very thin. We suggest you use at least three fluids of differing viscosity so you can see where the uncertainty is greatest.

6. Again start with your surface in a horizontal position. Pour a generous amount of honey onto the surface, enough to float your smooth block in. Place the block in the puddle and use a ruler to measure the height of the block's top above the table. Subtract the thickness of the block to estimate the thickness of the fluid film.
7. Incline the surface to the angle measured by static friction as above, and use a stopwatch or ordinary clock with a seconds hand to time the block's movement over a short course of 5-10 cm. As the fluid spreads it will decrease in thickness and the block's motion will slow, so you can't let it travel too far.
8. Return the surface to horizontal, remove the block, then scrape the honey back into a collected puddle (you might need to add some more honey to reset the experiment). Replace the block in the puddle and repeat the height-for-thickness measurement as per step 6.
9. Now incline the surface to the shallower angle measured by active friction as above, and repeat the timed-distance measurement of step 7. You should notice that the motion is slower at the shallower angle. Make sure the course distance is the same, as we can expect similar amounts of spreading and reduction of fluid thickness over similar distances.

10. Clean up the honey, and repeat steps 6 to 9 with first cooking oil, then water. You should notice a significant difference in film thickness, and a range of different behaviours approximating a constant velocity. You may find the film thickness difficult to estimate or distinguish from zero for both these fluids, and you may see effects that can be attributed to surface tension.
11. Lastly measure your block's sides S_1 and S_2 and calculate its surface area by $A = S_1 S_2$.

You can now use your measurements to work out the viscosity of the three fluids, by a similar process of balancing shear weight forces against viscous forces.

The block's shear weight force can be described as a shear *stress*, a form of pressure where the normal component of weight force is divided by the block's surface area:

$$\tau = W \cos \theta / A \quad [\text{units N/m}^2 \text{ or Pa}]$$

This shear force equates to the motion velocity and film thickness by *Newton's Law of Viscosity*:

$$\tau = \mu dV/dy \sim \mu V/t$$

Where V is velocity in metres/second, t is thickness in metres, and μ is a viscosity coefficient having the units of N.s/m^2 or Pa.s . While the shear stress depends only on the angle of the incline and the weight of the block, the viscosity is a property of each fluid at the measurement temperature and can be measured by its physical response in terms of the velocity gradient within the fluid, approximated by flow velocity V and thickness t .

Conclusions

This a partly inductive (observational) and partly deductive (analytic) exercise. Your careful observation of the system's behaviour in the three different modes is more important than your measurement of static, active and fluid friction parameters.

The dry static friction parameter tends to be larger than the dry active, resulting in a range of motion behaviours including acceleration, deceleration and constant velocity. Both static and active friction factors tend to be numerically larger than viscosity.

However, there is a major difference in the physics of dry and wet friction, possibly best reflected in the fact that viscosity has units of pressure and time, while dry friction factors are dimensionless numbers with no units, ie they are simply conversion coefficients that equate a normal force to a friction force.

It will be seen later in this course that viscosity is an important component of a dimensionless number that equates fluid momentum forces to viscous friction forces in the complex mode of flow known as *turbulence*.

1.7 Surface Tension

Surface Tension is a material property, also effected by temperature, that determines the behaviour of the fluid at interfaces with gaseous and solid boundaries. A fluid will work to minimise or maximise its surface area, depending on the relative balance of adhesive (sticking to other surfaces) and cohesive (sticking to itself) forces.

Surface tension is a determinant in a range of physical behaviours including:

1. Capillary action, whereby a fluid column will climb narrow tubes or apertures including the spaces between particles such as soil grains.
2. Adhesion, where viscous shear forces come into balance with surface tension forces and thin layers of fluid no longer flow under shear stress.
3. Bubble and droplet pressure, where a fluid surface or film creates high internal pressures inside droplets or bubbles.
4. Wetting and waterproofing, in which droplets either suspend (with minimum surface area) or spread (to maximum surface area) on a surface.

You will need:

- A small, flat, metal object with at least one smooth surface, such as the lid from a jar.
- Some candle wax.
- A selection of liquids with different wetting characteristics such as water, oil, gasoline and thermometric mercury.

MATERIALS HAZARD WARNING: Petrol and some oils are flammable and toxic. Mercury is toxic and produces a toxic vapour, hazardous on contact with skin, mucous membranes or eyes. Always use mercury in a well-ventilated open space and avoid handling it directly with unprotected skin.

Wetting

First establish the wetting characteristics of your liquids and the smooth surface:

1. Place the lid on a level surface and separately introduce small quantities of each liquid onto the surface. Clean and dry the surface between liquids. Try to make drops of different diameters, and note the fluid thickness at each diameter.
2. Note whether each liquid forms a distinct droplet on the surface or spreads into a shallow film or puddle. If a droplet forms, estimate the angle of the liquid to solid interface, or at least note whether it is acute (a prominently rounded droplet) or obtuse (a flatter droplet like a small mound).
3. Consider the balance of fluid weight forces, which will tend to cause the liquid to flow to its lowest level, against surface tension forces, which tend to suspend the liquid at different levels above the minimum potential energy state.

The thickness of a droplet determines the static pressure and hence weight force, while the liquid-solid interface angle partly determines the surface tension force in conjunction with ionic and molecular bond forces.

The force balance can be described in terms of a mean static pressure over the profile area of a droplet (effectively its diameter D times its thickness t) balanced against a surface tension force on the profile edge length:

$$\rho g t / 2 \cdot Dt = \sigma D \cos \theta$$

Where $\rho g t / 2$ is the mean pressure at a film thickness t , Dt is the area over which this acts, and $\sigma D \cos \theta$ is the surface tension force on that diameter at the angle θ .

You will have noted that droplets of different diameter tend to form the same thickness, so we would expect the diameter to cancel, as indeed it does:

$$\sigma = \rho g t^2 / 2 \cos \theta$$

You could use this to calculate the surface tension of each of your liquids, and compare them to published values. There will be large uncertainties in your measurements, so don't worry too much if the results are not ideally accurate.

Waterproofing

Now prepare the surface for entirely different wetting characteristics:

4. Melt the candle wax onto the smooth surface. If possible, heat the surface gently on a stove hotplate so that the wax melts down to an ideally smooth level surface.
5. Repeat steps 1 to 3 for each of your liquids, noting any differences in behaviour. In particular note whether a liquid changes from wetting to droplet-forming or vice versa. Measure contact angles and thickness and calculate the surface tension parameter again for each liquid.

Conclusions

A comparison of wet and waterproofed behaviours should suggest to you the idea that surface tension, supposedly a property of a fluid, is in fact a property of the interface or relationship between different substances. This depends on the condition (temperature, corrosion, protective coating) of both substances and not on the fluid alone.

1.8 Non-Newtonian Fluids

Non-Newtonian fluids differ from conventional fluids in their response to shear stress. In a conventional or Newtonian fluid, viscosity is a constant dependant only on temperature and chemical composition, however in a non-Newtonian fluid there is an additional dependence on stress.

For example many types of paint are “stress-thinning” (dilatant): they are viscous enough to cling to a brush, but become thinner due to shear stress from the brushing action. Conversely some particle suspensions such as toothpaste and cornflour are “stress-thickening” (pseudo-plastic): they are thin enough to pour, but the action of stirring or brushing causes a thickening so that they adhere more strongly.

You will need:

- A packet of cornflour (or corn starch) – not wheat starch or other cornflour substitute.
- A mixing bowl

Walking on milk (actually cornflour suspension or corn starch)

Here’s where you get to perform a popular game-show stunt using cornflour suspension, a pseudo-plastic or stress-thickening liquid:

1. Mix the cornflour with water slowly, stirring and testing its consistency as you go. It shouldn’t be made too stiff or too runny.
2. Test the consistency of the mix by taking the spoon out and striking the surface of the suspension. You should be surprised to see it hit the surface with a “thunk” as though it is firm and solid, then sink gently in as though liquid.
3. Pour out a small amount of the suspension into your cupped palm. It should pour easily and sit in your hand like milk. Now *quickly* work the liquid with both hands, rolling it into a ball and with a smooth action, *bounce* it on the table.
4. Use your fingers to “walk” on the liquid in the bowl.
5. Show/amaze your friends (unless they’ve already seen it, as is becoming increasingly common among gameshow-mad generation z)

Section 2: Statics

2.1 Buoyancy and Stability

How to balance a model sail boat

Model sailing boats differ from full scale prototypes (a term favoured by engineers to indicate they are building something new to test their designs) in the way their mass and area dimensions scale down: Mass is proportional to volume, which varies as the *cube* of the model scale, while sail area is proportional to the *square* of the model scale.

As a result there is a changing balance between overturning forces (due to wind pressure on sail areas) and righting forces (due to ballast mass and displacement volume). A working model sail boat must be balanced by additional “ballast” or weight forces, often provided by an unrealistically deep keel with lead weights on the end.

This information by itself would be enough for you understand the principle involved. If you're into model building, you can use these principles to measure the buoyancy-stability characteristics of a model boat and design it for a given “heel” or angle at which wind pressure causes the boat to lean over at speed.

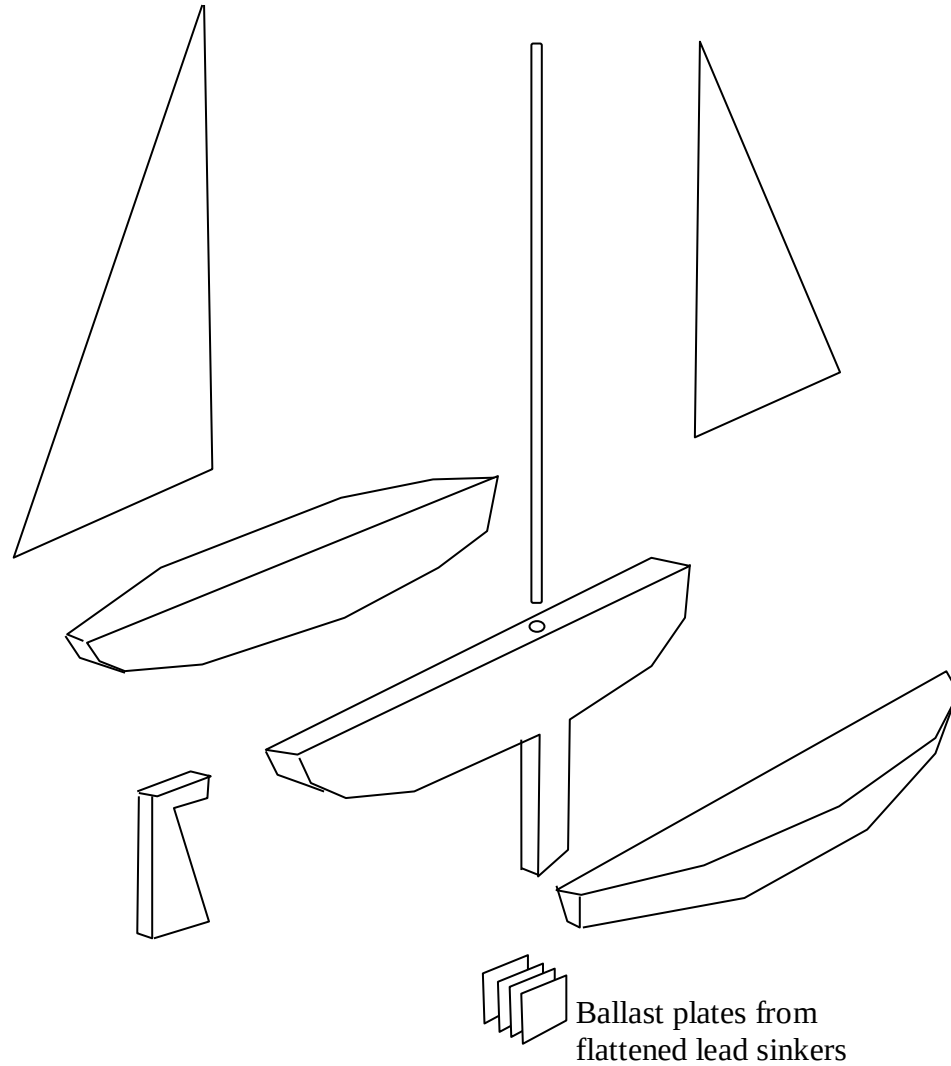
You will need:

- Materials for the construction of a model sail boat: balsa for soft hull components, pine board for rudder and keel, pine dowel for masts, nylon cloth and string for sails and rigging, marine varnish for waterproofing, a drill, glue and nails etc for assembly
- A plumb bob consisting of a weight attached to a string, for a vertical reference line
- A protractor for measuring angles
- Kitchen scales for measuring masses
- Some lead weights such as ordinary fishing sinkers.
- Sticky tape or gaffa tape

Build the boat

This is easier said than done, and there's no room for too much detail here. You might start by looking up designs on the internet and doing some rough sketches.

1. Form the main hull shape by carving and sanding two symmetrical halves out of balsa wood, and gluing them together on a heavier keel shape cut from pine.
2. Drill a hole in the keel and insert a dowel mast.
3. Shape and attach a rudder to the back end of the keel
4. Cut and attach some sails to the mast and deck. Measure the height and area of the sails.
5. Flatten some lead sinkers with a hammer, and tape them onto the tip of the keel.
6. Waterproof the boat with marine varnish



Nylon, balsa, pine, and lead components of a simple model boat. These materials are chosen partly for their strength, and more importantly for their weight characteristics.

Balance the boat

With all these components assembled you should be able to place the boat in water and find that it floats upright and has a natural balance. If it tends to tip over you will need to add more ballast mass to the tip of the keel. On the other hand if it sits too low in the water you might need to remove some ballast. With a balance attained, check and adjust the vertical on the mast by comparing against a plumb bob:

7. Hang the plumb bob from a ceiling fitting (*not* an electric light) and line your eye with the mast and plumb bob.
8. Make sure the sails are hanging in the centreline. If the mast visibly leans to one side, place a small lead weight on the other side, away from the centreline of the deck, and adjust its position until the mast is vertical. If you can't get the mast vertical you may need to use a heavier deck weight.
9. Move the weight forward or back to correct any pitch to the stern (back end) or bow (front end).

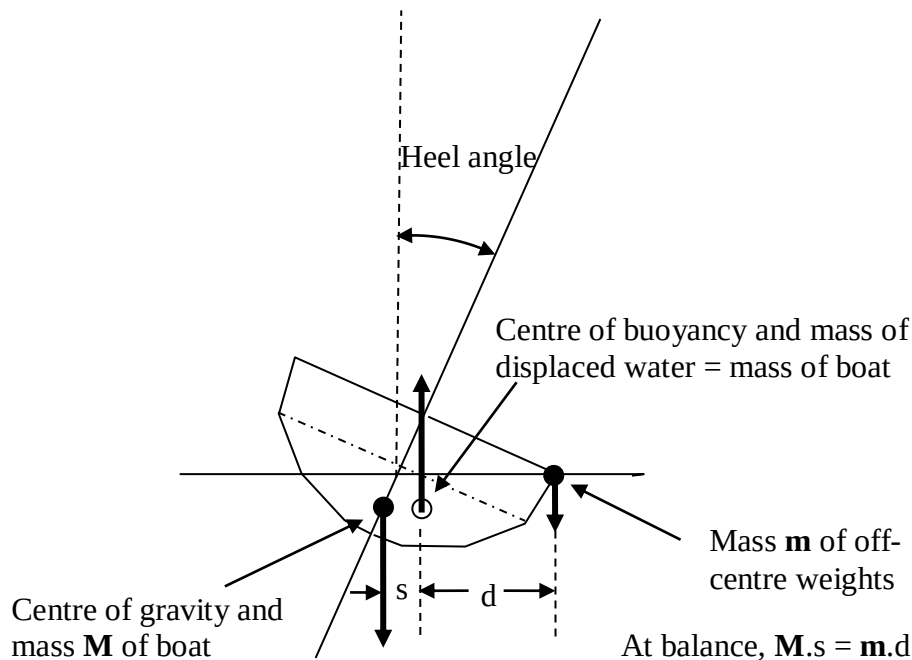
10. Attach the weight at the balance point with glue or putty.

Test the boat's buoyant stability.

This is where you measure and refine the boat's response to applied off-balance loads, such as crew weight or lateral wind load.

11. Place a moderate lead weight (two or three sinkers say) on the edge of the deck and use the plumb bob and protractor to measure the angle to which the mast heels over.
12. Record the angle of heel and the mass of the additional weights. Also measure the distance from the centreline to the edge of the deck where the weights are placed.
13. Repeat for at least three sets of heavier weights (add one or two sinkers each time) and record the heel angle as a function of mass.

We can now visualise some theory as to how the boat's mass and buoyant forces interact (see the figure): As the boat heels over due to the applied eccentric load, the geometry of the submerged volume changes and the centre of buoyancy (the centroid of the submerged volume) moves laterally. The boat's centre of mass (excluding the additional masses) does not move and as a result creates a torque or moment about the centre of buoyancy equal and opposite to the torque created by the eccentric load.



We don't need to know the centre of mass or buoyancy, since we now have some raw data on how the boat responds to an eccentric load. We can compare this to the off-centre load created by wind pressure, and decide if the heel is excessive or about right for a given windspeed.

To a good approximation, the torque due to an eccentric mass can be calculated as

$$T = m.d$$

Where m is the mass and d the distance from the centreline of the eccentric load.

This can be equated to an eccentric torque due to wind of velocity V :

$$T = 1.2V^2A.h/6$$

Where A is the sail area and h is the sail height. This formula computes the drag force on the sails and multiplies it by $1/3$ of the sail height, which is the location of the (triangular) sails' centroid of area, where the overall drag force applies. This gives a wind torque tending to heel the boat, much like an eccentric weight load.

So for a given wind load we can look up (from the load test data) an equivalent eccentric torque and resulting heel angle. For dynamic reasons we don't want this to exceed certain values: say for a 2 m/s wind speed we wouldn't want more than 15° of heel. If your design heels more than that amount, add more ballast to the tip of the keel.

Conclusions

This exercise is a big job, and definitely one for the real engineers among you: kids at heart who grew up making and testing model planes, boats, trains, etc, and aren't afraid to do the hard work to make a model that can be tested to destruction if necessary.

There are some simple approximations in the theory that you could improve by locating the centre of mass. A simple method that is practicable for model boats (if not for full scale prototypes) is to balance the boat horizontally on a ruler. Try it and see what this suggests to you.



2.2 Forces on Plane Surfaces

Pressure-sensitive sluice gate hinges

The section on fluid statics and structural reservoir design should impress on you one major point, that the torque or moment on a water-retaining structure is more important than the force alone. Locating the centre of force is a first essential step in determining the overturning or force-opening moment of a structure such as a wall or sluice gate.

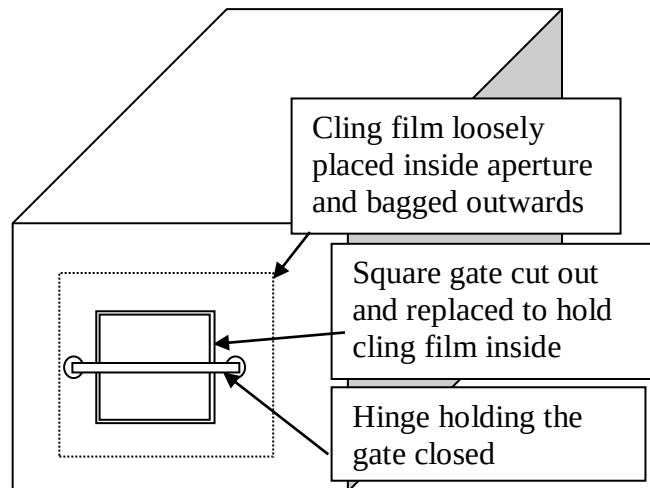
This exercise is designed to help you visualise the location of a the centre of hydrostatic force, and to see how a structural design that fails to locate that point might fail due to unbalanced moments.

You will need:

- A plastic kitchen storage container or tub, about 12 to 15 litres will do, square or rectangular in plan and with vertical or near-vertical sides.
- A sharp hardware knife such as a Stanley knife (warning, razor sharp).
- A candle
- A roll of plastic cling film such as Glad Wrap TM.
- Some blu-tac
- A permanent marker pen and a pencil

Making a sluice gate

1. Draw a square in the middle third of one wall of the tub.
2. Light the candle and use it to heat the Stanley knife blade. This will make it easier to cut the plastic, and less likely to slip and cut your skin.
3. Cut out the marked square, creating an aperture or gate in the tub. Re-heat the knife to keep it cutting easily.
4. Cover the square aperture from the inside with one layer of cling wrap, spreading the film loosely to at least a few inches around the hole.
5. Loosen the film even further by prodding and lifting it from the inside, making it baggy and concave in the aperture. The film should be able to flop out of the aperture by about 1 cm.
6. Take the square of plastic you have cut from the tub and put it back in its aperture from the outside, so that it holds the cling film in. Hold it in place with a pencil blu-tacked in place halfway up the outside of the gate:



Locating the centre of pressure

Now we can fill the tub with water and observe a few things about the role of pressure in the balancing of the gate:

7. Fill the tub with water to a level well above the gate. Water pressure will increase behind the gate, and the position of the resultant force will change as the water level rises. You should see the gate trying to rotate about the hinge as the centre of pressure changes. You may need to reinforce the hinge to stop it springing off.
8. With the tub almost full, reposition the hinge (pencil and blu-tack) to a point about a quarter of the way from the top of the aperture. What happens?
9. Reposition the hinge again to a point about a quarter of the way from the bottom of the aperture. Again, what happens?

You should see the gate trying to open by rotating about the hinge, first from the bottom and then from the top of the aperture. As you move the hinge vertically it moves in and out of balance with the hydrostatic pressure distribution, creating unresolved moments that are only contained by the cling film.

You should be able to physically locate the centre of the pressure distribution by positioning the hinge just slightly below the centre of the gate, until the gate appears to be in balance and no longer bulges open at either end. The exact depth of the pressure centre is given by:

$$h_p = h_c + \frac{I_c \sin^2 \theta}{h_c A}$$

Where h_p is the depth to the centroid of the gate, $A=s^2$ is the area of the gate, $I_c=s^4/12$ is the moment of area of the gate, and θ is the angle of the wall to the horizontal.

Conclusions

This exercise should graphically demonstrate to you the importance of force-opening moments in fluid retaining structural designs. A gate may be capable of supporting a

hydrostatic force, but not a hydrostatic moment, and may fail as a result of the moment rather than the force.

At the very least, by observing the changing balance of forces as you move the hinge from place to place, you should now be convinced that the resultant force from a pressure distribution exists at a definite location. The balance of forces depends not only on the magnitude but the position of that force.

2.3 Forces on Curved Surfaces

This exercise is designed to demonstrate that the hydrostatic pressure force on a curved surface is exactly equal to the same pressure acting on an equivalent plane-projection or silhouette of the curved surface.

This result permits so many useful simplifications in fluid structural design that it should be given an important-sounding name, but most people don't even know who discovered it. Was it Toricelli? Boyle? Archimedes himself perhaps?

After this experience you can forget the principles involved, never mind the name, and embark a much simpler professional practice in two dimensions rather than three.

You will need:

- A plastic bucket
- A sink plug
- A hollow rubber ball a little bigger in diameter than the plug
- A Stanley knife and candle for hot cutting
- A 10 mm syringe blank (no needle, just the barrel and plunger)

There's a hole in the bucket

1. Measure the sink plug's inside diameter, that is the size of the hole the plug is designed to fit into.
2. Draw a circle of this diameter on the base of the bucket
3. Using a hot Stanley knife (see Forces on plane surfaces, above), cut a neat circular hole in the bucket floor. Put the plug into this plughole.
4. Hang the bucket from a washing line and fill it with water. If it leaks, you haven't done a very good job with that hole! Do it again properly.
5. Open the syringe in air to 10 mm and block the nozzle with blu-tack.
6. From underneath the bucket, press the syringe plunger against the sink plug and push upwards. Record the deflection of the syringe at which the plug is forced up and out of its hole (see Pressure and Density, Boyle's law for a description of this simple force scale).

The lifting force corresponds to a hydrostatic pressure force of $F = \rho g H A$ where H is the depth of the bucket and $A = \pi D^2/4$ is the area of the plughole, *not* the area of the inside of the plug.

We should now be able to demonstrate the same force, to within experimental uncertainty, using the rubber ball as a plug:

7. Drill or puncture a hole in the ball and hold it underwater, squeezing the air out so that it fills with water (this step is to remove the airspace and eliminate excess buoyant forces on the ball)
8. Empty the bucket and place the rubber ball in the plughole.
9. Hang the bucket up again and fill it, holding the ball in place until downwards pressure forces squeeze the ball and create a seal. If the bucket leaks you may need to clean the hole up with the knife, to make sure it is perfectly circular with no jagged edges.
10. As per step 6 use the syringe to push up on the ball valve from underneath, noting again the deflection at which the ball is forced up and out of the plug hole.

Conclusions

You should find that the same force is required to open the (spherical) ball plug as to open the (flat) sink plug, and that in both cases it is the area of the plughole, not of the plug or ball, that determines the hydrostatic pressure force.

That should save you some time doing engineering calculations!

Section 3: Dynamics

3.1 Emptying the bath

It's a constant surprise (to me) that many science and engineering students have never heard of the principle of a *syphon*, or if they have they still have no idea how it works.

You can probably tell that *this is the one exercise you really must attempt, especially if you don't know what I'm talking about.*

You will need:

- A full bathtub
- A window
- Some hose pipe from the garden or college garden shed (put it back when you've finished)
- A house brick

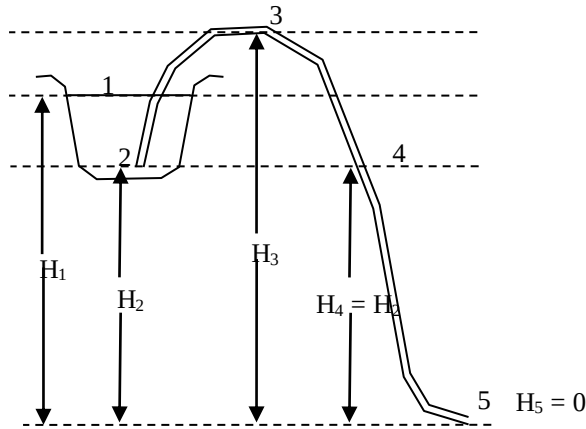
The total energy at two ends of a flowing pipe are the same, after allowing for frictional and other losses. Over short lengths these losses are fairly small and we can assume the total energy due to pressure, velocity and potential energy is constant.

If you put one end of a hosepipe in a bathtub and throw the other end out of the window, probably nothing will happen. To get the water to flow up the pipe to at least the level of the window, you need a source of pump energy.

But what happens once the water has been pumped out of the window and has flowed down the other side to at least the level of water in the bath? Do you have to keep pumping?

1. Put one end of your hose securely (put a brick on it) in the full tub and throw the other end out of the window (if you live upstairs, please be careful)
2. Go outside and *suck* on the end of the hose. There will be some resistance at first, until the water level rises past the window, then it will get gradually easier. Once the water has flowed down to at least the level of the bathtub, there will be no resistance and you can stop sucking.
3. Watch the hose for a few moments more ... and here comes the water, flowing all by itself! How can that be possible?!
4. Let the hose run until the bath empties. Water the garden. Think about the environment. When the hose encounters air, the flow will soon stop.

Let's write Bernoulli's equation at a few points in time and space and see what emerges:



Point 1: $E = H_1 + 0 + 0$

(velocity and pressure terms are zero)

Point 2: $E = H_2 + 0 + P_2/\rho g = H_2 + 0 + (\rho g(H_1 - H_2))/\rho g = H_1$

(with no losses, $E_2 = E_1$)

Point 3: $E = H_3 + V^2/2g + P_3/\rho g = H_1$

So $V^2/2g + P_3/\rho g = H_1 - H_3 < 0$

(total velocity and pressure head are negative; but since V^2 is positive, P_3 must be negative ie this point is in suction)

Point 4: $E = H_4 + V^2/2g + P_4/\rho g = H_1$

So $V^2/2g + P_4/\rho g = H_1 - H_2 > 0$

(Pressure returns to positive: the flow is driven by pressure from at least here on)

Point 5: $E = 0 + V^2/2g + 0 = H_1$

(with no losses the pipeline flows at a velocity proportional to $\sqrt{H_1}$)

Since losses are driven by flow, in reality there is an initiation stage in which flow accelerates and losses increase. When losses and energy balance out at the pipe outlet, the flow stabilizes at a velocity slightly less than that calculated with no losses.

The suction at point 3 is provided by:

- a) You, until the flow passes point 4, and then
- b) The negative pressure generated inside a column of water suspended from a height such as a window (see Pressure and Density, Breaking the column).

Conclusions

The main point that should accord with your experience is that there is a clear transition from negative to positive pressure between points 3 and 4. We can see this without calculating any actual values, but by inspecting the diagram: $H_1 - H_3 < 0$ and $H_1 - H_2 > 0$. That's all we need.

3.2 Orifice to Weir Outlets: Transitional Flow

Naturally as a child you were fascinated by the emptying of the bathtub: the amazing vortex, the scary sucking noise, the final bubbling denouement.

As a university student you may even have gone as far as emptying the bath out of the window, just to see what happens. It's clearly one of the great joys in life.

Let's apply the deductive scientific method to this universal experience and see what we can discover.

You will need (you may have some of this from the pressure sensitive sluice gate):

- A plastic kitchen storage container or tub, about 12 to 15 litres will do, square or rectangular in plan and with vertical or near-vertical sides.
- A sharp hardware knife such as a Stanley knife.
- A candle
- A 2 cm length of 16 mm clear PVC tubing
- A tube of silicon waterproof sealant
- Some sticky tape and an ultrafine-point permanent marker
- A stopwatch
- A ruler

As a reservoir empties into a low-level outlet the flow undergoes a transition from *orifice* flow, when the water is well above the level of the outlet, to *weir* flow when the water is near or below the level of the outlet. Orifice flow is governed by Reynolds number, hydrostatic pressure and the area of the outlet, while weir flow is governed by open channel effects comprising the Froude number and the top water surface width.

The transition is partly characterised by the formation of whirlpool vortices, which may occur singly or in pairs. In effect the water surface is drawn down into the orifice, creating the physical conditions for a transition between submerged and surface flow behaviours.

1. Use the hot knife method (see pressure sensitive sluice gate) to cut a 20 mm diameter circular hole about 1/3 of the way up one wall of the tub.
2. Insert the 16 mm PVC tubing into the hole so that one end is flush (level) with the inside surface of the tub. Seal it in place with silicon sealant and allow to dry.
3. Place a strip of sticky tape vertically on the outside wall of the tub. Mark a reference point near the top of the strip with the marker pen
4. Fill the tub, allowing the projecting tube to overflow into a sink or drain.
5. Turn off the tap, allowing the tub to continue draining through the orifice.
6. When the surface falls to your reference mark, start the stopwatch and count off 5-second intervals as the water level falls. At each interval, mark the water surface level on the tape strip.
7. Measure and record the height of each marked dot above the *invert* (bottom of the outlet pipe) with a ruler.

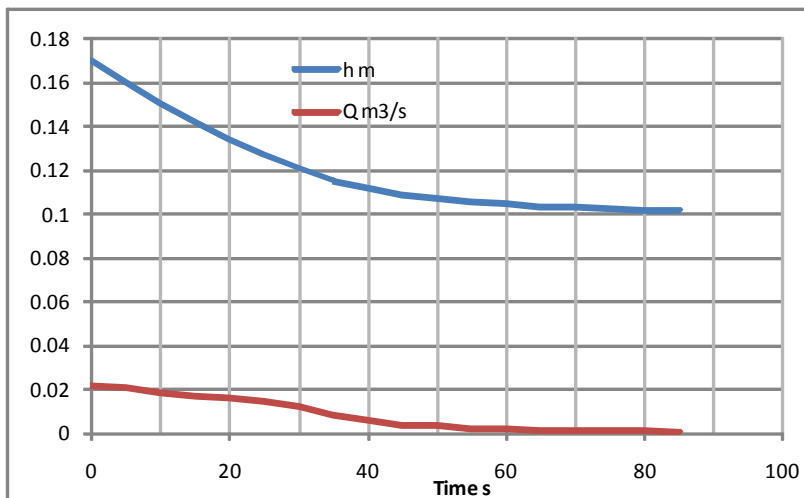
- You will need to be consistent about which point of the water surface you mark: I recommend you focus on the silvery line formed by the bottom surface of the water viewed from the outside. The top of the meniscus varies and is less accurate.
- Your pen will need to keep up with the water level, as it drops rapidly and 5-second intervals go fast when you're observing a changing system.
- Avoid dividing your attention between the stopwatch and the water surface. Constantly changing focus is hard, distracting and a source of error. Hold the stopwatch in view so you can see it out of the corner of your eye as you focus on the water surface. Better yet, get a friend to count for you, so you can anticipate and mark on the 5s and 10s.
- You will probably need to do this a few times to get a clean line, and you should do it still a few more times to get enough data for a smooth line.

The output dots are a time-series of water levels that can be read as a profile of the falling water level's velocity: the distance between any two points, divided by the time interval of 5 seconds, is equal to the average velocity of the water level between those points:

$$V = \Delta h / \Delta t$$

For simplicity we can ascribe these mean velocities to each point rather than to intervals between points. This provides an estimate of the flowrate out of the tub at each moment, since $Q = VA$ where V is the water level velocity and A is the water surface area at that level. By continuity in an incompressible fluid, the same flowrate is occurring at the mouth of the 16 mm outlet.

A typical output is shown in this graph:



The research question is, can you identify in the data a clear transition from orifice flow to weir flow?

8. Measure the surface area of the tub as a function of height. Most plastic storage tubs of the type you're likely to use have sloping walls, so you could measure the base width and length at the top and bottom of the tub and extrapolate linearly between them.
9. Process the data in a spreadsheet as a time series of height measurements h above the outlet invert. Divide each height difference by 5 (seconds) to obtain the velocity V at the first of each pair of heights. Calculate the area A as a linear extrapolated function of height. Calculate the flowrate $Q = VA$.
10. Write formulae for orifice flow and weir flow as a function of head above the invert:

Orifice:
$$Q = C_d \frac{\pi D^2}{4} \sqrt{2gH} \quad C_d \sim 0.5 - 0.6$$

Weir: See lecture 14 for a Bernoulli derivation of weir flowrates

11. Create a separate lookup table of parameters. Use absolute \$A\$1 references in your formulae so that all data points use the same parameters. This way you only need to change the parameters in the lookup table, not throughout the entire sheet.
12. Vary parameters until you get a good match of orifice and weir models to different parts of the data.

You should be able to identify a range of points in the middle of the data where neither orifice nor weir models agree well with the data. This indicates that the flow in that range is in transition from orifice to weir flow.

Now, the most important part of this exercise is to *go back to the tub and observe the flow closely in that transitional range*. What do you see that is different?

Conclusions

This helps to explain why emptying the bath was so much fun as a child!

3.3 Pipeline pressure: visualising a hydraulic grade line

The purpose of this exercise is to give you some direct insights into how pressure varies with flowrate and diameter along a pipeline, and in particular to see that pressure *always* falls to zero (gauge, or atmospheric pressure) at the outlet from a pipe.

Zero gauge pressure at a pipe outlet is the basis of integral methods that account for pressure gains in the upstream direction, starting from an easy zero at the downstream end. This exercise will show you the zero firsthand.

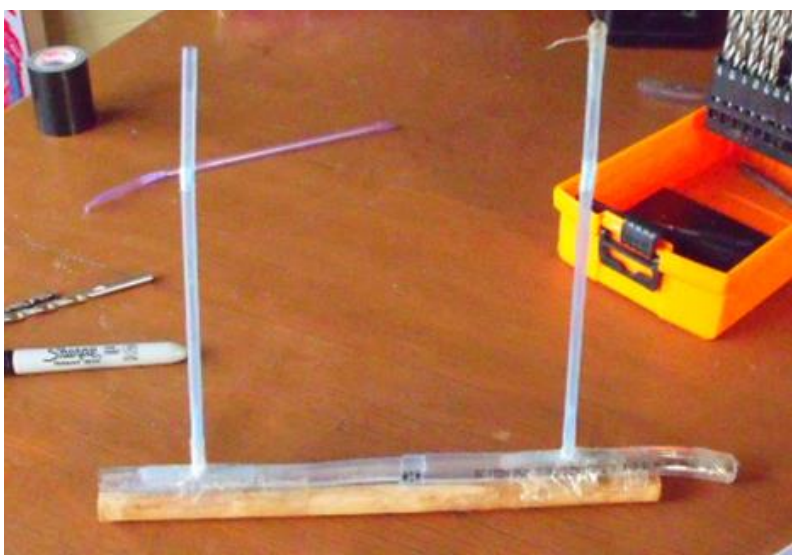
You will need:

- Two 20 cm lengths of 12 mm diameter dowel (cylindrical timber pieces)
- 1 x 25 cm length of 5 mm clear PVC tubing
- 1 x 25 cm length of 10 mm clear PVC tubing
- A tap and connected hosepipe.
- A packet of drinking straws
- A tube of silicon sealant
- A roll of sticky tape
- A 3 mm drill bit

An open vertical *standpipe* attached to a pressurised pipeline will fill to the level of the hydraulic grade line. The HGL includes the potential and pressure energy terms of Bernoulli's equation but does not include the velocity head. The total energy line TEL is just the HGL plus the velocity head.

1. Tape the two dowels together to make a base
2. Tape the 5 mm tubing to the base, leaving 5 cm free at one end. Attach that end to the hosepipe (as usual you may need to experiment with tape to get it to fit)
3. Drill two holes in the tubing, one at either end of the base. The hole at the open end of the tubing should be about 1 cm from the end.
4. Take a straw and split one end to about 2 cm. Lift back the split ends to form an open mouth. Cut the split end off to about 2 cm and fit it into the end of another straw. See illustration overleaf.
5. Place the split end over a drilled hole and cover it with silicon sealant. Tape over the whole attachment, creating a standpipe base attached to the tubing.
6. Repeat with another straw for the second standpipe base
7. Create one tall standpipe by attaching straws to the inside (hose end) standpipe base: squeeze one end of a straw and push it into the end of another straw, then tape around the join. Continue up a wall or pole as high as you can reach.
8. The standpipe base at the open end of the tubing doesn't need to be any taller. (why not?)

See the following photo gallery:



Attaching plastic pipes so that they don't leak is a minor art. Straws are an easier fit than PVC tubing because the thin material is more flexible (cheaper too)



Now turn the tap on and see what happens. Turn it up and see how high up the standpipe you can make the HGL rise. Recall that the HGL is a measure of the pressure in the tubing. Multiply the height by about 10,000 to get the pressure. You just made yourself a pipeline pressure gauge!

What's happening with the short standpipe? Can you see any water in there at all? It should be a pretty convincing zero.

Between the main standpipe and the outlet there can be several metres of fall over only a few cm of pipe length. See how large a hydraulic gradient or friction slope you can generate.

Build another gauge using the 10 mm PVC tubing. Plug it in and note the different pressures (and losses) compared to the 5 mm tubing. Note again the resounding zero at the outlet. Try different combinations of diameter within the gauge and note the losses.

Conclusions

Did you ever see a *nothing* so interesting and surprising as the zero at the end of a pipe?

3.4 Bottle Rocket Science

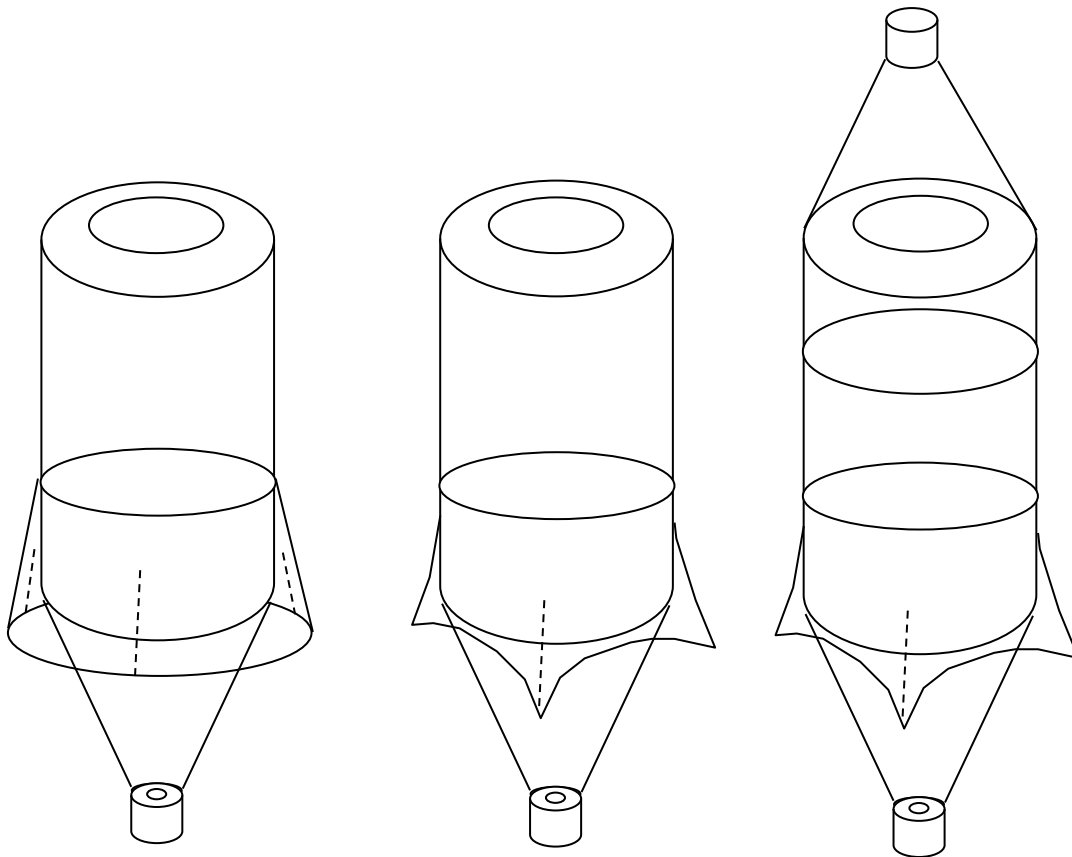
With your simple hydraulic jack properly constructed, you should have the skills to create an adequate pressure seal for a compressed air and water bottle rocket.

Rocket science per se is fluid mechanics plus thermal materials science plus remote sensing plus control systems theory plus celestial mechanics. Complicated, yes. Building a bottle rocket provides objective lessons across a smaller range of topics: energy conservation, pressure, momentum, thrust, drag, aerodynamics and ballistics just to name the theory. You can also learn some practical skills in precision engineering with pressurised components, highly recommended for engineering technology students.

You will need:

- Three 1.25 litre PET soda bottles with plain unmarked cylindrical barrels
 - A rice tub, that is a round Chinese food container (the sort the rice comes in)
 - A cork, real or plastic, from a wine (not champagne) bottle
 - A strong epoxy glue such as Araldite
 - A contact glue such as shoe glue
 - A good strong tape such as gaffer tape
 - An old bike tyre with working valve
 - A drill with 6, 8, 9 and 12 mm bits
 - A 6mm round file
 - A flat file
 - At least 10 cm length of 8 mm diameter brass or steel pipe
 - About 6 cm length of 5 mm diameter clear PVC tubing
 - A good bike pump
 - A rubber foot from a stool or table leg
 - A 100 mm diameter cardboard post pack roll with plastic end caps
 - Three 30 cm lengths of 12 mm dowel
 - A 200 x 200 mm piece of pine board
1. Cut the cork in half and drill a 9 mm hole through the exact centre of one half. Test your nozzle base by fitting the 8 mm pipe into it and rolling it along the table. If there is any discernible wobble in the pipe, try again with the other half of the cork.
 2. Cut the 8 mm pipe down to a 1 cm length nozzle. The ends of the nozzle must be absolutely square. Use the files to clean up any rough edges. Stand it up on a mirror: if you can see any kink in the reflection, try again with another piece.

3. Fit the 8 mm nozzle into the drilled cork, and fit the cork flush inside the neck of a bottle.
4. Drill an 8 mm hole in the exact centre of a PET bottle top. Screw this onto the bottle. Make sure the plastic doesn't cover any part of the nozzle mouth, but does cover the pipe cross section to hold the nozzle in.
5. Cut the bottom off the Chinese container and fit the tub onto the nozzle end of the bottle to make a skirt. It has to be the right size container to fit onto the bottle.
6. Cut the skirt in three places. Fold and strongly tape these to form three fins.
7. Cut the top off the end of the other bottle and fit it onto the top of the rocket as a nose cone. Glue about 60 cents in 10 cent pieces inside the cap for ballast (as in ballistic mass) and screw the nose cap on.



Difficult to draw in three dimensions, but not too hard to construct. This is the launch vehicle.

Now we need to make a quick release pressure seal and valve to attach to the bike pump:

PRESSURE AND PROJECTILE WARNING: PET bottles can sustain much larger pressures than the target of 100 PSI, but may rupture if they are faulty or old. **WEAR SAFETY GOGGLES**, long sleeved clothes and gardening gloves to avoid injury by flying shards of PET under high pressure. **DO NOT POINT THE ROCKET NOSE OR NOZZLE AT ANYONE WHILE IT IS PRESSURISED.**
(www.une.edu.au/policies/pdf/fieldwork.pdf).

8. Drill a 6 mm hole in the base of the rubber table foot. Tape the outside of the foot with about a half dozen layers of gaffer tape.
9. Cut the valve from the bike tube with a few cm around it to spare.
10. Trim and fit the valve into the table foot with the screw end out.
11. Fit this onto the nozzle cap of the rocket and attach the pump. Aim for 100 PSI pressure by using a pump gauge or suitable calculation with Boyle's law. If the foot pops off before you get to 100 PSI, smear some contact glue onto the outside face of the nozzle cap and leave it to dry. Try again with the new high friction connection. Repeat with more layers of glue until the seal can take a 100 PSI load before popping off.

Lastly we need a launch pad and fuelling vehicle:

12. Cut a 40 mm hole in the centre of a post pack end cap and glue or tape it onto the post pack tube. Keep the other cap to make a storage pack for your rocket.
13. Strongly tape the three pieces of dowel onto the cardboard tube to make three legs about 20 cm long.
14. Drill three 12 mm holes into the pine board and glue the dowel legs into the base.
15. Drill a 6 mm hole in the cap of the last PET bottle. Fit the 5 mm PVC tubing into it. Fill the PET fuelling vehicle with water and fit the cap.

Bottle Rocket now ready for launching.

Take the rocket and launch gear to an open area such as a suburban cricket pitch. Watch the game and wait for tea time, for a receptive audience.

PUT ON SAFETY GOGGLES AND GLOVES. Fill the rocket with about 600 ml of water by inserting the 5 mm fuel line into the nozzle and squeezing the fuel vehicle about half empty. Attach the quick release pressure seal to the nozzle cap. Slide the rocket into the launch tube, nozzle projecting through the end cap, and stand it on the pitch. Referees will now begin to approach. Attach the bike pump and be ready to deflect questions. There may be concerns about water on the pitch. Keep pumping. Warn spectators to stand back. At the last minute look up, or you'll miss it.

This rocket is designed to achieve an initial velocity of about 25 m/s from a maximum acceleration of about 9 gees, and should reach a height of at least 120 metres. There will be no water on the pitch: the water propellant is ejected in a light mist to about 50 m

above the ground. The impression from a distance is of an alien spacecraft doing a jump into hyperspace. You should get at least a stunned silence followed by a smattering of light applause. There may be more curious questions from onlookers.

Conclusions

To make a better bottle rocket than this you will need to do some time-simulation calculations with orifice flowrates, momentum transfer and fuel consumption. Try varying the diameter of the nozzle to get different mass flow rates and propellant ejection times. Try different fuel loads and ballistic masses. See if you can work out a trigger for the quick release pressure seal.

Again, DO NOT POINT THE ROCKET AT ANYONE. The acceleration and thrust are surprisingly large, enough to bruise or put an eye out.

Section 4: Open Channel Hydraulics

4.1 Build a flume

A *flume* is an artificial open channel, often designed with an articulated floor and walls to allow different long- and cross-sections to be set up.

The mini-flume described in this exercise provides a variable bed slope and some simple obstructions leading to sub- and supercritical flow, with backwaters and hydraulic jumps as interesting dynamic features.

You will need:

- The tub with PVC pipe outlet from the Orifice to Weir Outlet exercise
- The lid from the tub
- Some blu-tack

Open channel hydraulics are an aspects of fluid mechanics with almost limitless variability in behaviour. One very interesting feature of open channel flow is that it is an almost exact analogue of compressible fluid flow, with the detailed behaviour of variable gas density reflected in the analogous depth of water, where changes in bed width are equivalent to duct diameter. Remarkably, sub- and supercritical open channel flow are equivalent to sub and supersonic gas flow. So by studying open channel flow, you're understanding gas flow a whole lot better.

1. Cut the long edges off the lid of the storage tub and glue them together back to back to form an open channel. Do the same with the short edges.
2. Assemble the two lengths of channel with blu-tack and attach them to the pipe outlet from the tub. The channel joints are rudimentary but flexible enough to provide sudden changes in slope.
3. Fill the tub and allow the outlet to flow into the channel.
4. Manipulate the channel in any way you can think of: make it flatter, or steeper, straighter or more bent, with convex or concave vertical bends, or with lumps or wedges of blu-tack to create humps or narrows. Try to trigger transitions to and from sub and supercritical flow by races and jumps. Build little weirs and sluice gates. Think about and record (notes, photography, video) what you see.
5. Try calculating Manning's equation for normal depth, or critical depth at measured flowrates. How accurate is your theory?

**Conclusions**

Open channel hydraulics is far too broad a field to easily set out as an exercise. We hope you find it a fascinating experience to explore in your own way, with these few suggestions and the study guide for ideas.



APPENDIX: PHYS207 INTERNAL LABORATORY MANUAL

INTRODUCTION

Welcome to the laboratory component of the Physics 207 Fluid Mechanics unit. We hope that you find the laboratory sessions interesting and that they add to your knowledge base in physics and engineering. The laboratory work is designed with several aims in mind but the main one is to act as an extension to lectures in order to help you understand and see for yourself that the theories do work within reasonable ranges of experimental uncertainty.

Some extra aims of the laboratory sessions include:

- To teach you some of the techniques used in making measurements.
- To develop your skill in observation and in the interpretation of measurements.
- To develop your ability to assess uncertainty in measurement.
- To give you a first hand acquaintance with some fluid dynamics phenomena.
- To teach you good practice in recording the results to measurements and how to maintain a laboratory work book.
- To teach you how to write a technical report.

For internal students the laboratory sessions commence at 2.00 pm and should finish by 5.00 pm. Internal labs take place once per week on a designated day and attendance at all laboratory sessions is **compulsory**. If you miss a session due to illness then a doctor's certificate must be shown to the unit coordinator and you may be asked to make up the session at a later date. Laboratory sessions may not be scheduled for every week (eg week 1 and 13 do not usually have laboratory sessions) and so you must look at the physics noticeboard or check with the coordinator for details.

External students complete the laboratory part of the unit at the mandatory residential school.

You should bring the following to each laboratory session:

- A pen, a sharp pencil, a ruler, a protractor and a compass.
- A calculator. This should be able to calculate trigonometric functions, the exponential function and natural logarithms.
- This laboratory manual, your laboratory work book and relevant unit texts available on closed reserve.

Assessment will be based, in equal proportions, on:

- Your demonstrated ability to correctly handle laboratory equipment and follow procedures;

- The amount of effort that you have demonstrated in learning to use equipment and follow procedures, and;
- The quality of the written work submitted in your laboratory work book, including your recognition of errors and uncertainties.

The instructions for each experiment are written in a way that will lead you through the aim and theory of what is being examined. You should read through these *before* the lab commences and make sure that you understand them.

Before you begin working with the apparatus you *must* write a brief summary of the experiment's Aims, Theory and Methods in your lab work book. This ensures that you begin the procedure with the best possible understanding of what you are about to do.

- **Aims** – in plain language what do you intend to measure or apply.
- **Theory** – with technical jargon and formulae / derivations briefly explained.
- **Methods** – step-by-step procedures with sketched illustrations.

You may wish to also think about how you would design an experiment given the aims and theory that you have read. You might base this on the available apparatus or design an alternative experiment using other procedures and apparatus.

When you are ready, work through the procedure given for the experiment and take the measurements that are required. Use your lab work book to continue to write up a short summary of each stage of the experimental process:

- **Results** – brief descriptions of experimental outcomes, with tabulated measurements, calculations and uncertainty estimates, plus sketched diagrams and graphs to appropriate scale.
- **Conclusions** – relate your **Results** to your **Aims**, allowing for uncertainties and errors, and make relevant comments on the applicability of method to theory.

You will be expected to do this written work in the laboratory time so don't spend hours on your write-up.

It is important to enter all your measurements in your laboratory work book and note uncertainties that can be estimated as well as any mistakes that were made. Improvements to the experimental procedure that might reduce uncertainties or eliminate potential errors should be suggested in your conclusions.

Where asked to take measurements you are expected to give a value for the uncertainty in that measurement. For **calculated** quantities, additional calculations of uncertainty may be required. The next section describes these uncertainties and which ones we expect you to use and understand in the second-year laboratory.

NOTES ON UNCERTAINTY

Measurements cannot be made to any desired precision. There are a number of factors that contribute to this limitation, including faulty apparatus, apparatus with its zero of scale at a nonzero position, fluctuations in environmental conditions (such as temperature and pressure), actual variations that exist in the required physical quantity (eg. when you measure the position of an electron you will get several values around an average value due to the uncertainty principle), and the fact that measuring instruments have a limit to their precision (if you measure a distance of $1\frac{1}{2}$ meters with a ruler that has meter markings then this is less precise than for the case where a ruler with cm markings is used).

Every measurement has an uncertainty. The numerical value of the uncertainty is a measure of how far the stated value of the quantity deviates from the true value. Sometimes the word error is used in place of uncertainty. The types of uncertainties you will encounter in second year physics are:-

- 1) Systematic or regular uncertainties.
- 2) Statistical or random uncertainties.
- 3) Uncertainties due to limit of reading of instruments.

Systematic or regular uncertainties are produced by well defined causes and can in some cases be eliminated if they are detected. These may arise from the instrument or from the observer or they may be due to some environmental condition.

Such uncertainties are often constant in magnitude or they may vary in some regular way with other variables. Repeated observations and statistical analysis will not necessarily reveal them.

For example, if a micrometer does not read zero when the jaws are closed, all readings will be too large or too small – but this is easy to detect by a simple calibration check at the start; on the other hand if a barometer vacuum is imperfect, every pressure reading will be too small and the impurity may be difficult to detect by casual inspection.

These uncertainties should be eliminated as far as possible, for example, by inspecting and allowing for the zero error of the micrometer or making a standard correction for the imperfect vacuum in a barometer. If they cannot be eliminated then a note should be made of possible causes of uncertainty.

Statistical or random uncertainties arise from various unknown sources and may include the actual variations that occur naturally in the magnitude of the measured quantity. The occurrence of this type of uncertainty is usually revealed by repeated observations. If it is possible to repeat a measurement many times, and if the repetitions give different results, then a good estimate of the uncertainty is half of the maximum deviation. About ten repetitions are needed for this to be reliable.

Instrumental or limit of reading uncertainties. The limit of reading of an instrument is the smallest quantity which can be reliably measured using the instrument's scale. For example, the smallest division of a typical metre rule is 1 mm. The limit of reading of this instrument is generally taken to be ± 0.5 mm. Note, however that differences of 0.2 mm may be reliably measured by someone with good eyesight and a 0.5 mm scale division.

NOTE—almost all measurements are in fact two measurements: one to verify the zero of the instrument's scale and one to make the measurement. Both have uncertainty, but only the systematic error of the zero point can be eliminated. The net uncertainty remaining is due to the limit of reading the instrument, typically half the finest scale division.

In the second year laboratory we expect you to keep track of the uncertainties due to the limit of reading of instruments and to eliminate or comment on the other two. This means that every time you make a measurement we expect you to also quote a value for the uncertainty in this measurement in terms of the smallest quantity that can be reliably measured, plus any relevant comments on (possible) systematic and (observable) random uncertainties.

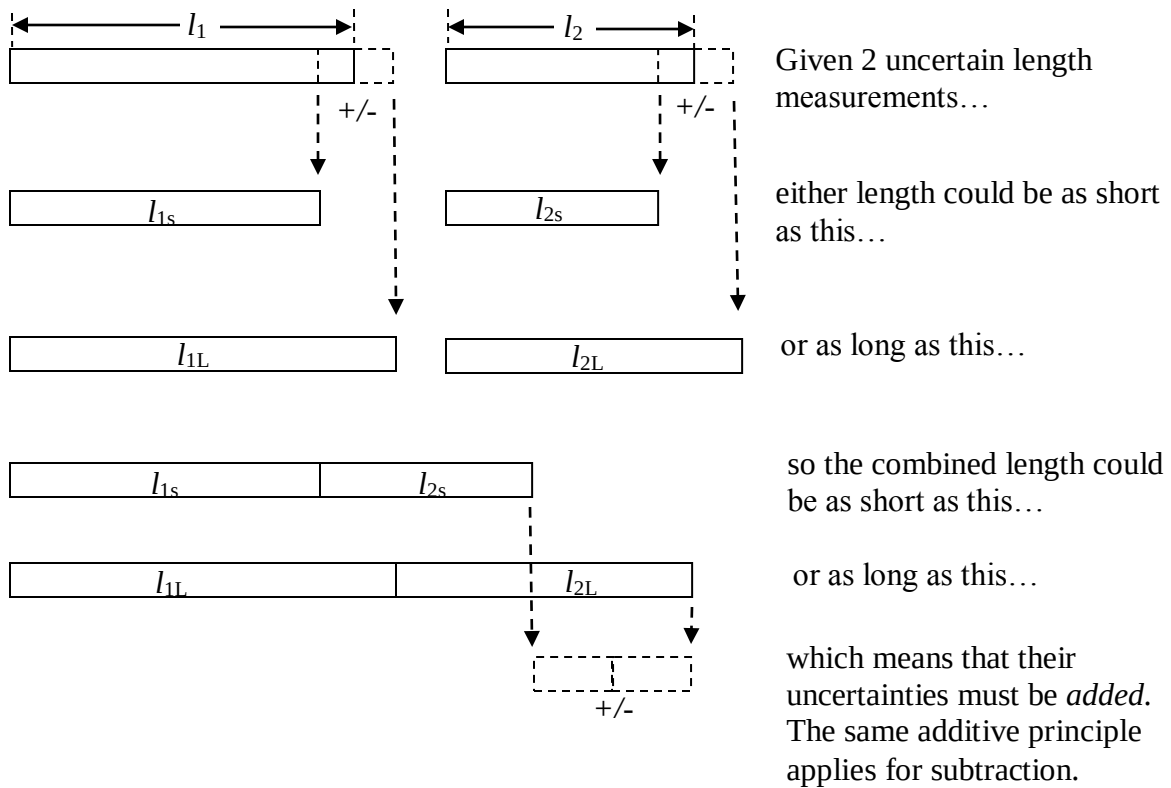
Sometimes you will be asked to calculate a value (such as total mass = $m_1 + m_2$) and quote the uncertainty on this calculated value. You would do this using the uncertainties on each measured quantity. There are standard methods for doing this, depending on the calculation.

The next section shows you how to calculate the maximum uncertainties when combining measured values for which you know the uncertainty.

Combinations of Uncertainties

Uncertain measurements that are used in calculations produce numerical results that are themselves uncertain. Combined measurements are always more uncertain than the original measurements due to the compounding of errors.

For example, if two uncertain length measurements are combined, either of the two actual lengths may be longer or shorter than their recorded values and the combined lengths may also be either longer or shorter, up to the combined uncertainties:



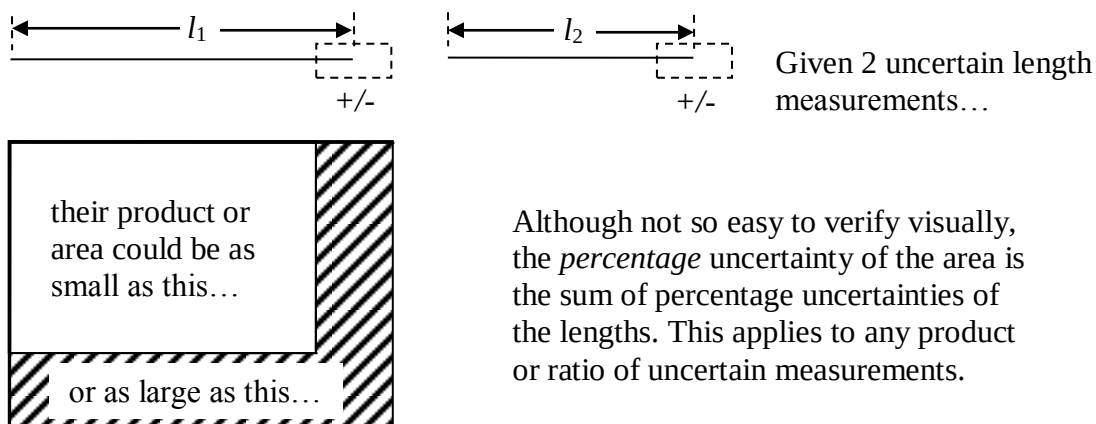
For example

If $l_1 = 80 \pm 0.5 \text{ mm}$
 and $l_2 = 40 \pm 0.5 \text{ mm}$
 then $l_1 + l_2 = 120 \pm 1 \text{ mm}$
 or $m_1 - m_2 = 40 \pm 1 \text{ mm}$

Rule 1. The uncertainty in a sum or difference is the **sum** of the uncertainties

Note that uncertainties are never subtracted.

A rule for multiplication and division of uncertain quantities can also be derived by a diagram:



For example

If $l_1 = 80 \pm 0.5 \text{ mm}$ ($0.5 / 80 = 0.625\%$)
 and $l_2 = 40 \pm 0.5 \text{ mm}$ ($0.5 / 40 = 1.25\%$)
 then $l_1 \times l_2 = 3200 \pm 60 \text{ mm}^2$ ($0.625 + 1.25 = 1.875\%$ of $3200 \text{ mm}^2 = 60 \text{ mm}^2$)

ie the area could be as small as $79.5 \times 39.5 = 3140.25$
 or as large as $80.5 \times 40.5 = 3260.25$
 so $3260.25 - 3140.25 = 120 \text{ mm}^2$ or $3200 \pm 60 \text{ mm}^2$

Rule 2. The percentage uncertainty in a product or quotient is the sum of the individual percentage uncertainties.

The final case to be considered is that of functions. You will not be expected to calculate the uncertainties for these in the second year laboratory but we place them here for your general interest.

An example would be to suppose an angle θ is measured to be $\theta = 0.520 \pm 0.010$ radians. What is the uncertainty in $\sin \theta$?

Rule 3. The uncertainty in $f(x)$ is $f'(x) \delta x$ where $f'(x)$ is the first derivative of $f(x)$ and δx is the uncertainty in x .

Rule 3 comes from the definition of a differential. If $y = f(x)$ then

$$f'(x) = \lim_{\delta x \rightarrow 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

Now suppose that δx is small, but not quite zero. We interpret δx as the uncertainty in x . Then

$$f'(x)\delta x \sim f(x + \delta x) - f(x)$$

The right-hand side of this expression is just the uncertainty in the function. That is, if

$$y = f(x)$$

then

$$\delta y = f(x + \delta x) - f(x)$$

This demonstrates rule 3, which can be used to prove rules 1 and 2.

In the above example, $\theta = 0.520 \pm 0.010$ radians, so the uncertainty in $\sin \theta$ is $(\cos 0.52) \times 0.01 = 0.009$ since $d(\sin \theta) / d\theta = \cos \theta$. Therefore $\sin \theta = 0.497 \pm 0.009$.

It follows from Rule 3 that the **percentage** uncertainty in x^n is n times the **percentage** uncertainty in x . For example the percentage uncertainty in $\sqrt{x}$ is half that in x .

Notice that calculations using uncertain measured inputs can become very uncertain indeed. This tends to motivate improvements in either the precision of experimental methods or the simplicity of theoretical formulae, or both.

Setting out Uncertainty Calculations

Calculations where several operations are combined may cause confusion. A tabular form of calculation assists considerably and should be used for all uncertainty calculations.

Example:

$$V = xy(p + q)$$

By measurement, we find that $x = 1.32 \pm 0.04$ m, $y = 0.470 \pm 0.020$ m, $p = 14.1 \pm 0.5$ m, $q = 13.2 \pm 0.5$ m. We want to calculate V and its uncertainty.

The uncertainty table below is filled in by the following steps.

1. Insert the experimental data, taking care that all quantities are expressed in SI units.
2. Calculate $(p + q)$ and its uncertainty from Rule 1.
3. To calculate $xy(p + q)$ and its uncertainty by Rule 2, the percentage uncertainties of $(p + q)$, x and y are required.

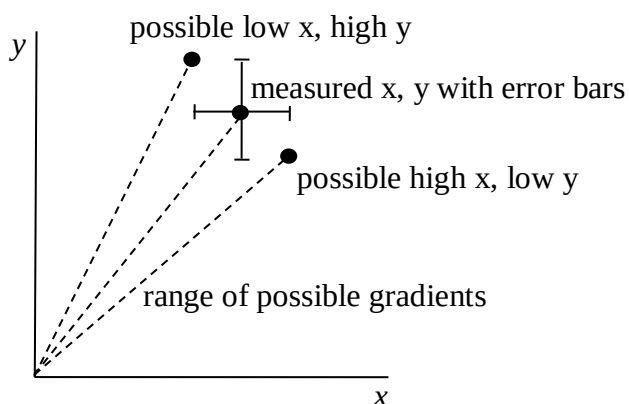
4. Calculate $xy(p + q)$.
5. Calculate the percentage uncertainty in $xy(p + q)$ from Rule 2 and finally the absolute uncertainty.

Quantity	Value	Uncertainty	% uncertainty
x	① 1.32	① 0.04	③ 3.0
y	① 0.470	① 0.020	③ 4.3
p	① 14.1	① 0.5	
q	① 13.2	① 0.5	
$p + q$	② 27.3	② 1.0	③ 3.7
$xy(p + q)$	④ 16.9	⑤ 1.9	⑤ 11

The answer is therefore $V = 16.9 \pm 1.9 \text{ m}^3$

Uncertainties on graphs

You may also be required to plot values on a graph, calculate a value from the gradient of this graph and quote an uncertainty for this value. You therefore need to know how to calculate the uncertainty in the best-fit line. This is done by first plotting your points on the graph with what are known as error bars. These are lines drawn through the plotted point indicating the span of values that a measured quantity could have. The size of the error bars indicates the uncertainty in that measurement (in both x and y). So if you were plotting a point at $x = 1 \pm 0.2\text{m}$ and $y = 2 \pm 0.2\text{m}$ you would plot a point at $x = 1\text{m}$, $y = 2\text{m}$ and place a line of length 0.2m to either side of it in x and in y . You would then draw a best fit line from which to calculate your measured value for the gradient and a worst fit line (that is still contained within the error bars) to calculate the maximum deviation from this value. This is the uncertainty in the gradient.



Quoting uncertainties and reporting results

The rules for quoting uncertainties and reporting results are:

- Quote the result to a precision that matches the uncertainty.

<i>Example</i>	$35.83 \pm 0.06 \text{ m}$	correct
	$35.8 \pm 0.06 \text{ m}$	incorrect
	$35.8294 \pm 0.06 \text{ m}$	incorrect

- If a power of ten is needed, use the same power for the uncertainty as for the result. This makes the statement of result easier to read.

Example

$(2.174 \pm 0.011) \times 10^{-7} \text{ J}$	correct
$2.174 \times 10^{-7} \pm 1.1 \times 10^{-9} \text{ J}$	incorrect

- ALWAYS include the units.

NOTES ON THE DRAWING OF GRAPHS

1. Every graph must have a title or informative caption. It should be self contained, so that a reader does not have to dig around in the text to discover what the graph is about.
2. Choose a scale which fills the grid as far as possible, but always use multiples of 1, 2 or 5 for the major divisions. Other choices make it difficult to plot points.
3. ALWAYS label the axes with the name of the quantity being plotted.

SHOW THE UNITS.

4. Data points must be clearly visible. Use small crosses or dots inside circles.
5. The graph should be a smooth curve, almost always a straight line, which lies evenly between the plotted points with some on either side. Do not sacrifice smoothness by forcing the line to pass through every point. Do not attach undue significance to the origin; it is subject to the same experimental uncertainties as any other point.
6. If the points lie approximately on a straight line, and if there is sufficient reason to expect that a straight line is a correct representation of the data, then draw the line of best fit using a ruler and a **sharp** pencil. A clear plastic ruler is best.
7. If you need the slope of a line or other information, then you may print it on the graph. **NOTE:** a slope usually has **UNITS**.

The graph below illustrates the above points

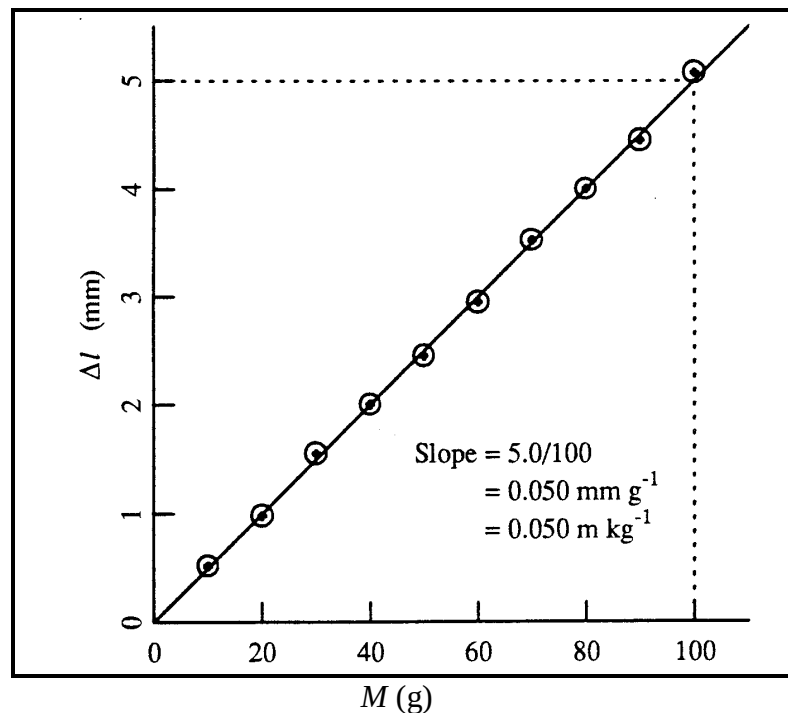


Figure 1. Cumulative extension Δl of a thin nichrome wire as a function of the load from a mass M applied to one end.

GOOD EXPERIMENTAL PROCEDURE

- Record all measurements directly into your work book. Recording observations on scraps of paper and subsequently copying into the work book is very bad practice.
- As far as practicable, make several observations of each quantity. This minimises the effects of mistakes and it assists with the determination of the uncertainty in the measurement.
- ALWAYS record exactly what you read from an instrument. DO NOT perform unit conversions or any other calculations before writing the result down.
- It is usually best to record observations in the form of a table. Each column of the table must have a heading, INCLUDING UNITS AND UNCERTAINTIES. A tabular form can shorten the time taken in doing calculations, since they can then be performed in a systematic way.

Example

We measure height h as a function of time t and need to plot a graph of h^2 against $1/t$. We expect the graph to be a straight line.

The results should be recorded as follows:

H (cm)	h^2 (m ²)	t (min)	$\frac{1}{t}$ (sec ⁻¹)	Comments

The units in columns 1 and 3 are the units in which the respective instruments are calibrated. The final comment column is useful for recording other necessary experimental conditions or other comments on the reading. Note anything that you think might affect the accuracy of a measurement.

- Handle apparatus carefully. You may be required to pay for the repair of apparatus damaged through misuse.
- At the end of the afternoon, the apparatus must be left tidily in the condition in which you found it.

HOW TO PRESENT A TECHNICAL REPORT

A report must contain the following sections:

- Title of the experiment and date that it was conducted.
- Aims of the experiment.
- Theoretical background.

Do not just copy the theory from the experiment description. Paraphrase it. We are looking for evidence that you understand what you are reporting on.

- Method, experimental observations, and comments.

The presentation of your work should be clearly and logically arranged so that the reader can easily follow what you have done. You should write in the past tense (for example “the object was weighed on electronic scales”). Do not reproduce large quantities of the experimental notes, merely state, in your own words what was done. The comments on your work should include the following:

- A brief explanation of what was done at each stage of the experiment.
 - Relevant detailed information not included in the experimental notes.
 - A description of mistakes you may have made and how they were corrected, or their effect if it was not possible to correct them.
- Results and calculations. The following should be included:
 - Your results should be presented in tabular format, with uncertainties quoted where appropriate. Tables can also be used for calculations where appropriate. Graphs and calculations should be included in this section.
 - An uncertainty analysis where asked. All direct measurements must be stated with an uncertainty.
 - The final, result should be quoted and compared with tabulated data, giving reasons for any discrepancy.
 - Conclusions.

Irrespective of whether an experiment was successful or not, you **MUST** write a conclusion, or more usually the plural conclusions. As well as providing a final statement of the results obtained, it affords an opportunity for a critical appraisal of the experiment. In general, you should address the following points:

- Summarise the experiment in terms of the fundamental principles of physics demonstrated.
- State the final result(s) and the estimated uncertainty or major source of uncertainty, including the appropriate units.
- Compare your result with tabulated data, citing the reference.
- Discuss any discrepancy between the experimental and tabulated results, highlighting the major sources of uncertainty.
- If possible, suggest ways of improving the experiment by reducing the uncertainty factors and removing or reducing difficulties.

FREQUENTLY USED SYMBOLS

There are many symbols used in physics. Here is a list of some that you should become familiar with.

Greek alphabet

Learn to recognise the Greek letters and practice writing them legibly.

	α	alpha		ν	nu
	β	beta	Ξ	ξ	xi
	γ	gamma		$\omicron$	omicron
Δ	δ	delta		π	pi
	ϵ	epsilon		ρ	rho
	ζ	zeta	Σ	σ	sigma
	η	eta		τ	tau
	θ	theta		υ	upsilon
	ι	iota	Φ	ϕ or φ	phi
	κ	kappa		χ	chi
Λ	λ	lambda	Ψ	ψ	psi
	μ	mu	Ω	ω	omega

SI prefixes

Use these as often as possible. You should learn the prefixes down to tera and femto.

kilo	k	$\times 10^3$	milli	m	$\times 10^{-3}$
mega	M	$\times 10^6$	micro	μ	$\times 10^{-6}$
giga	G	$\times 10^9$	nano	n	$\times 10^{-9}$
tera	T	$\times 10^{12}$	pico	p	$\times 10^{-12}$
peta	P	$\times 10^{15}$	femto	f	$\times 10^{-15}$
exa	E	$\times 10^{18}$	atto	a	$\times 10^{-18}$
zetta	Z	$\times 10^{21}$	zepto	z	$\times 10^{-21}$
yotta	Y	$\times 10^{24}$	yocto	y	$\times 10^{-24}$