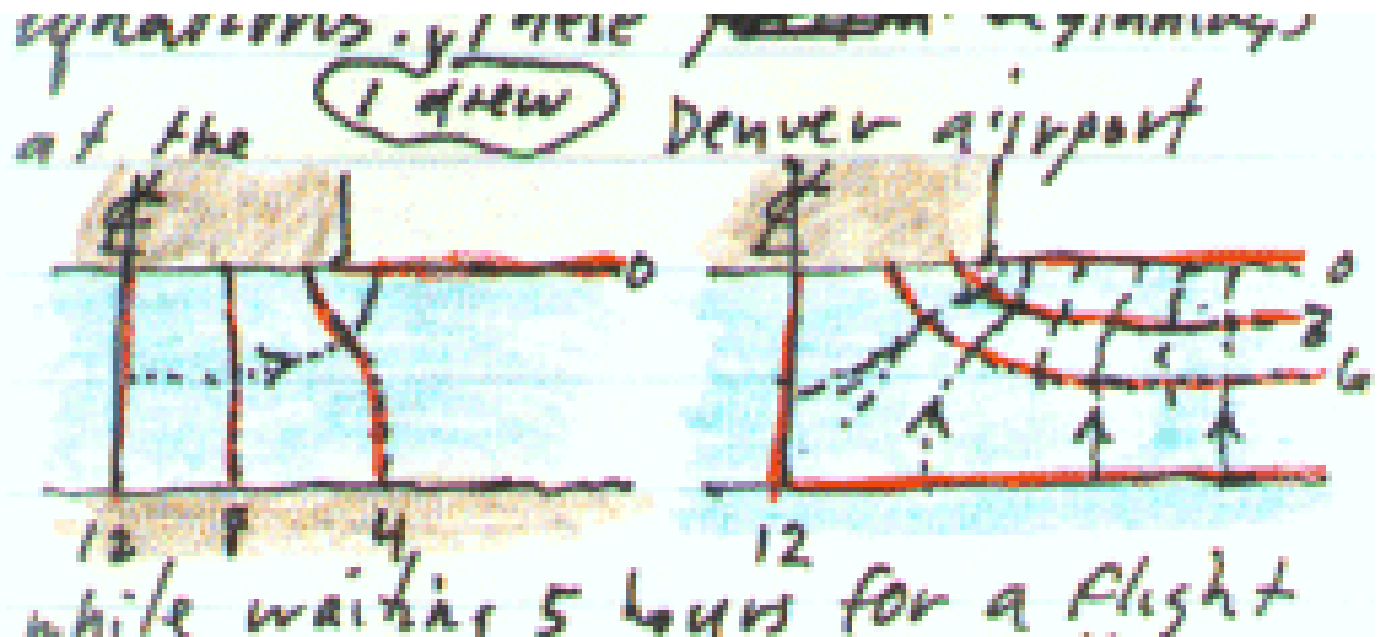
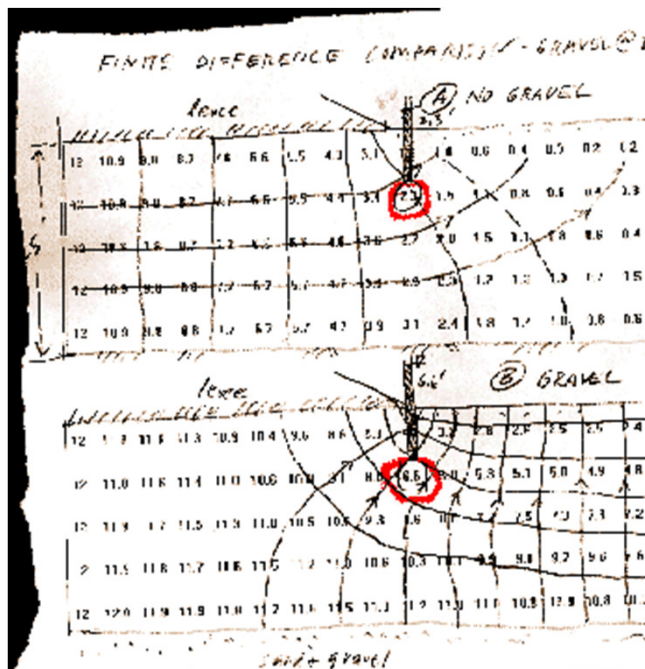


Lecture 9 Seepage Flow Nets

Text: Das (2010). Chapter 8.

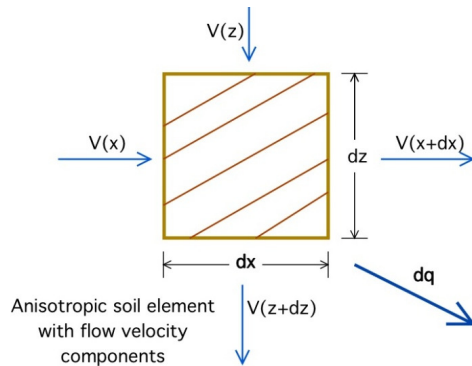
Darcy's law is generally adequate for one dimensional flow analysis, ie where the geometry of the flow is not complex and where simplifying assumptions carry no extra risk. This is rarely the case for realistic engineering design work, other than as a first estimate.

For 2nd-order analysis, a hand-drawn 2-dimensional diagram known as a *flow net*, using theory based on continuity and permeability, is a standard design detail. These are often prepared as a preliminary to a full design analysis using 3d computer models, and are essential for developing your understanding of groundwater flow.



Flow Net Potential Theory

Darcy's Law and Continuity (in the fluid mechanics sense) provide sufficient conditions for construction of a flow net diagram:



Continuity : Inflow = Outflow : $Q = VA$ or $q = VI$

q is the flow per unit depth into the page, I is a length such as dx or dz at right angles to the flow, V is the discharge velocity as per Darcy's law.

$$dq = V_x dz + V_z dx = V_{(x+dx)} dz + V_{(z+dz)} dx$$

$$\text{so } (V_x - V_{(x+dx)}) dz = (V_{(z+dz)} - V_z) dx$$

$$\text{but } (V_{(z+dz)} - V_z) = \partial V_z$$

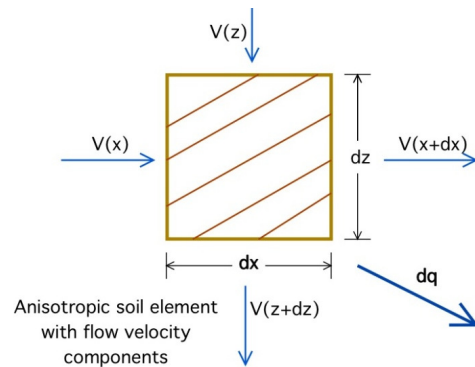
$$\text{and } (V_x - V_{(x+dx)}) = -\partial V_x$$

$$\text{so } -\frac{\partial V_x}{\partial x} = \frac{\partial V_z}{\partial z} \quad \text{or} \quad \boxed{\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} = 0}$$

The highlighted expression is a simplified form of the differential continuity equation, which is equivalent to the more familiar $Q = VA$

Flow Net Potential Theory

Darcy' law enables us to replace unknown velocities with known or measurable headloss rates and permeabilities:



$$V_x = -k_x \frac{\partial h}{\partial x} \quad \text{and} \quad V_z = -k_z \frac{\partial h}{\partial z}$$

Darcy's law: the minus signs indicate +ve velocity for decreasing head (negative hydraulic slope)

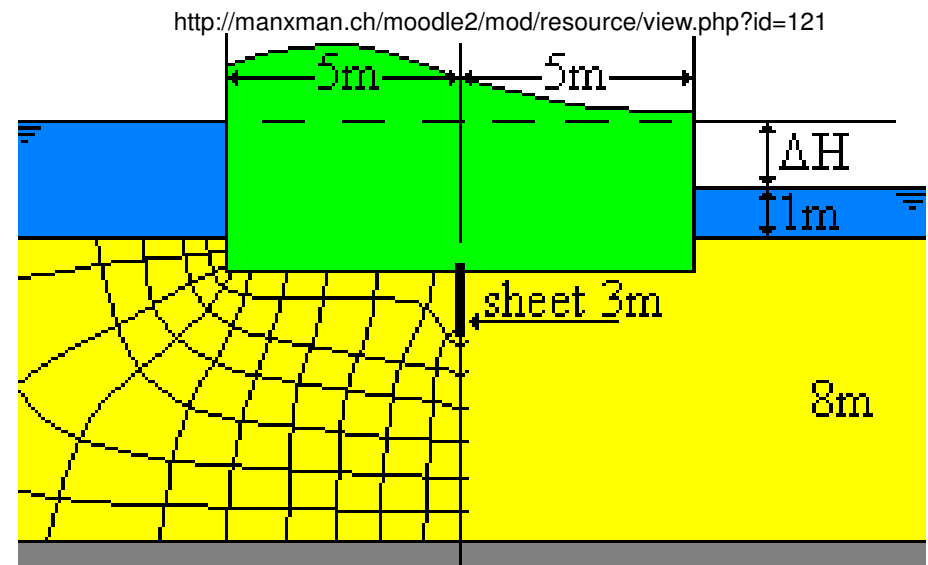
$$\text{so} \quad \frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} = 0$$

$$\text{becomes} \quad k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

The highlighted expression is a modified form of the *Laplace equation*, a 2-dimensional differential equation which is consistent with both Darcy's law and continuity.

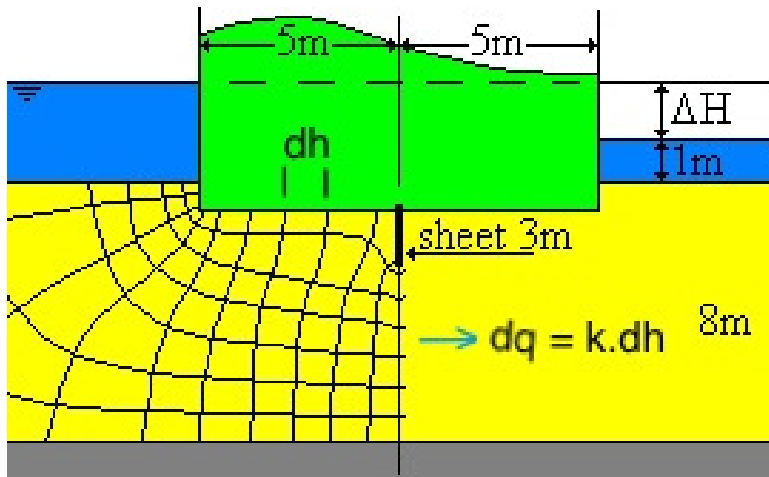
When $k_x = k_z = k$ (isotropic soil), the Laplace equation has a useful property of *orthogonality*, that is the contours of constant head are always at right angles to the streamlines.

As such the equation can be solved graphically, by sketching orthogonal flow nets with the contours always intersecting at right angles.



Flow Net Streamlines

Streamlines are never crossed by the flow. Therefore the flow within each stream tube (between pairs of streamlines) is constant.



<http://manxman.ch/moodle2/mod/resource/view.php?id=121>

For simplicity we draw lines of constant head at constant *increments* of head dh , and streamlines at constant increments of discharge dq .

The flow net becomes orthogonal when we also let $dq = k.dh$

Then the total headloss ΔH and total flow per unit width q are simply functions of the number of lines we include in the sketch:

$$\Delta H = N_{\phi} dh$$

$$q = N_{\psi} dq$$

where N_{ϕ} is the number of equipotential intervals (between lines of constant head) and N_{ψ} is the number of streamtubes (intervals between streamlines). (Greek letters ϕ phi and ψ psi will be explained in a moment)

$$\text{but } dq = k.dh$$

$$\text{so } q = N_{\psi} dh = k \frac{N_{\psi}}{N_{\phi}} \Delta H$$

A well drawn flow net with a known headloss ΔH , conductivity k and streamlines (ψ) orthogonal to potential lines (ϕ) gives a useful indication of the seepage flowrate.

Flow Net Graphical Method

A graphical flow net is sketched and re-sketched until the lines intersect at right angles.

1. Draw a long section of the surface and groundwater channels including any infrastructure such as dams or sheet piles.

2. Draw water surfaces at upstream and downstream ends of the channel

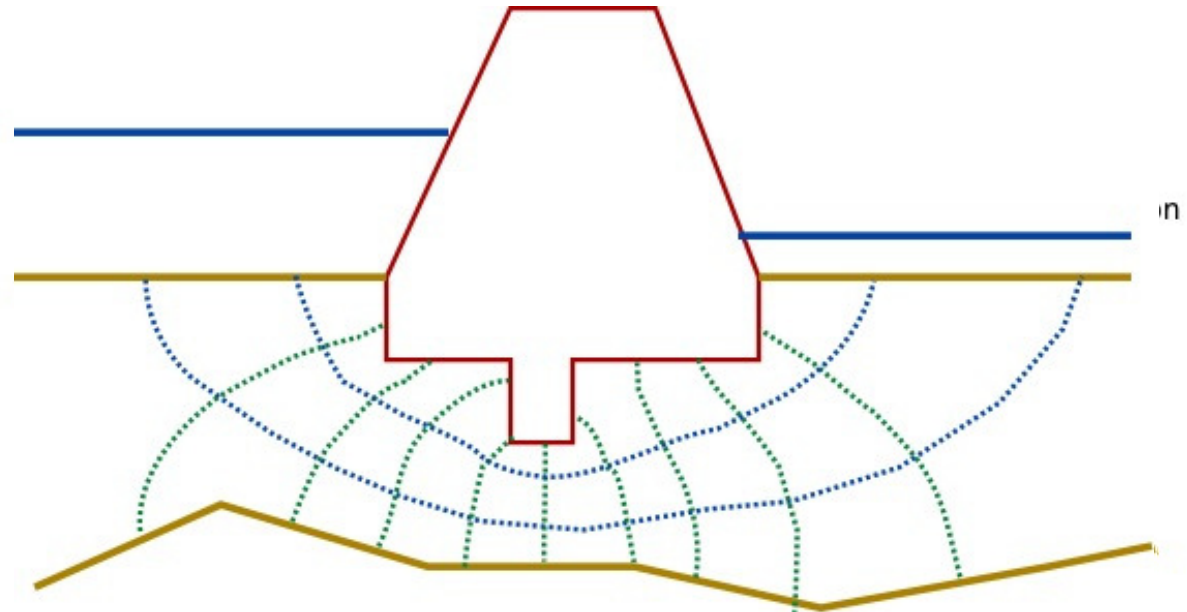
3. Identify head and headloss at start and endpoints of the flow path

4. Identify any boundary conditions such as impermeable strata (bedrock)

5. Sketch a primary streamline dividing the groundwater channel into two flow tubes. The streamline is perpendicular to any lines of constant head in the channel, eg the up- and downstream ends. THIS IS A GUESS. Use symmetry to position the line well.

6. Sketch a few lines of constant head to divide the flow path into say four distinct bands. Head lines are perpendicular to boundaries and streamlines

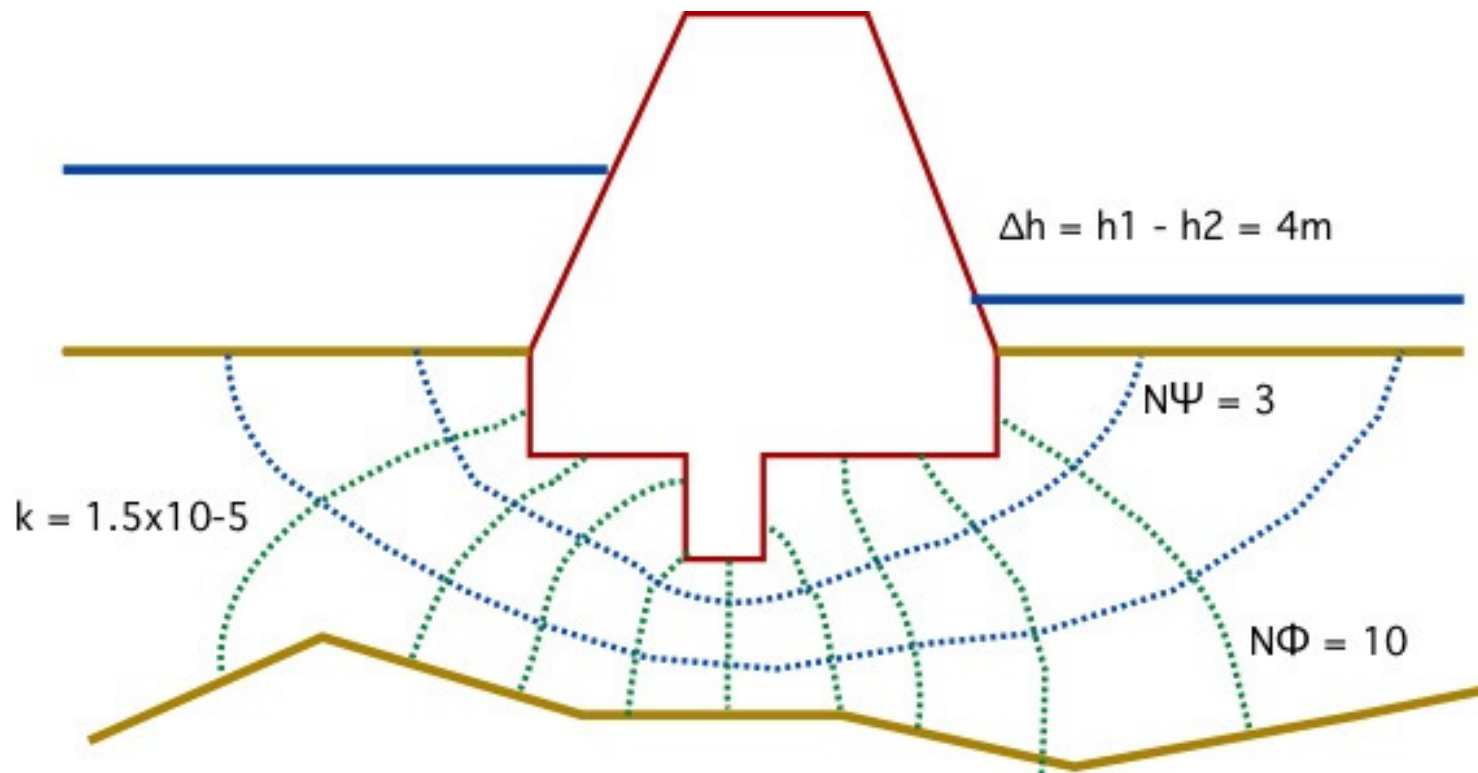
7. Continue sketching streamlines and potential lines, erasing and correcting where necessary to obtain an orthogonal-looking flow net.



A certain amount of approximation usually remains at the end. Graphical software is useful for speeding this up

Flow Net Graphical Method

To a reasonable level of accuracy, $q = k \frac{N_\psi}{N_\phi} \Delta H$



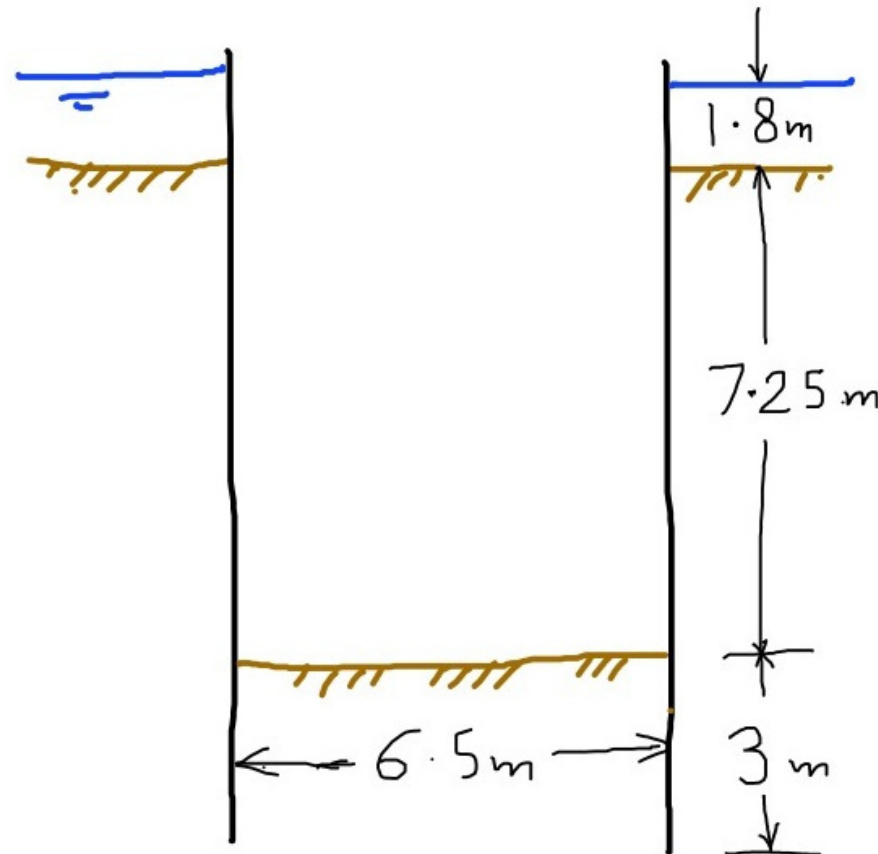
$$q = 1.5 \times 10^{-5} \times 4 \times 3 / 10$$

$$= 1.8 \times 10^{-5} \text{ m}^2/\text{s}$$

Flow Net Graphical Method

Example: The long cofferdam shown can be considered to have 2-dimensional seepage flow. Determine the seepage per metre width into the dam if the water level is maintained at the excavated lower level. Assume isotropic soil, $k = 0.015 \text{ m/s}$, $\Delta H = 9.05 \text{ m}$

$$q = k \frac{N_\psi}{N_\phi} \Delta H$$



Flow Net Graphical Method

Solution: No of potential steps = 15; No of flow tubes = 7

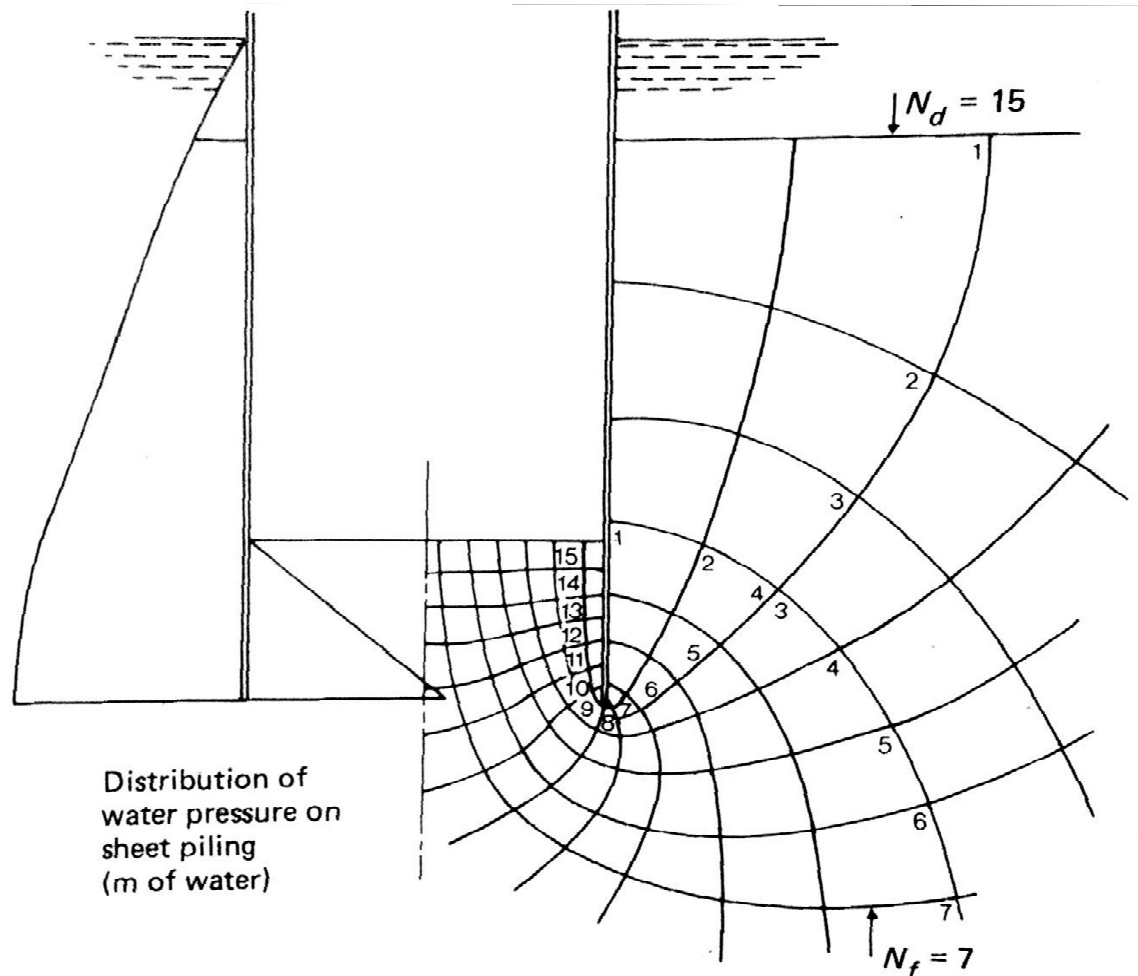
Seepage:

$$q = 0.015 \times 9.05 \times 7/15$$

$$= 0.063 \text{ m}^3/\text{s}/\text{m}$$

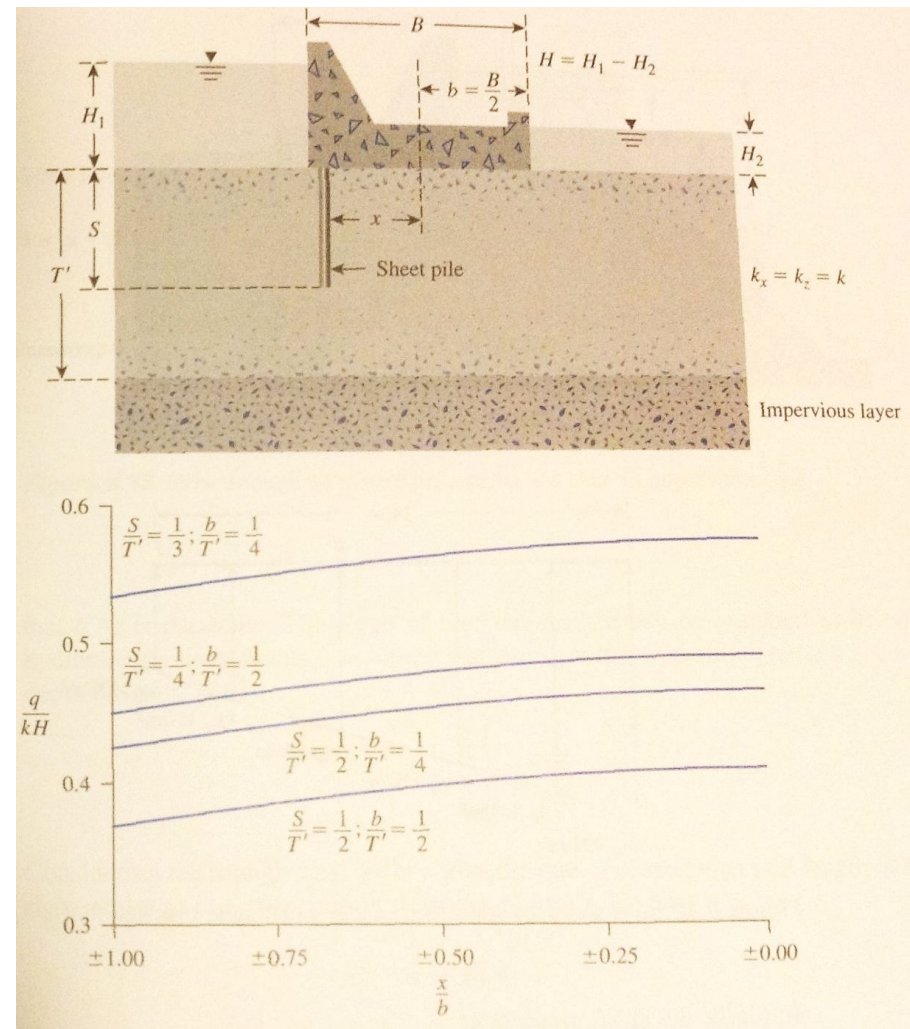
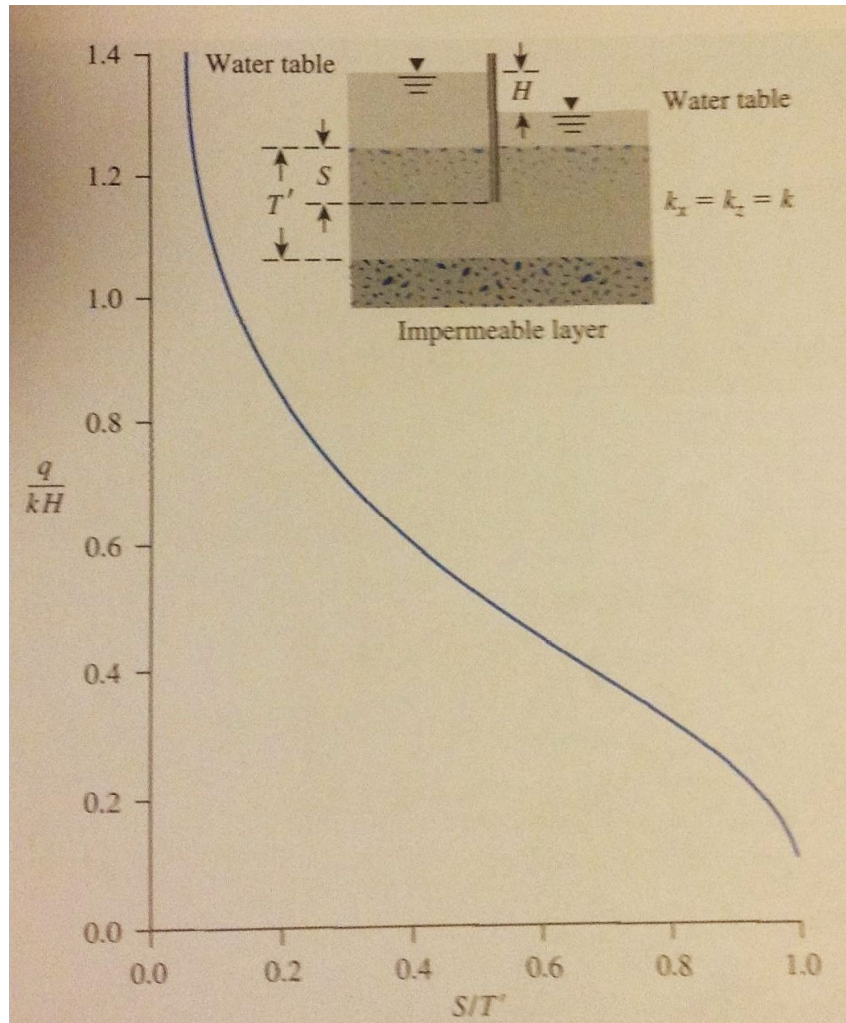
This is for half the dam ...

$$q = 0.126 \text{ m}^3/\text{s}/\text{m}$$



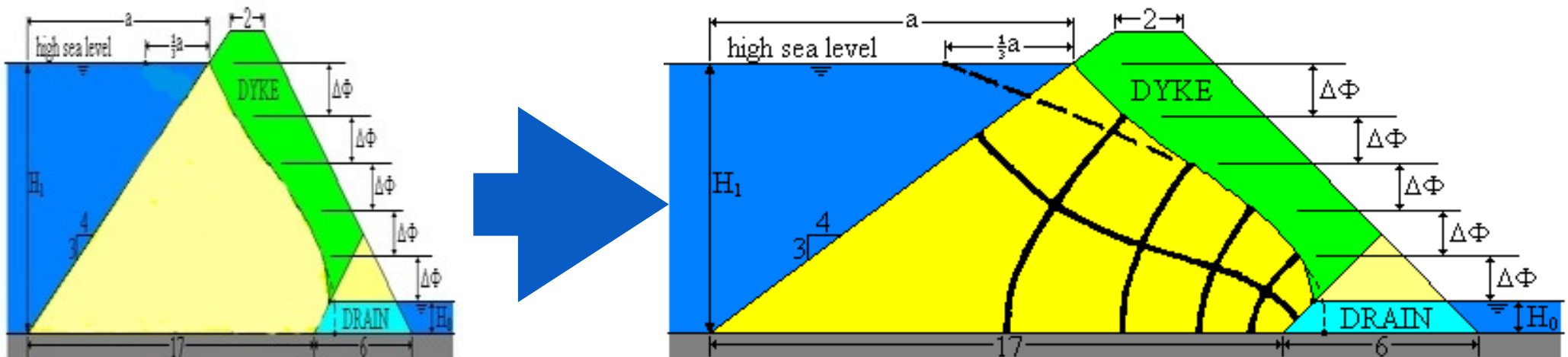
Integrated Graphical Solutions

A considerable amount of work has been put into integrating mathematical solutions of flownets into graphical solutions requiring only the use of tables or graphs. Two examples are shown:



Flow Nets in Anisotropic Soils

When k_x and k_z are unequal, the graphical method can be applied by coordinate transformation; "stretching" of the x axis.



<http://manxman.ch/moodle2/mod/resource/view.php?id=121>

Channel geometry is transformed by a horizontal scale factor (about 2 in this case) then the orthogonal flow net is drawn as before.

The scale factor is:

$$x' = x \sqrt{\frac{k_z}{k_x}}$$

The seepage is then:

$$q = \sqrt{k_z k_x} \frac{N_\psi}{N_\phi} \Delta H$$

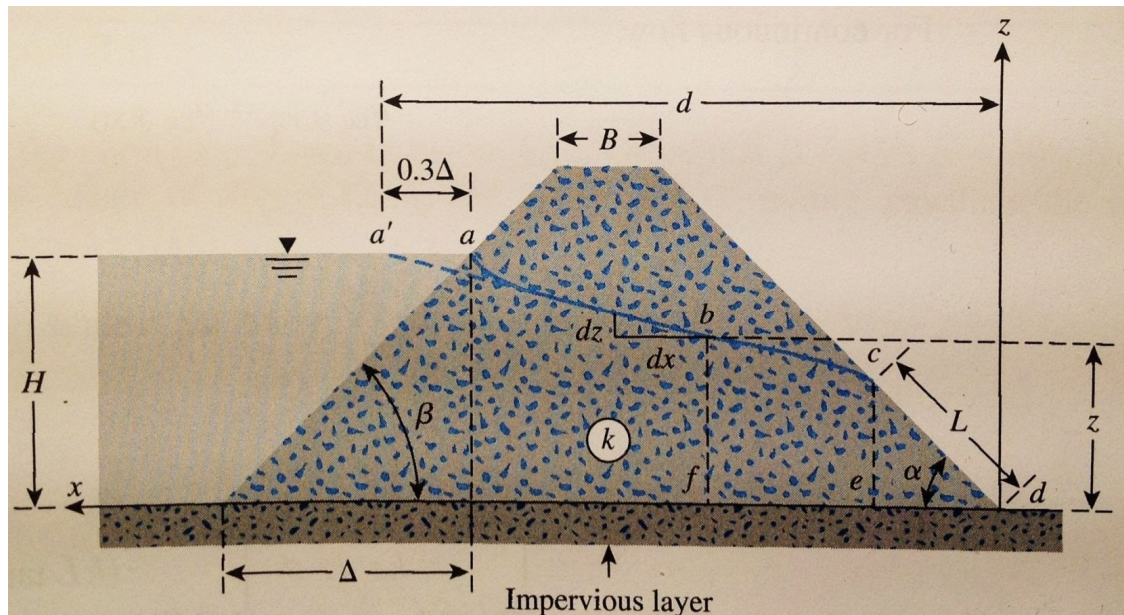
The method is modified:

1-4. Draw a longsection of the surface and groundwater channels ... *on the transformed x axis* ...

5-7. Draw an orthogonal flownet ... *on the transformed channel* ...

Earth Dam Seepage: Easy Solution

For the limited case of an earth dam constructed of isotropic soil on an impervious base (not an unrealistic assumption for disturbed or engineered soils), a simple analytic solution (Schaffernak's method) exists.



$$q = kiA$$

$$i = \frac{dz}{dx} = \tan(\alpha)$$

$$A = L \sin(\alpha)$$

$$q = kL \tan(\alpha) \sin(\alpha)$$

This simple method is adequate for exactly this type of earth dam, which is not uncommon in light urban or agricultural engineering. The face length L (a design unknown) can be estimated for assumed water levels (the 0.3Δ correction is due to Casagrande):

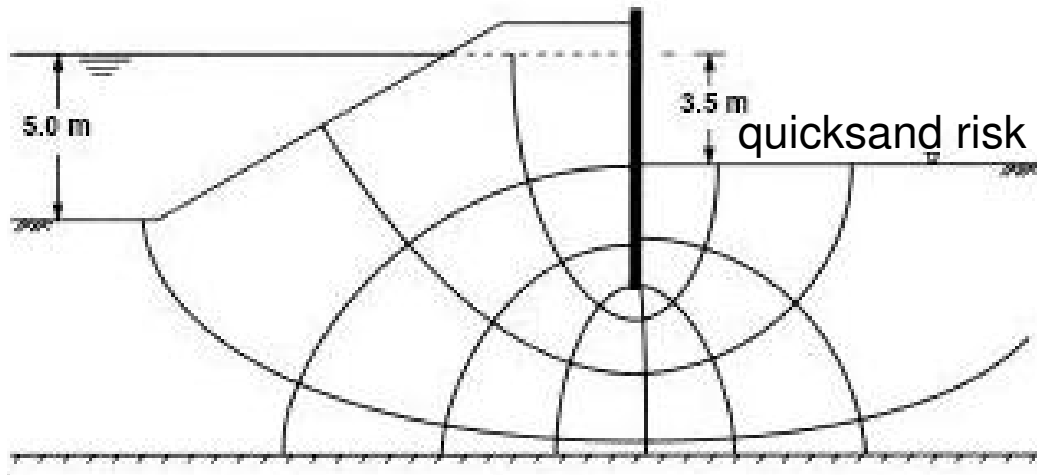
$$L = \frac{d}{\cos(\alpha)} - \sqrt{\frac{d^2}{\cos^2 \alpha} - \frac{H^2}{\sin^2 \alpha}}$$

$$d = W - \frac{0.7H}{\tan\beta}$$

W is the width from heel to toe of the base

Soil Stress due to Seepage

Groundwater flow causes frictional drag on soil particles. Rising groundwater may create sufficient lift to suspend particles in a "quick" condition.



Since $V = k \cdot dh/dl$ we find higher velocities wherever the friction slope (hydraulic gradient) is higher.

The friction slope is indicated by the spacing of the flownet grid; closer spacing indicates a higher rate of headloss, therefore higher velocity.

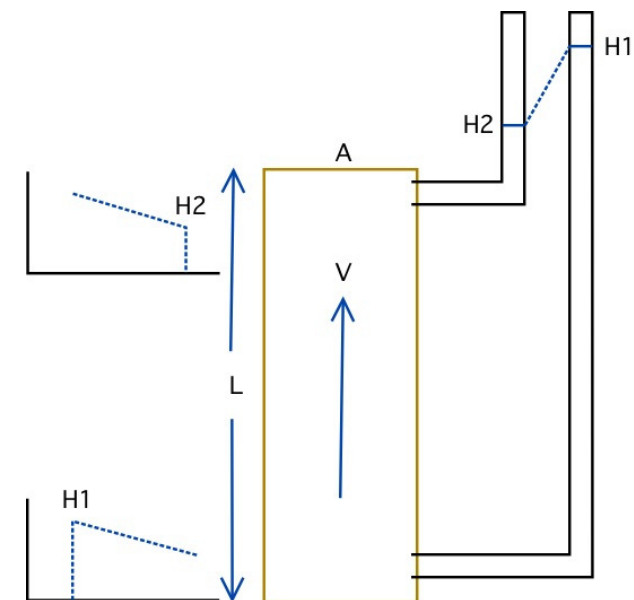
A rising flow is balanced against the submerged (buoyant or effective) weight of a soil column

Effective soil stress:
$$\sigma' = \gamma' L = \gamma_w \left(\frac{G - 1}{1 + e} \right) L$$

Pressure drop due to friction:
$$\Delta P = \gamma_w \Delta H = \gamma_w \frac{dH}{dL} L = \gamma_w i L$$

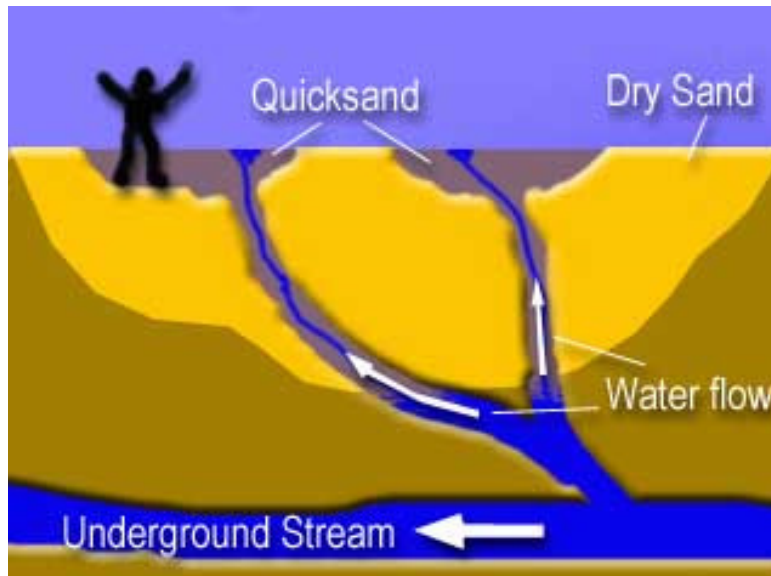
Stress = Pressure at critical hydraulic gradient:

$$\sigma' = \Delta P \quad : \quad i_c = \frac{G - 1}{1 + e}$$



Soil Stress due to Seepage

There is a critical hydraulic gradient which in rising flows will cause uplift of the soil:



<http://www.unmuseum.org/quicksand.htm>

$$i_c = \frac{G - 1}{1 + e}$$

This phenomenon goes by a number of names: "Quicksand"; "Piping" or "Boiling"

Soil is lifted and transported by the flow, with a loss of structural integrity of soil.

Factors of safety against piping ensure that hydraulic gradients in rising flows are well less than critical:

$$FSP = \frac{i_c}{i_{max}} \geq 1.5$$

For many soils, $G \sim 2.7$ and $e \sim 0.7$ so $i_c \sim 1$

Geotextile filters can be layered into soil wherever piping may occur. These are heavy fabric blankets that hold the soil in place while allowing flow to seep through.

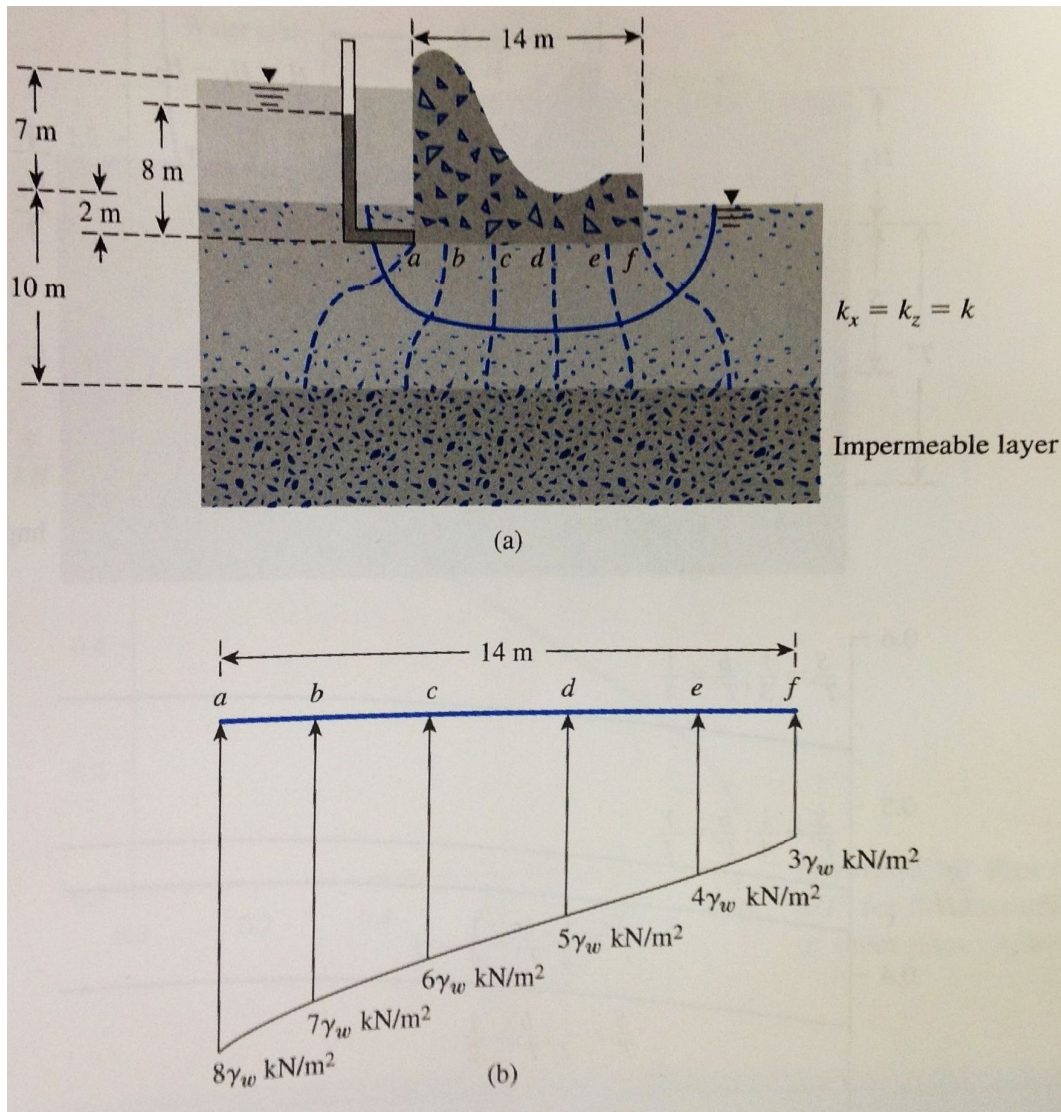
Specifications for filters were originally developed by Terzaghi. See the reading for more information.

Garah Dam Failure 1981



Uplift Pressure under Hydraulic Structures

Flow nets can be used to determine the uplift pressure at the base of a hydraulic structure.



There are 7 potential drops (N_ϕ) in the flownet, and the total difference in head across the net is 7 m. Each drop is 1 m.

The head at each potential line is converted directly to pressure:

$$P_a = (7+2-1)\rho g = 8\text{kPa}$$

$$P_b = (7+2-2)\rho g = 7\text{kPa} \dots \text{etc}$$

The uplift force is found by integrating pressure over area:

$$F_{ab} = \frac{P_a + P_b}{2} WL_{ab}$$

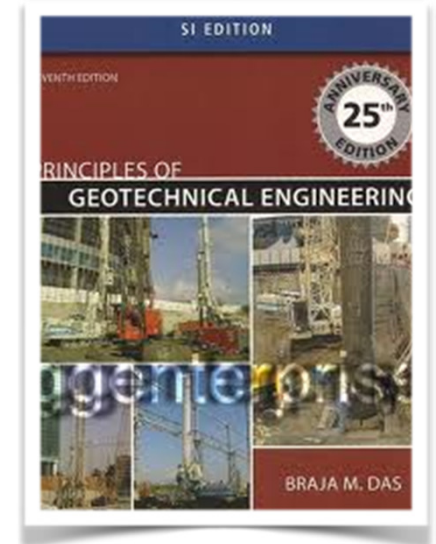
$$F = F_{ab} + F_{bc} + \dots + F_{ef}$$

Uplift failure is a significant factor in dam design.

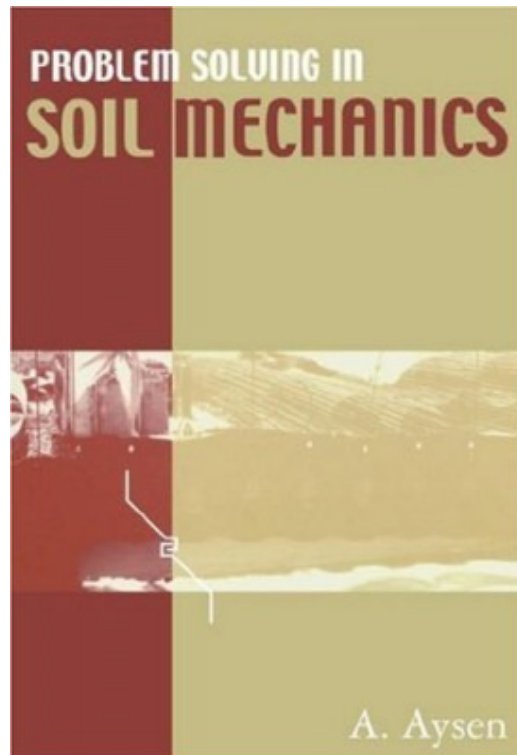
Readings

Chapter 8, Das.

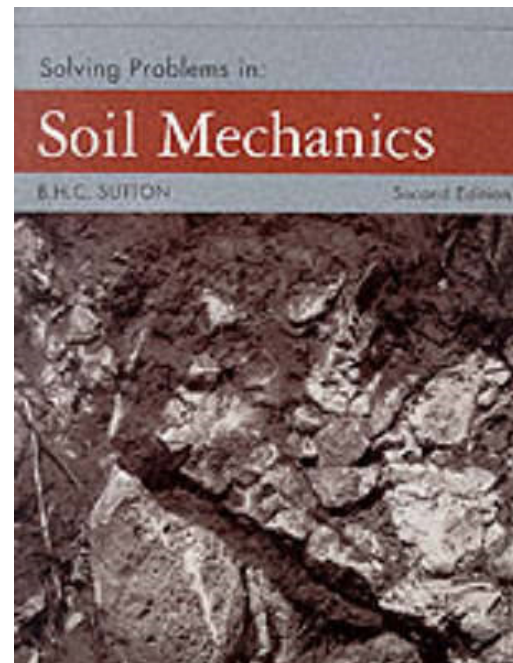
Internet research: Flownets, potential flow, quicksand.



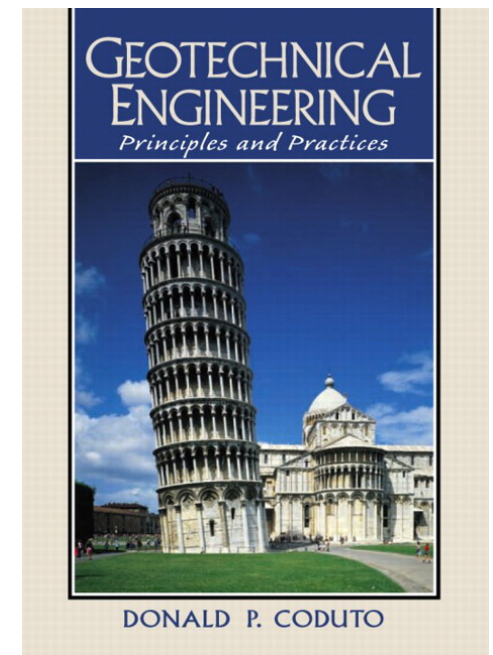
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