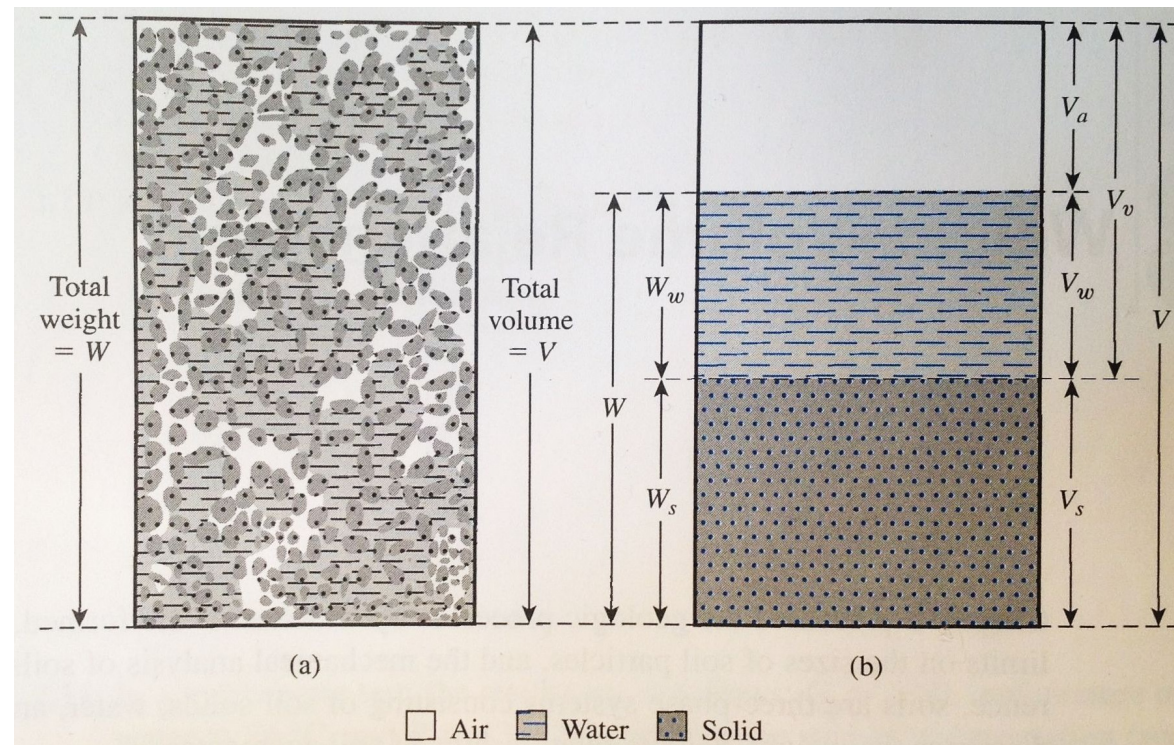


Porosity and Density

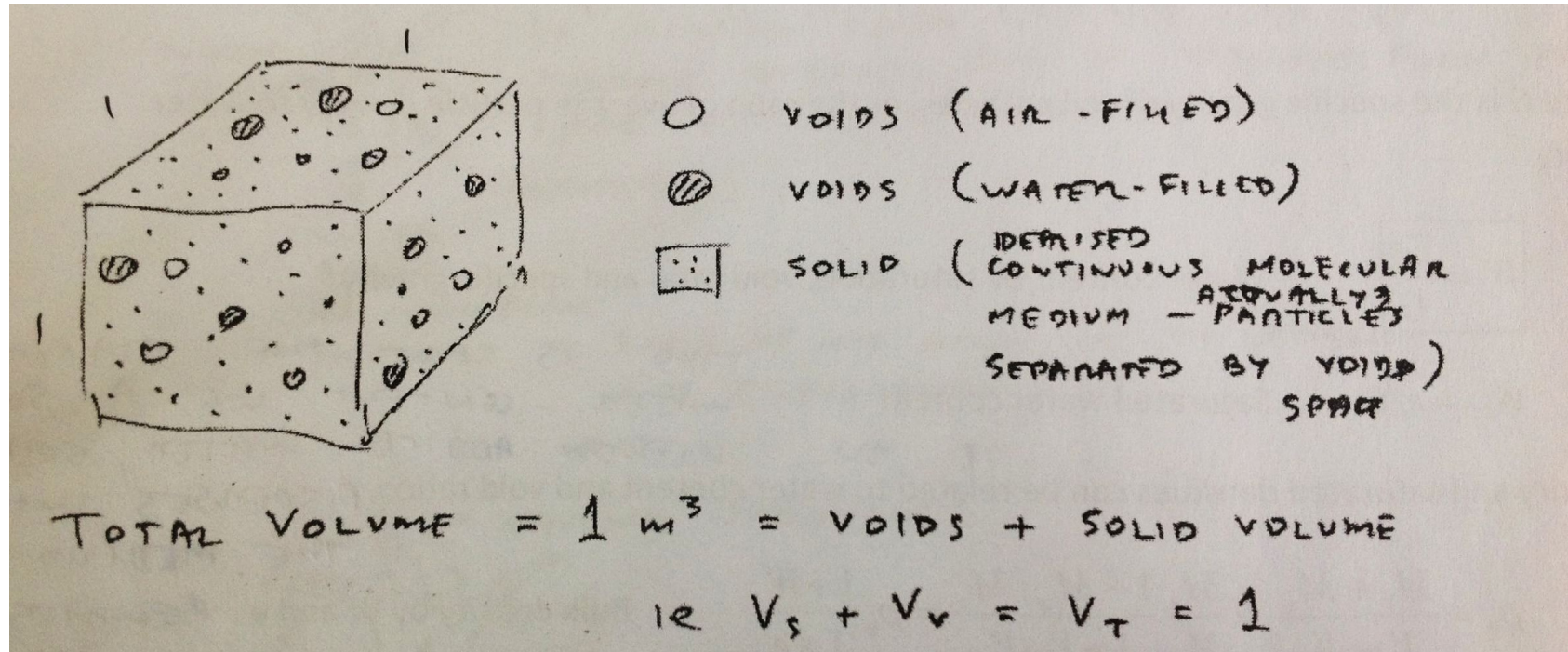
Text: Das. (2010). *Principles of Geotechnical Engineering*, Ch. 3

Figure 3.1 shows an element of soil of volume of Volume V and weight W as it would exist in a natural state. To develop weight-volume or density relationships for the soil we must separate the three phases, that is solid water and air as shown.



Porosity and Density

Consider a cubic metre of soil:



It would be unusual in practice to find a one cubic meter block of soil with uniform properties. The unit volume idealisation is useful as a framework for the following derivations.

The illustration also does not indicate the connectivity of the pore space within the soil matrix. In real soils, the pore space is effectively continuous, while the solid material is a 'skeleton' of separate particles.

Porosity

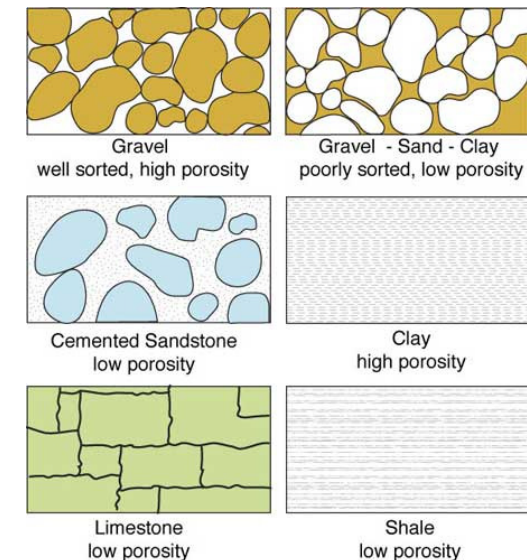
The *porosity* of soil is the proportion of void volume to the total volume.
The porosity of a unit volume of soil equals the volume of voids:

$$n = \frac{V_v}{V_t} = \frac{V_v}{1} = V_v < 1$$

Porosity n ; proportion by volume of voids including air and water volumes.

$$n = V_v = V_a + V_w$$

Porosity *decreases* with drying and compaction. Low porosity may be a good or a bad thing:



Void Ratio

The *void ratio* is a measure of porosity relative to the solid volume rather than the total volume. This is often easier to measure, and as it is related to porosity it provides a means of measuring porosity directly.

$$e = \frac{V_v}{V_s} = \frac{V_v}{V_t - V_v} = \frac{n}{1 - n}$$

So given a measurement of the void ratio the porosity is:

$$e = \frac{n}{1 - n}$$

$$e - en = n$$

$$e = n(1 + e)$$

$$n = \frac{e}{1 + e}$$

The void ratio is thus a kind of inverse of the porosity: while porosity is always less than 1, the void ratio can be greater or less than 1.

Saturation Ratio and Moisture Content

The *saturation ratio* is a measure of the proportion of water in the voids. If all voids are water-filled the saturation is 100%; if the voids are dry it is zero:

$$S = \frac{V_w}{V_v} = \frac{V_w}{n} \times 100\%$$

The saturation ratio is closely related to the *moisture content*, which can be measured directly:

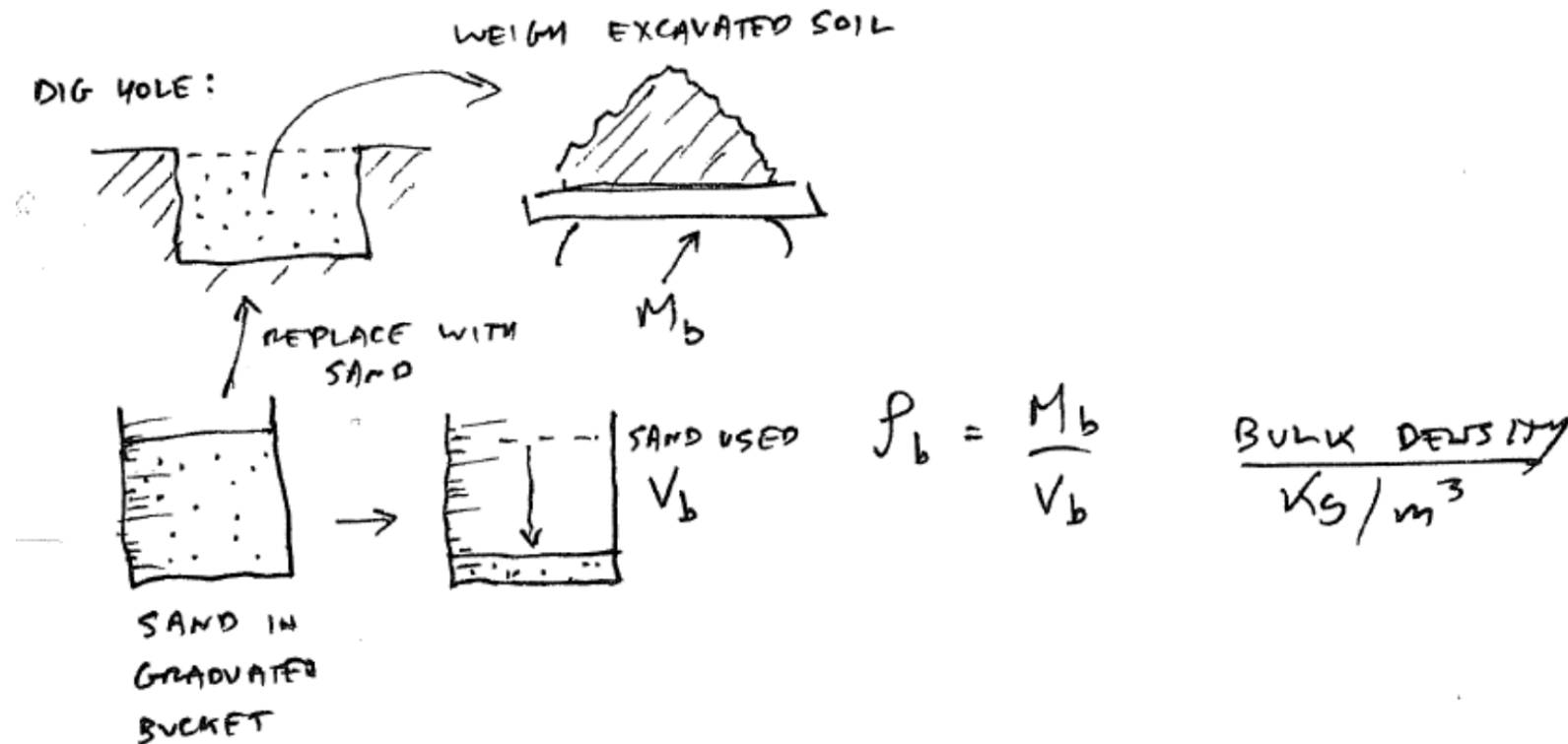
- A bulk sample of soil at field moisture is weighed (M_b = bulk mass), dried and weighed again (M_s = solid mass).
- The mass of water in the soil is inferred from the change in mass of the dried sample: $M_w = M_b - M_s$
- The moisture content (or water content) W is defined as the ratio of the mass of water to the mass of dry soil:

$$W = \frac{M_w}{M_s} = \frac{M_b - M_s}{M_s} \times 100\%$$

Bulk Density

Bulk density is defined as the mass per total volume of a sample of soil, usually considered in the "field" state, ie with insitu moisture and air voids. Bulk density is in fact defined in terms of how it is measured. A useful field method is the sand-replacement method:

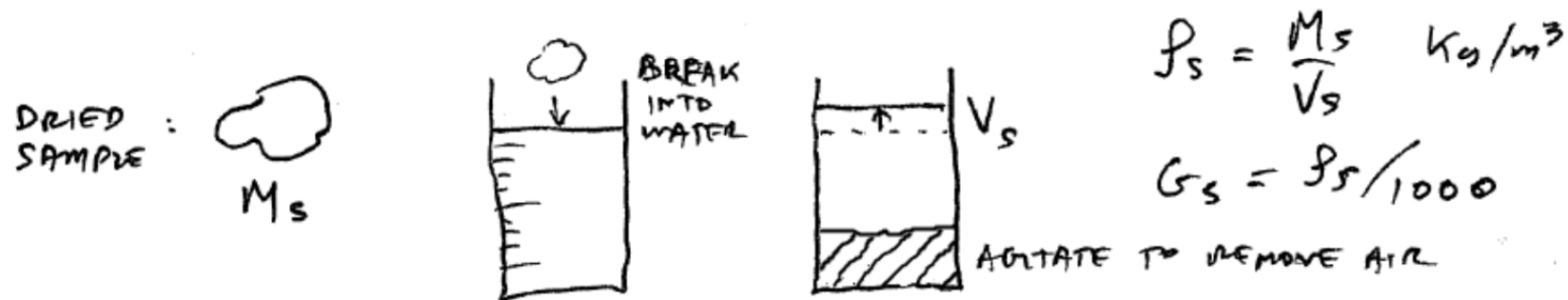
A 2 – 3 kg mass of soil is excavated by hand spade and weighed immediately. The hole is filled with a measured volume of sand, and the bulk density of the soil in-situ is calculated:



Specific Gravity and Particle Density

Specific gravity is defined as the ratio of the density of a continuous solid to that of water. This can be measured directly from a dried soil sample:

~500 g Samples from the field drying test are broken up (lightly ground if necessary to break up smaller aggregates but not coarse particles) and dispersed in a measured volume of water, stirring and agitating to remove all trapped air. The change in volume reflects the volume of soil particles only. Sedimentation does not need to be complete for the full volume (suspended and sedimented) to show. Soil dry mass has already been recorded:



where G_s is specific gravity, the ratio of particle density to water density.

The specific gravity of most soil components are fairly similar. In the absence of experimental data it is often acceptable to assume the SG of soil particles is about 2.7, ie a solid density ρ_s of 2700 kg/m³.

Density and Porosity: Key Relationship #1

Bulk and particle densities, combined with moisture content as measured above, can be used to estimate the porosity of a bulk soil sample:

$$\rho_b = \frac{M_s + M_w}{V_s + V_v} = \frac{M_s}{V_s} \frac{1 + M_w/M_s}{1 + V_v/V_s} = \rho_s \frac{1 + W}{1 + e}$$

This gives the void ratio based on measured values of ρ_b , ρ_s and W :

$$\rho_b = \rho_s \frac{1 + W}{1 + e}$$
$$e = \frac{\rho_s}{\rho_b} (1 + W) - 1$$

$$\rho_b = \rho_s \frac{1 + W}{1 + e}$$

The porosity is derived from the void ratio as before:

$$n = \frac{e}{1 + e}$$

Moisture and Porosity: Key Relationship #2

Measured values of moisture content W , specific gravity G_s (or G) and void ratio e , can be used to estimate the saturation ratio:

$$W = \frac{M_w}{M_s} = \frac{\rho_w V_w}{\rho_s V_s} = \frac{S n}{G(V_t - V_v)} = \frac{S n}{G(1 - n)} = \frac{S e}{G}$$

This gives the saturation ratio based on measured values of G , W and e :

$$S = \frac{GW}{e}$$

This is a useful relationship to remember! In fully saturated soil $S = 100\%$ and the relationship is even more useful.

Dry Density

Dry density is defined as the mass of dry soil per bulk volume. This is a useful measure of strength and is often quoted in preference to bulk density.

The dry density of a soil that is completely dry in the field (rare) could be measured directly as per the sand replacement method.

In practice, field samples in the form of clods or peds are dried and weighed, but their individual volume may not reflect the true volume of a dry soil mass, since the insitu condition of dry soil is often filled with drying shrinkage cracks.

Dry density can be estimated from the previously derived relationships, but again this may not reflect drying shrinkage.

$$\rho_b = \rho_s \frac{1 + W}{1 + e}$$

$$\rho_d = \rho_s \frac{1 + 0}{1 + e} = \frac{\rho_b}{1 + W} = \rho_s \frac{n}{e}$$

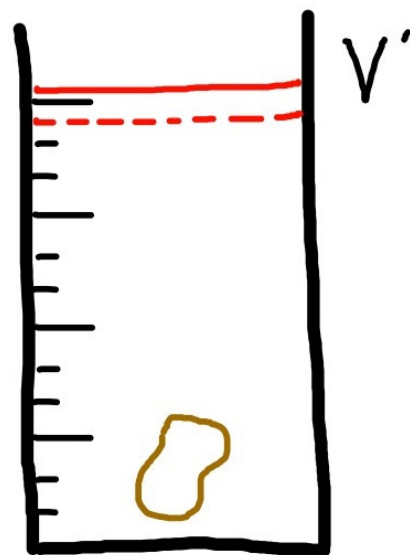
ρ_d is the estimated dry density assuming the void ratio is unchanged as moisture content reduces to zero

Dry Density of Clods/Peds

The dry density of individual dried clods can be measured by a volume immersion method.

Dried soil is unstable in water: clods quickly *slake* or break down in water, so their bulk volume is difficult to measure.

Soil does not react in the same way with oil, however. Immersion of a soil sample in oil provides a good measure of its bulk volume. Knowing the mass of a dry soil sample we can measure its dry density:



$$\rho_d = \frac{M_s}{V' - V_0}$$

Saturated Density

Saturated density is defined as the mass of saturated soil per bulk volume. Soil below the water table and in cases of excess surface runoff is saturated;

Saturated soil is partly supported by buoyancy and as such produces a lower bearing pressure compared to the same thickness of dry soil.

Despite this the actual density of saturated soil is higher than that of dry soil. This is one of the seeming paradoxes of buoyancy.

Again assuming that the wetting of soil produces no change in porosity, the saturated density can be calculated from:

$$\rho_{sat} = \rho_s \frac{1 + W_{sat}}{1 + e}$$

$$\text{but } W_{sat} = 100\% \times \frac{e}{G}$$

$$S = \frac{GW}{e}$$

$$\text{so } \rho_{sat} = \rho_s \frac{1 + e/G}{1 + e} = \frac{\rho_s}{G} \frac{G + e}{1 + e} = \rho_w \frac{G + e}{1 + e} = 1000 \frac{G + e}{1 + e}$$

Unit Weight

A *unit weight* is a type of density expressed in weight rather than mass units for calculations of force and pressure. For any of the densities defined so far, there is a corresponding unit weight, which is just the density multiplied by $g = 9.8$ or near enough to 10;

Dry unit weight $\gamma_d = g\rho_d$

Bulk unit weight $\gamma_b = g\rho_b$

Saturated unit weight $\gamma_s = g\rho_s$

Unit weight of water $\gamma_w = 9.8 \text{ kN/m}^3$

Unit weights are more commonly used than densities in engineering calculations because of the need to express loads in terms of force, not mass. Don't forget to multiply by 10! A selection of alternative calculations is provided in Table 3.1:

Unit Weight; Alternative Equations

Table 3.1 Various Forms of Relationships for γ , γ_d , and γ_{sat}

Moist unit weight (γ)		Dry unit weight (γ_d)		Saturated unit weight (γ_{sat})	
Given	Relationship	Given	Relationship	Given	Relationship
w, G_s, e	$\frac{(1 + w)G_s\gamma_w}{1 + e}$	γ, w	$\frac{\gamma}{1 + w}$	G_s, e	$\frac{(G_s + e)\gamma_w}{1 + e}$
S, G_s, e	$\frac{(G_s + Se)\gamma_w}{1 + e}$	G_s, e	$\frac{G_s\gamma_w}{1 + e}$	G_s, n	$[(1 - n)G_s + n]\gamma_w$
w, G_s, S	$\frac{(1 + w)G_s\gamma_w}{1 + \frac{wG_s}{S}}$	G_s, n	$G_s\gamma_w(1 - n)$	G_s, w_{sat}	$\left(\frac{1 + w_{sat}}{1 + w_{sat}G_s}\right)G_s\gamma_w$
w, G_s, n	$G_s\gamma_w(1 - n)(1 + w)$	G_s, w, S	$\frac{G_s\gamma_w}{1 + \left(\frac{wG_s}{S}\right)}$	e, w_{sat}	$\left(\frac{e}{w_{sat}}\right)\left(\frac{1 + w_{sat}}{1 + e}\right)\gamma_w$
S, G_s, n	$G_s\gamma_w(1 - n) + nS\gamma_w$	e, w, S	$\frac{eS\gamma_w}{(1 + e)w}$	n, w_{sat}	$n\left(\frac{1 + w_{sat}}{w_{sat}}\right)\gamma_w$
		γ_{sat}, e	$\gamma_{sat} - \frac{e\gamma_w}{1 + e}$	γ_d, e	$\gamma_d + \left(\frac{e}{1 + e}\right)\gamma_w$
		γ_{sat}, n	$\gamma_{sat} - n\gamma_w$	γ_d, n	$\gamma_d + n\gamma_w$
		γ_{sat}, G_s	$\frac{(\gamma_{sat} - \gamma_w)G_s}{(G_s - 1)}$	γ_d, S	$\left(1 - \frac{1}{G_s}\right)\gamma_d + \gamma_w$
				γ_d, w_{sat}	$\gamma_d(1 + w_{sat})$

Soil mechanics is over-full of relationships between density, porosity and moisture! In any text you will find large numbers of different equations that calculate the same things using different inputs.

As practicing geotechnical engineers you would need to be able to derive at least one set of these equations from scratch.

Typical values

Table 3.2 Void Ratio, Moisture Content, and Dry Unit Weight for Some Typical Soils in a Natural State

Type of soil	Void ratio, e	Natural moisture content in a saturated state (%)	Dry unit weight, γ_d (kN/m ³)
Loose uniform sand	0.8	30	14.5
Dense uniform sand	0.45	16	18
Loose angular-grained silty sand	0.65	25	16
Dense angular-grained silty sand	0.4	15	19
Stiff clay	0.6	21	17
Soft clay	0.9–1.4	30–50	11.5–14.5
Loess	0.9	25	13.5
Soft organic clay	2.5–3.2	90–120	6–8
Glacial till	0.3	10	21

These values are indicative only: the actual range of porosity and moisture is naturally very large. Dry densities depend on the condition of the soil, its state of consolidation and whether any compaction work has been done on the soil layers.

Brief Summary

Porosity

$$n = V_v = V_a + V_w$$

Void Ratio

$$e = \frac{V_v}{V_s} = \frac{V_v}{V_t - V_v} = \frac{n}{1 - n} \quad n = \frac{e}{1 + e}$$

Saturation

$$S = \frac{V_w}{V_v} = \frac{V_w}{n} \times 100\%$$

Moisture content

$$W = \frac{M_w}{M_s} = \frac{M_b - M_s}{M_s} \times 100\% \quad S = \frac{GW}{e}$$

Bulk density

$$\rho_b = \rho_s \frac{1 + W}{1 + e}$$

$$\gamma = g\rho \quad \text{Unit weight}$$

Dry density

$$\rho_d = \rho_s \frac{1 + 0}{1 + e} = \frac{\rho_b}{1 + W} = \rho_s \frac{n}{e}$$

Saturated density

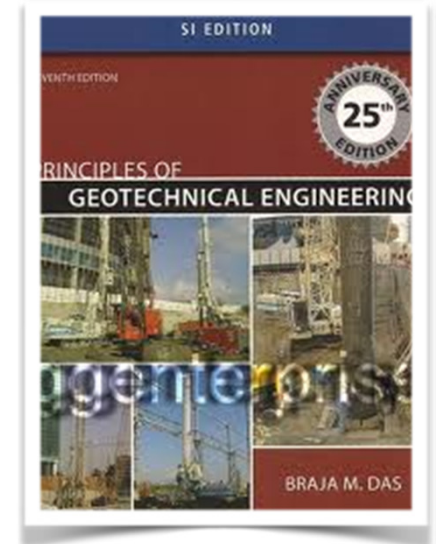
$$\rho_{sat} = \rho_s \frac{1 + e/G}{1 + e} = \frac{\rho_s}{G} \frac{G + e}{1 + e} = \rho_w \frac{G + e}{1 + e} = 1000 \frac{G + e}{1 + e}$$

Example 3.2 p. 60, Das

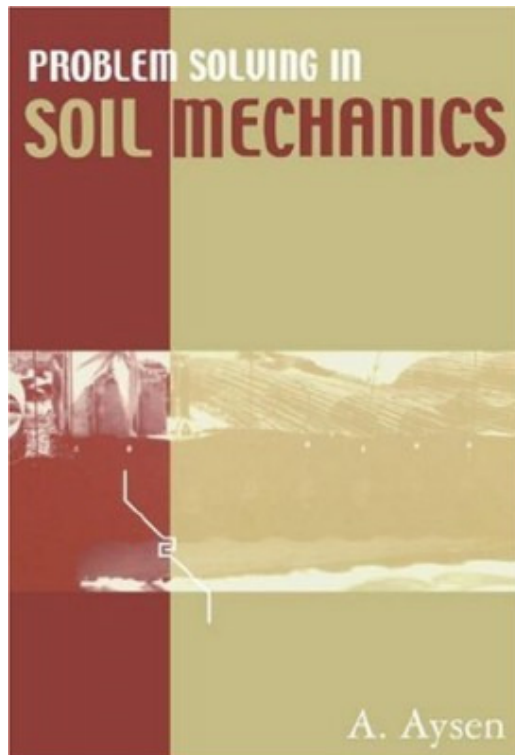
Work through this example in the textbook to ensure that you understand the concepts and can apply them.

Readings

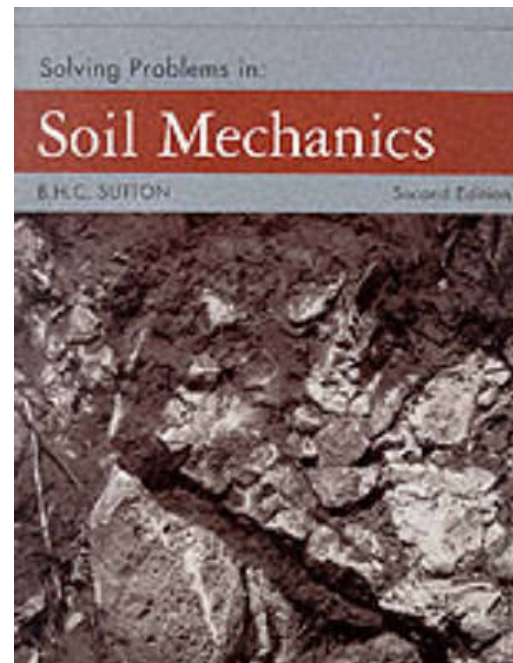
Das, Chapter 3.



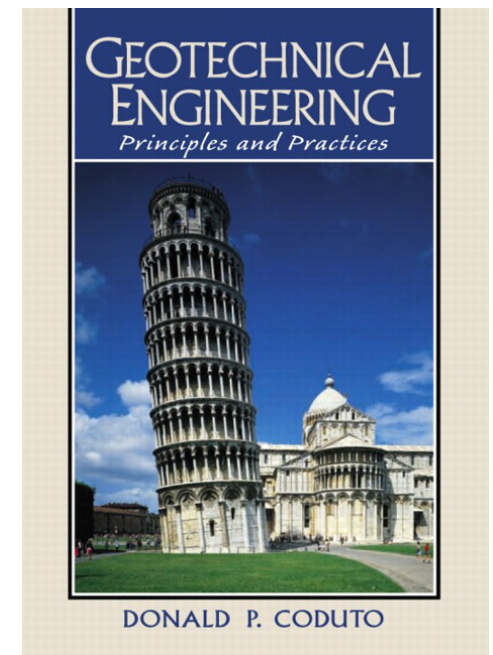
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