

Slope Stability

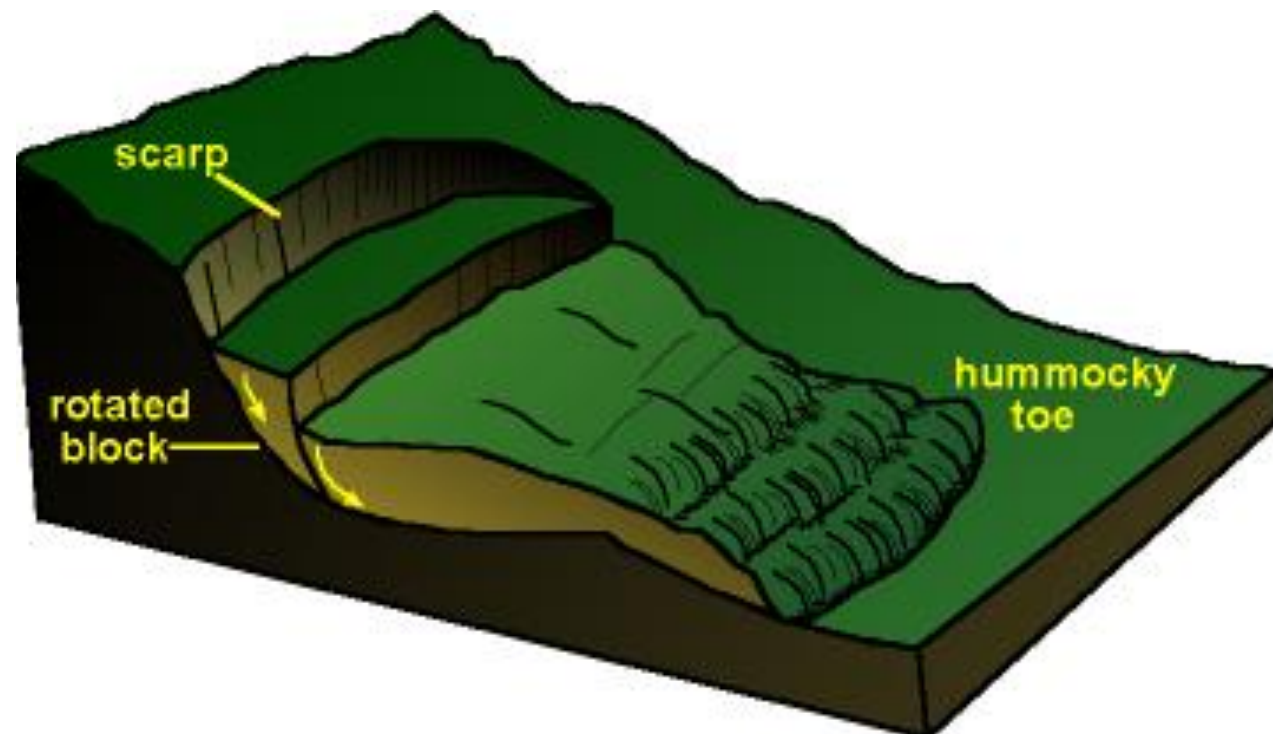
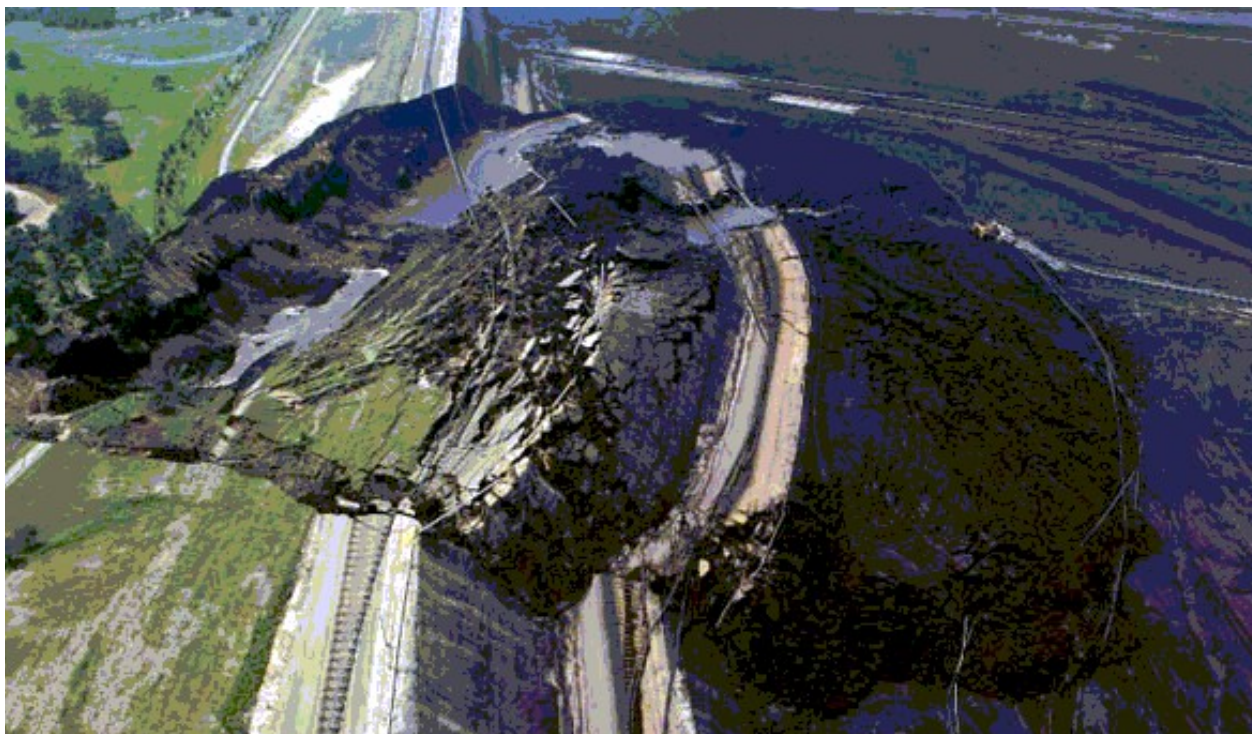
Text: Das, 2010. Chapter 15.

We have considered slope stability to a limited extent in developing the principles of shear strength and failure; namely the angle of repose of a cohesive or non-cohesive soil.

That analysis is limited in that the derivation assumes an infinite slope, which is a poor approximation even for large natural soil masses, and very unrealistic for engineered slopes and embankments. A slope's stability depends on details of the landscape above and below the slope and how the soil mass can move when overstressed.

Slope stability theory is a largely empirical science that uses some theoretical derivations but relies mostly on empirical data summaries and numerical methods which since the 1960s have been dominated by computer analysis.

The approach taken here is to discuss the main principles in general terms while providing detailed stepwise methods, highlighting the end result equations and graphical data summaries that computational methods rely on. More complete derivations are provided in the text.



Slope Stability: Modes of Failure

An exposed ground surface that stands at an angle to the horizontal is known as an *unrestrained slope*.

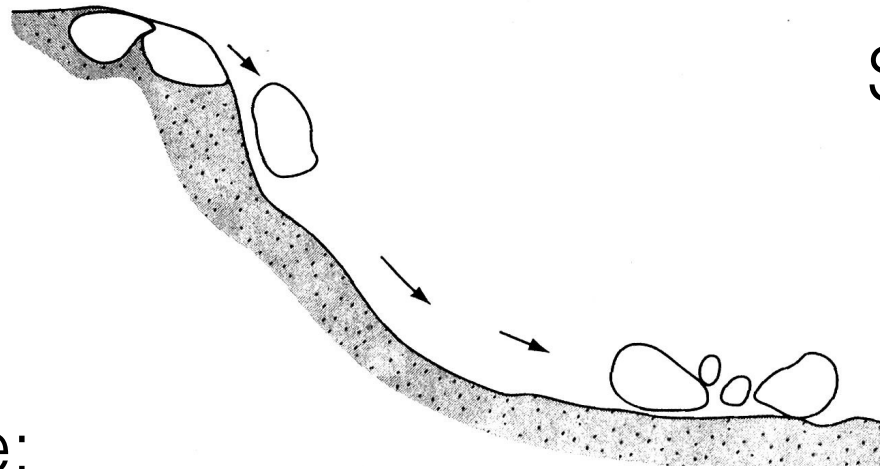
The slope can be natural or man-made. It can fail in various modes:

- Fall: This is the detachment of soil and/or rock fragments that fall down a slope.
- Topple: This is a forward rotation of the soil and/or rock mass about an axis below the centre of gravity
- Slide: This is the downward movement of a soil mass occurring on a surface of rupture which may be a plane or curved surface.
- Spread: This is a form of slide in which the soil separates into "hummocky" blocks
- Flow: This is a downward movement of soil mass similar to a viscous fluid

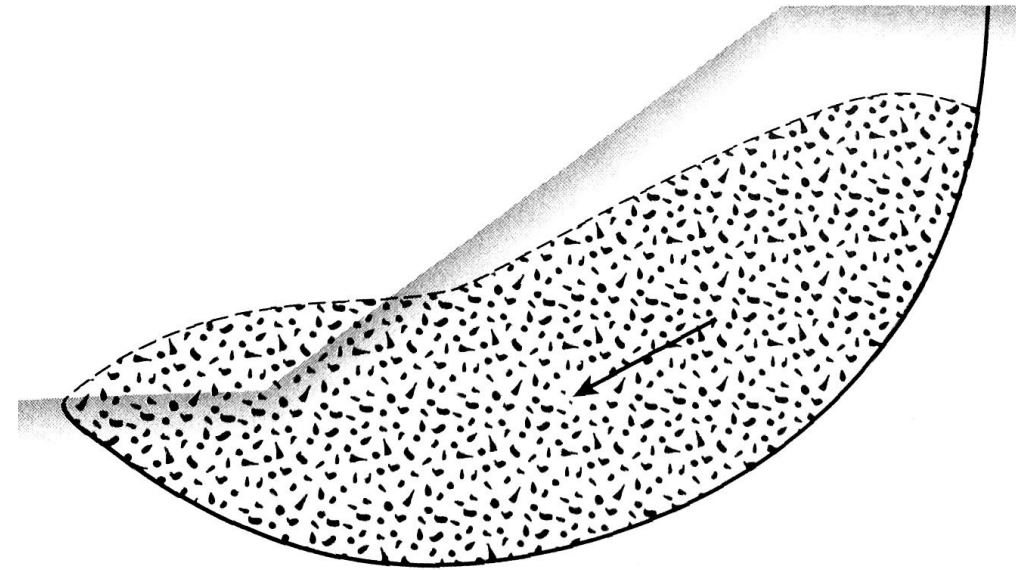
Fall and flow failures are collectively known as *translational* failures, while topple and spread failures are *rotational*. Slide failures may be translational or rotational.

Slope Stability: Modes of Failure

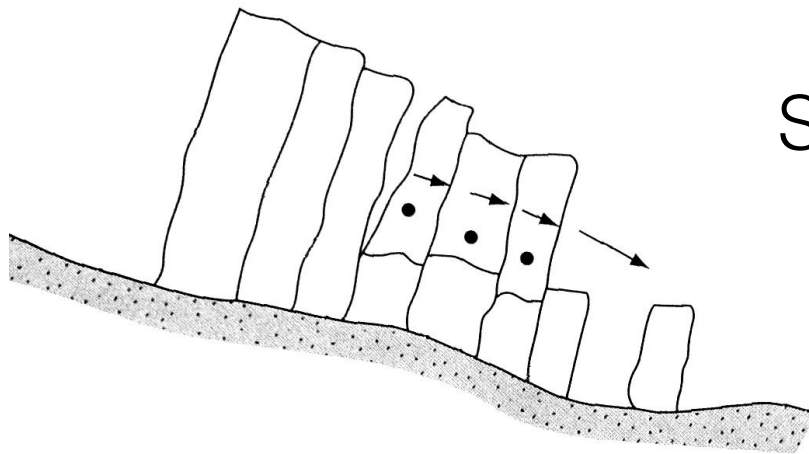
Fall:



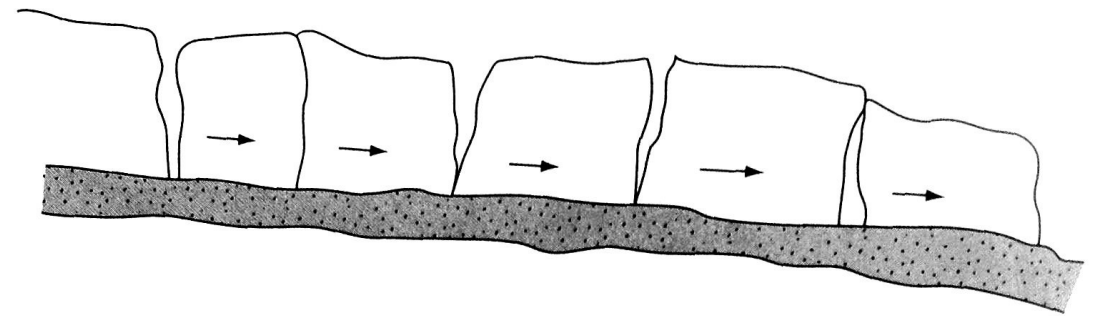
Slide:



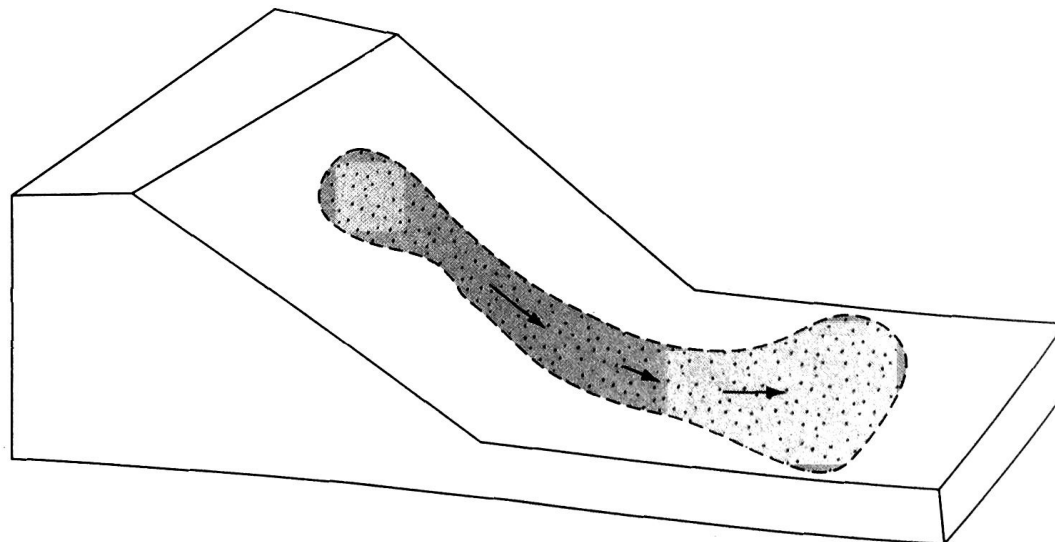
Topple:



Spread:



Flow:



Engineering methods emphasise the slide failure, which tends to precede spread and flow failures. Slide failure involves the entire volume of a slope rather than just its surface and is thus more serious than rockfall failure.

Slope Stability: Factors of Safety

Slope stability is usually defined in terms of a factor of safety with regard to strength, F_s .

F_s is defined as a ratio of the shear strength of a soil to the developed (or actual) shear stress:

$$F_s = \frac{\tau_f}{\tau_d}$$

F_s : factor of safety with regard to strength

τ_f : average shear strength

τ_d : average shear stress developed on a potential failure surface

The shear strength can be defined in terms of cohesive and frictional components:

$$\tau_f = c' + \sigma' \tan(\phi')$$

Likewise the developed shear stress can be defined in terms of cohesive and frictional components:

$$\tau_d = c'_d + \sigma' \tan(\phi'_d)$$

$$F_s = \frac{c' + \sigma' \tan(\phi')}{c'_d + \sigma' \tan(\phi'_d)}$$

Slope Stability: Factors of Safety

For simplicity the cohesive and frictional components of shear strength are assumed to fail simultaneously.

F_s is then the ratio of either the cohesive or frictional strengths and stresses:

$$F_s = \frac{c'}{c'_d} = \frac{\tan(\phi')}{\tan(\phi'_d)}$$

When F_s is equal to 1, the soil mass is in a state of impending failure.

Generally a value of $F_s > 1.5$ is acceptable for design of a stable slope.

(The components c'_d and ϕ'_d are variables rather than fixed soil values; they simplify the analysis of maximum stress on given trial surfaces and indicate the actual stress state)

Slope Stability: Critical Height

A factor of safety of 1 is often associated with a *Critical Height* H_{cr} , the maximum stable height possible for a given slope and failure envelope.

The critical height is a function of surface slope, strength parameters, saturation, seepage and the state of drained or undrained compression.

This very complex phenomena can nevertheless be demonstrated in principle by the simpler Mohr-Coulomb angle of repose analysis already presented:

$$\sin(\beta) - \cos(\beta) \tan(\phi') = \frac{c'}{\sigma'_o}$$

β : general surface slope angle: failure occurs at a critical depth of soil

σ'_o : failure stress on the horizontal plane at the critical depth

σ'_o is a function of the critical depth and surface slope:

$$\sigma'_o = \gamma H_{cr} \cos(\beta)$$

For slopes $\beta > \phi'$ there is a critical soil depth H_{cr} at which the slope will fail.

Slopes $\beta < \phi'$ are stable regardless of soil depth.

$$\text{or } H_{cr} = \frac{c'}{\gamma \cos^2(\beta) (\tan(\beta) - \tan(\phi'))}$$

Although the infinite slope is unrealistic, this general principle holds: any cohesive soil slope is stable up to a maximum critical height of embankment

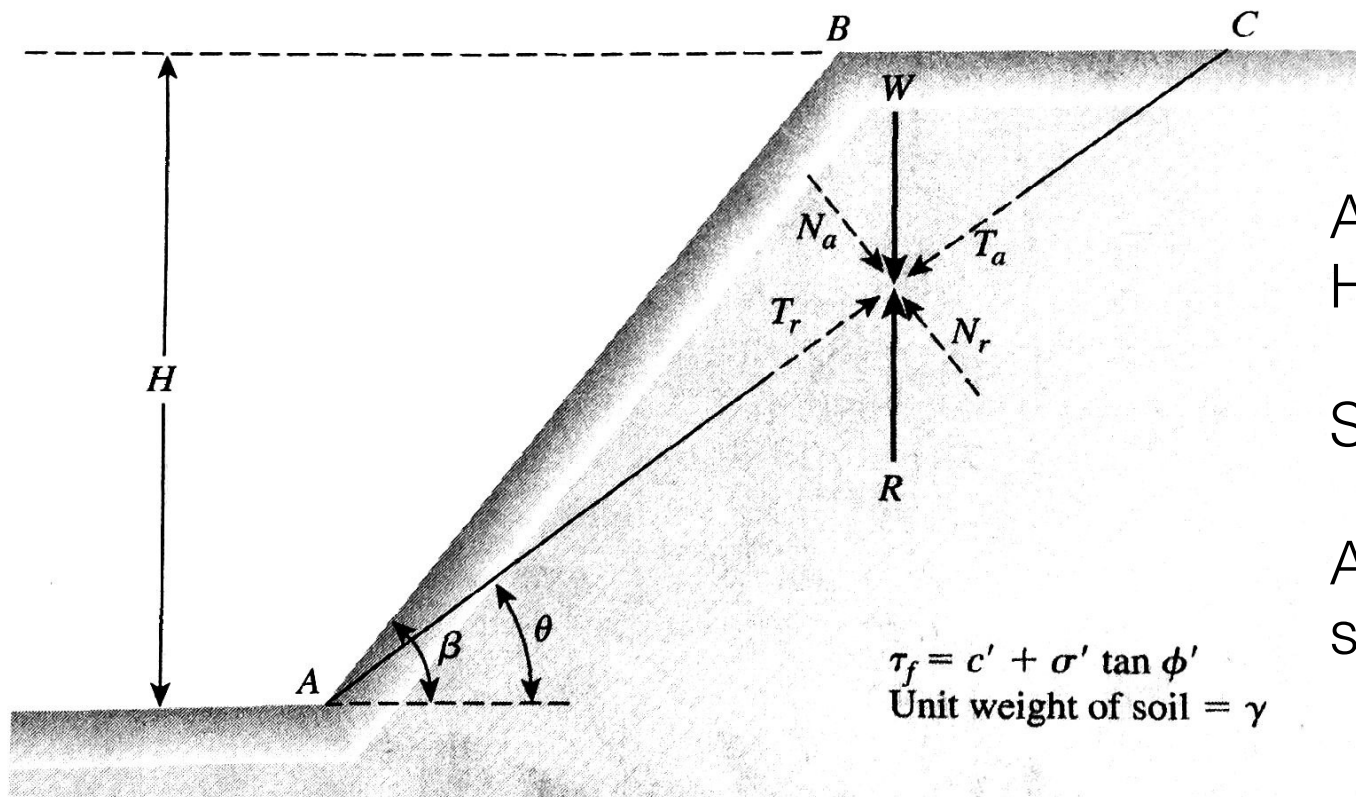
Critical Height: Finite Slopes

Finite slopes can be analysed by Culmann's method, assuming a plane shear failure surface as per the retaining wall method.

Culmann's method does not agree well with empirical data, except where the slope is near vertical or where planar strata are a pre-existing weakness in the soil.

The result is more accurate than infinite slope analysis however and provides a useful comparison as the next approximation:

$$H_{cr} = \frac{4c'}{\gamma} \left[\frac{\sin(\beta) \cos(\phi')}{1 - \cos(\beta - \phi')} \right]$$



Again, for slopes $\beta > \phi'$ there is a critical height H_{cr} at which the slope will fail.

Slopes $\beta < \phi'$ are stable regardless of lift height.

A more accurate analysis uses *curved* failure surfaces, usually segments of circular arcs.

Finite Slopes with Curved Failure Surfaces

Curved shear failure surfaces, known as *slip circles*, use the same principles in Culmann's method: balancing weight forces against shear resistance for a series of trial failure curves.

A slope may fail at the toe, or within the face of the slope, or at some point across the base of the slope:

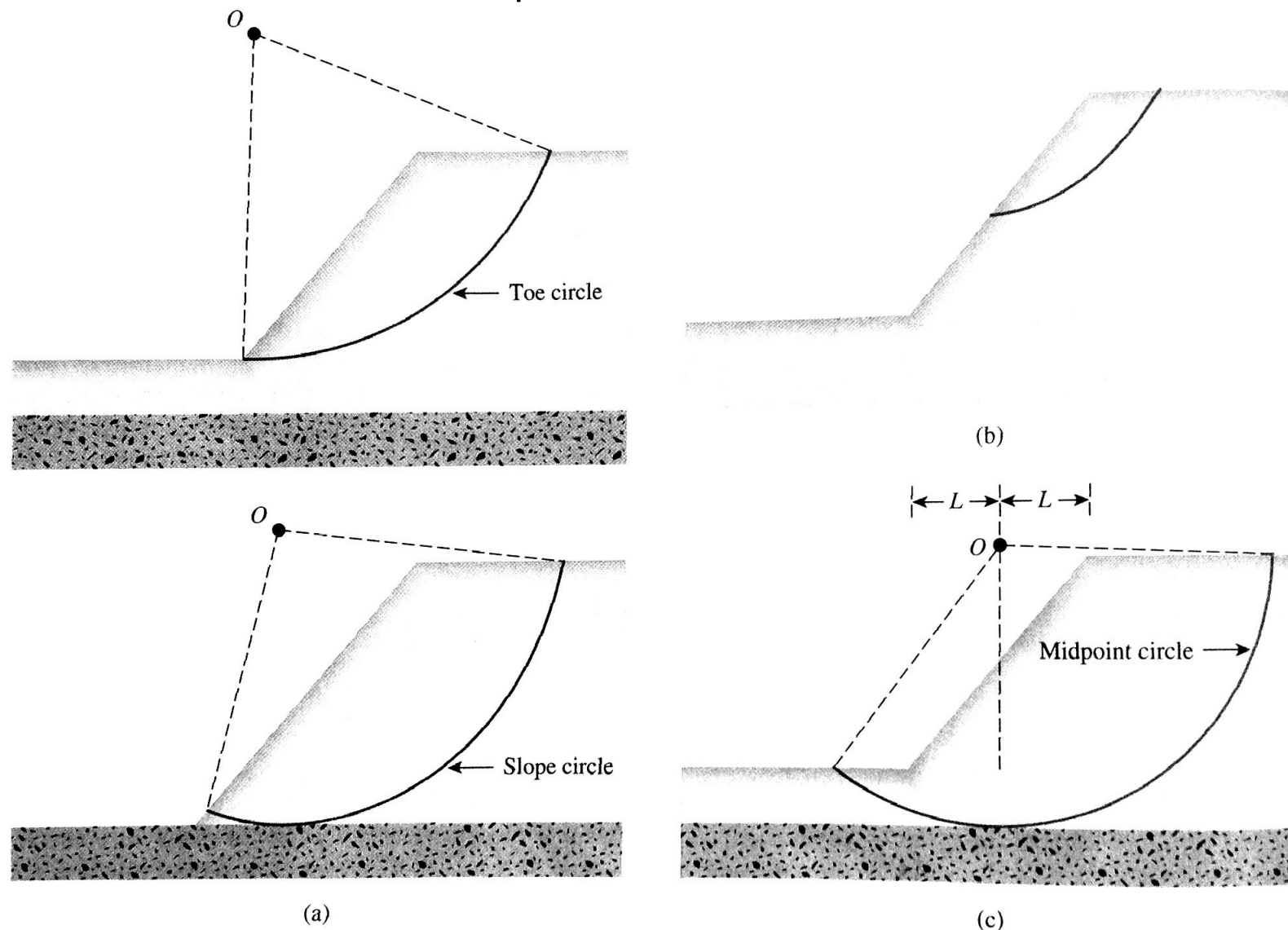


Figure 15.11 Modes of failure of finite slope: (a) slope failure; (b) shallow slope failure; (c) base failure

Finite Slopes with Curved Failure Surfaces

A number of different slip circle procedures are available; these can be divided into two main classes:

1. Mass Procedure: the mass of soil is treated as a single unit. This is useful when the soil can be assumed to be homogenous.
2. Method of Slices: the soil in the slip circle is divided into a series of vertical slices, the stability of each slice is evaluated separately and the results added together to give the factor of safety.

The method of slices is more accurate than the mass procedure in most natural soils.

Both classes of method are iterative, requiring solution for a range of trial slip circles and the worst case identified. Both methods are typically implemented as computer programs; they are rarely done by hand.

A number of different methods are used for different states of consolidation drainage in the soil, since this gives different forms of effective stress.

Mass Procedures: Drained and Undrained

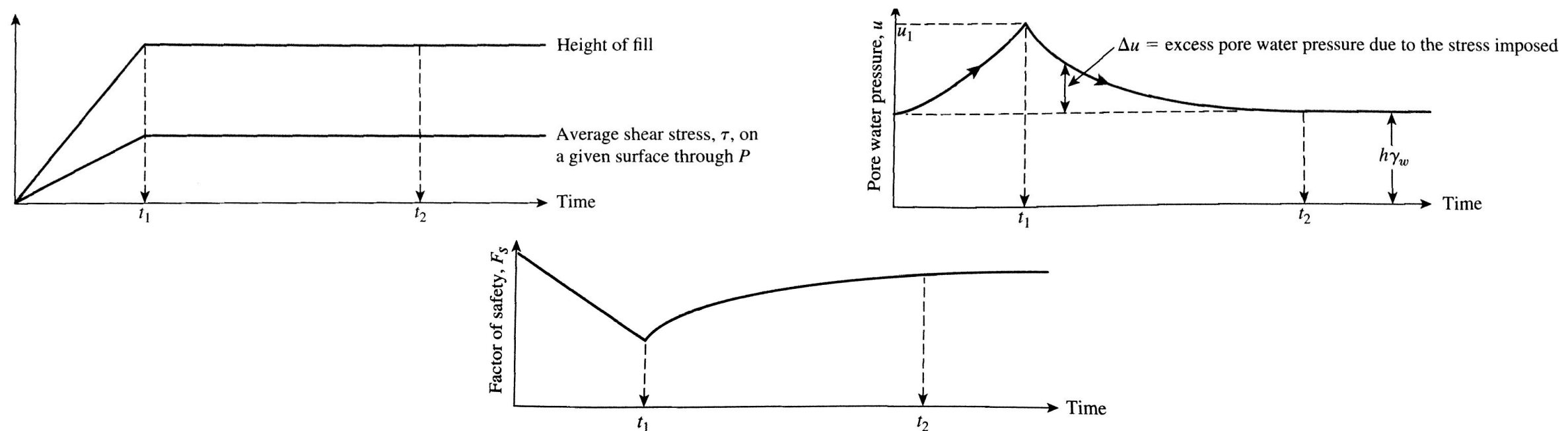
A constructed embankment consolidates slowly from undrained to drained, and therefore requires both forms of analysis for the short- and long-term.

The undrained condition uses total stress and the empirical result (from shear stress testing) that the total friction slope $\phi = 0$: the total shear strength is a constant c_u .

In the drained condition, possibly many years later, the shear strength is modelled by the Mohr-Coulomb $c' - \phi'$ failure envelope. $c' - \phi'$ soil tends to be stronger than $\phi = 0$ soil.

The factor of safety changes during construction and as a structure ages: a constructed embankment becomes gradually less safe as the height increases. The factor of safety is at a minimum as construction is completed.

As the embankment ages and pore pressure dissipates, effective stress develops in the soil so that more of the load is carried by consolidated soil particles; the factor of safety gradually increases.



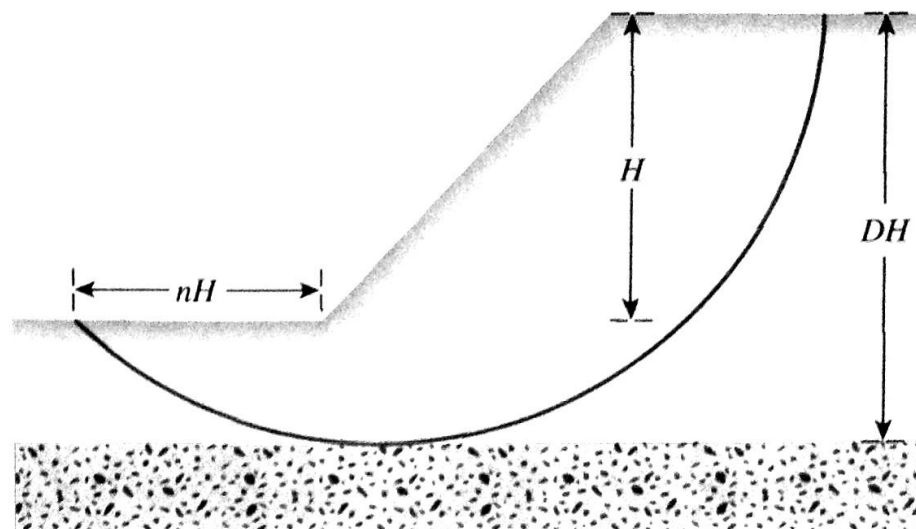
Mass Procedure: Undrained $\phi=0$ soil

Mass procedures use empirically measured *stability numbers* that represent the true value of the relationship between slope, friction and critical height.

In the preceding infinite and finite slope analyses the critical height is a function of the cohesion, unit weight and a stability number based on slope and friction angles:

$$H_{cr} = \frac{c'}{\gamma m} = \frac{c_u}{\gamma m}$$

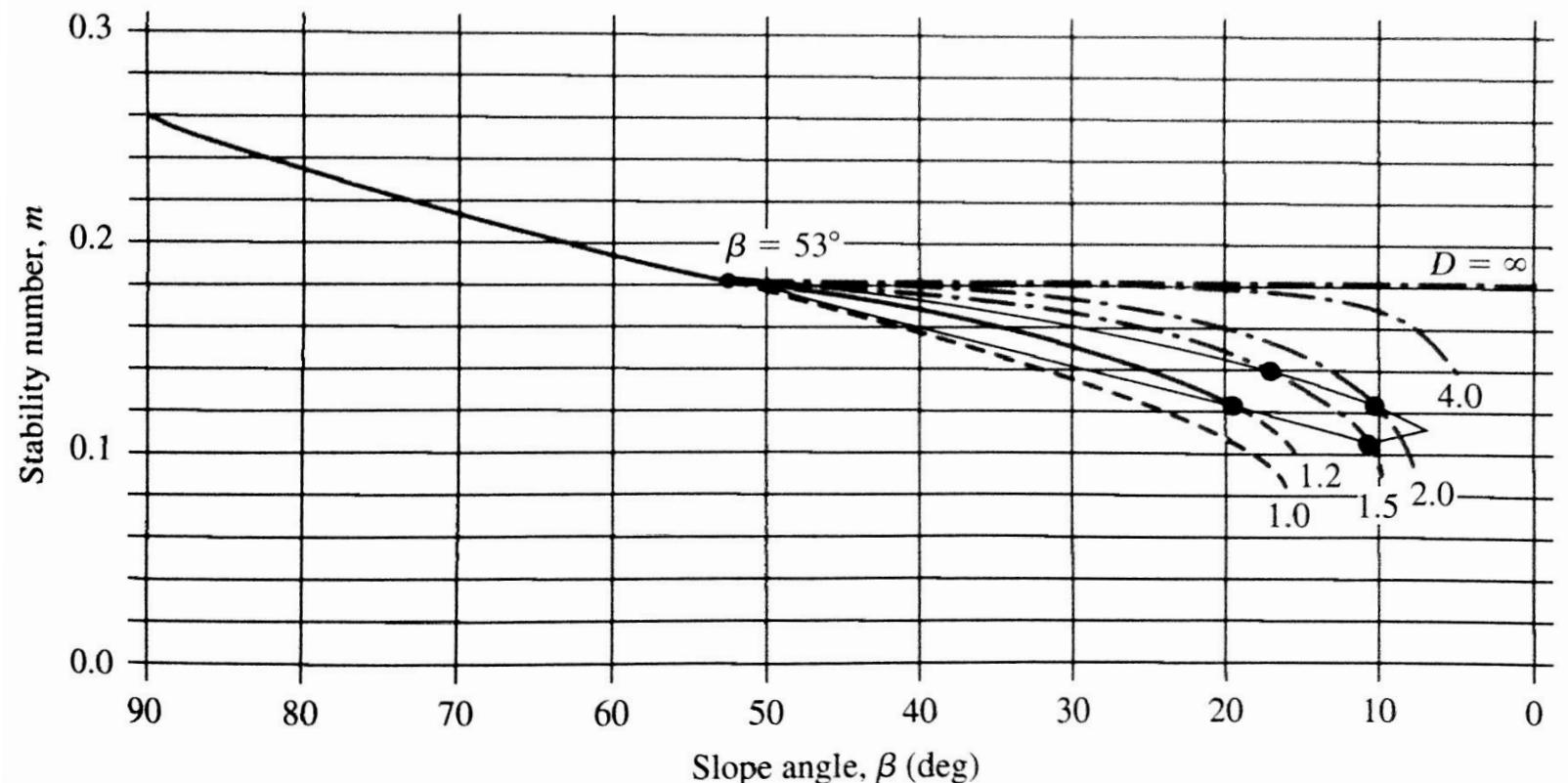
where m is the stability number. For $\phi=0$ analyses, the stability number has been plotted empirically and can be read directly for the critical slip circle.



For $\beta > 53^\circ$:
All circles are toe circles.

For $\beta < 53^\circ$:

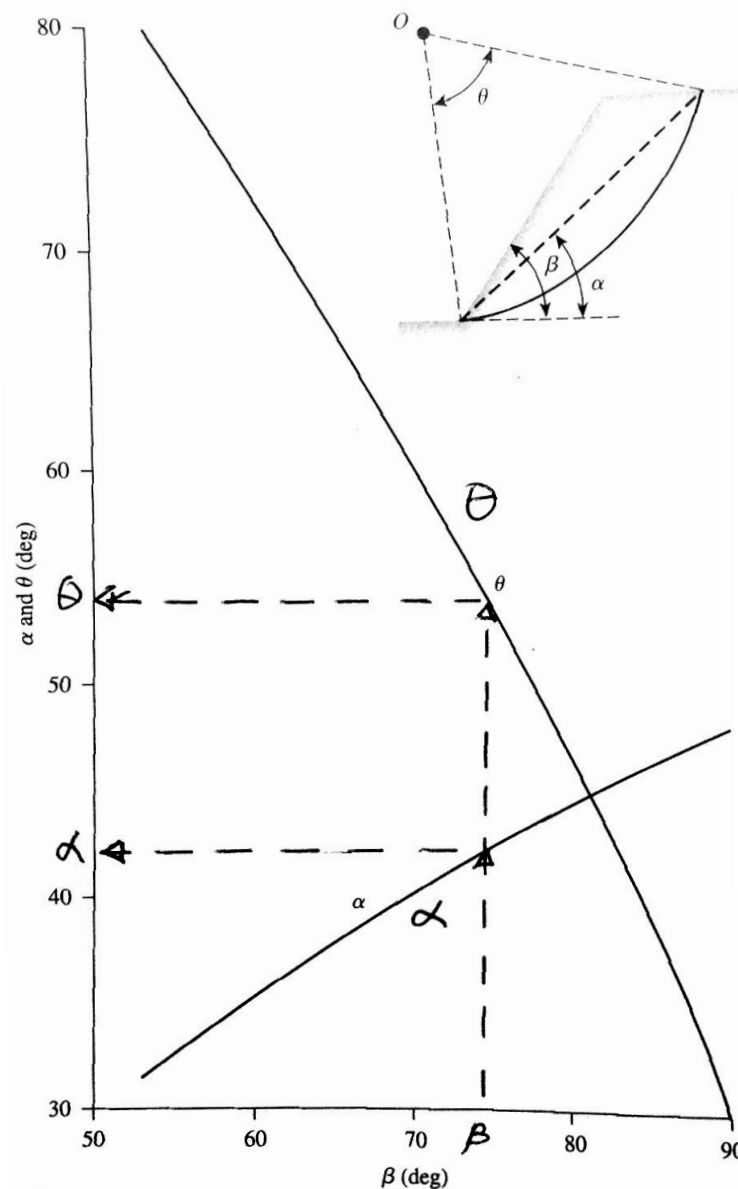
- Toe circle ———
- Midpoint circle - - - -
- Slope circle - - - -



Mass Procedure: Undrained $\phi=0$ soil

In addition to critical height, the $\phi = 0$ mass procedure provides an estimate of the location of the failure surface.

- For a slope angle $\beta > 53$ degrees, the critical circle is always a toe circle. The location of the centre of the circle is found by the angles α and θ :



- For a slope angle $\beta < 53$ degrees, the critical circle may be a toe, slope or midpoint circle:

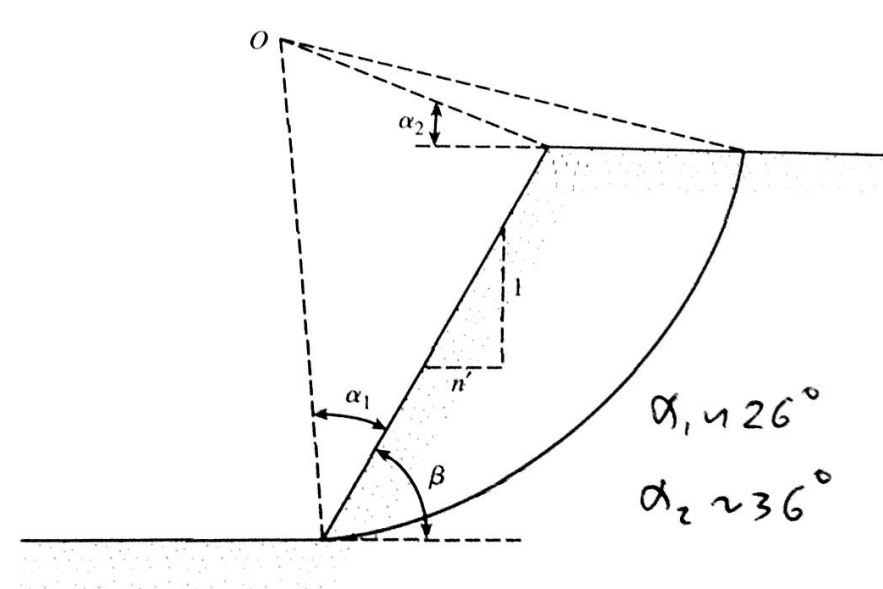
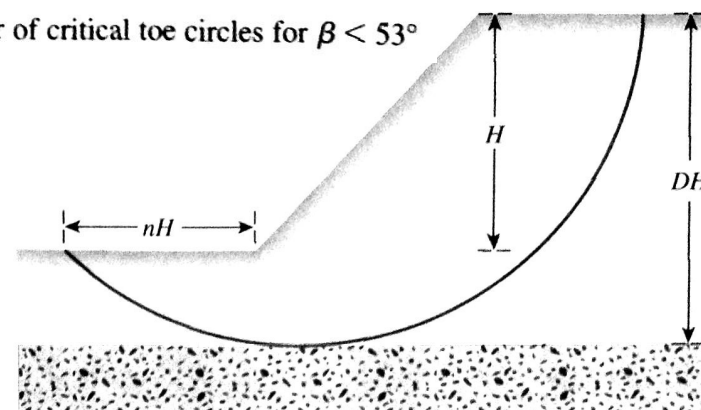
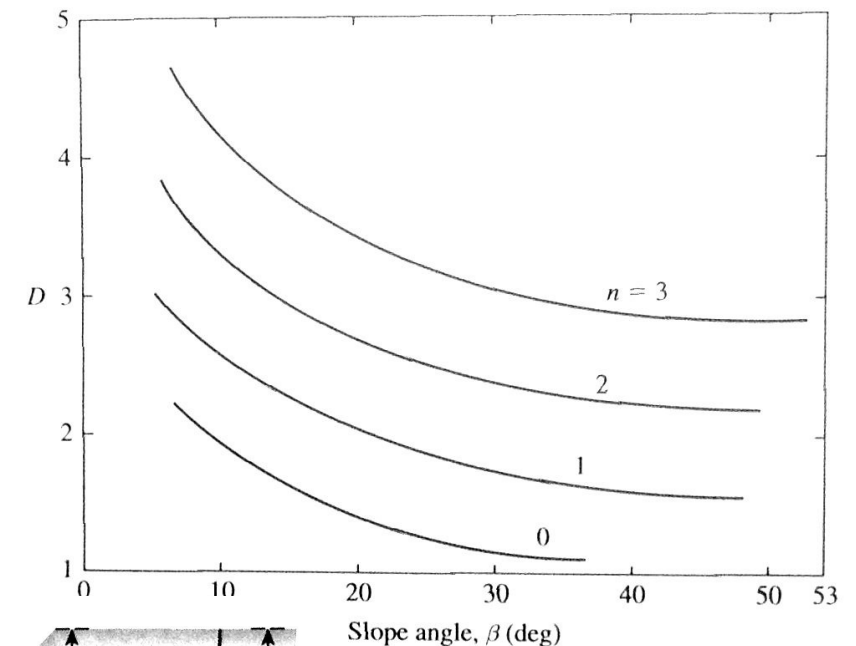


Figure 15.16 Location of the center of critical toe circles for $\beta < 53^\circ$



Mass Procedure: Undrained $\phi=0$ soil

Once the slip circle is located it can be plotted and measured either numerically or by drafting techniques for the factor of safety.

- The usual technique is to measure the areas and centroids of the slip circle, to find moments and resisting moments that are balanced as a factor of safety:

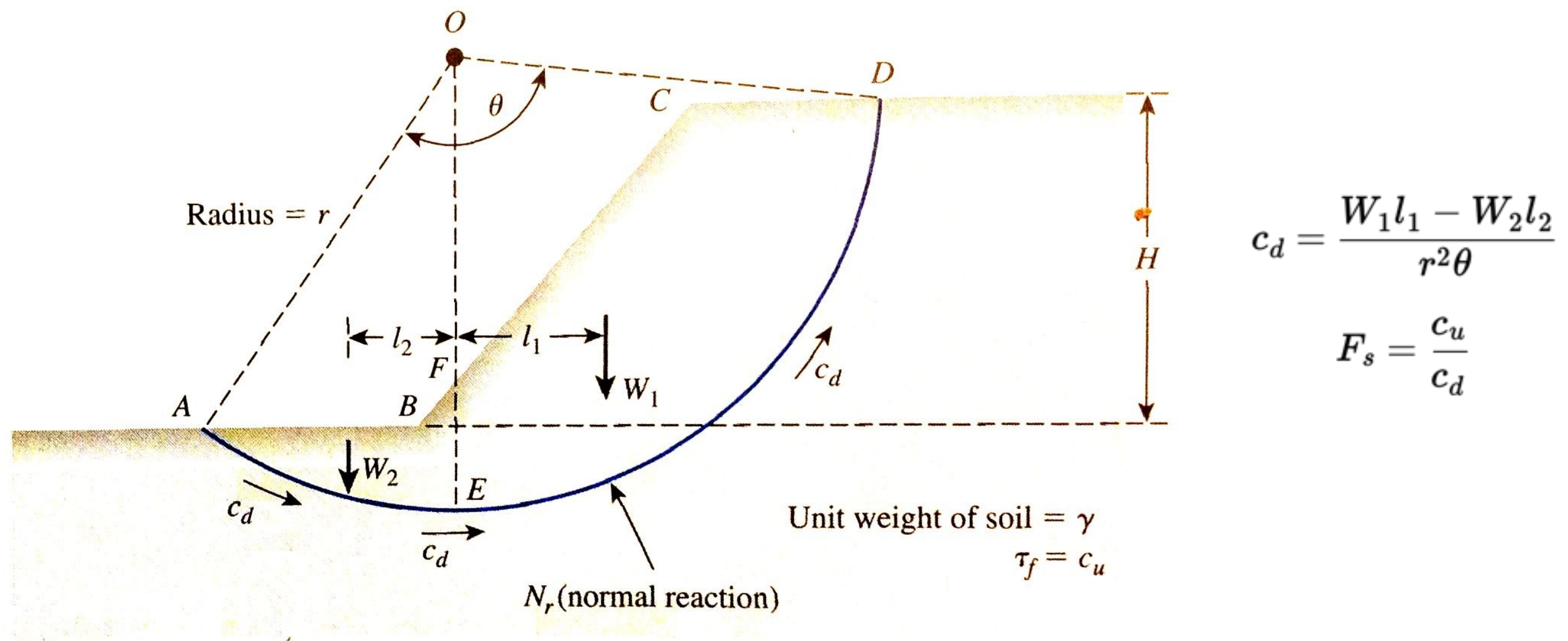


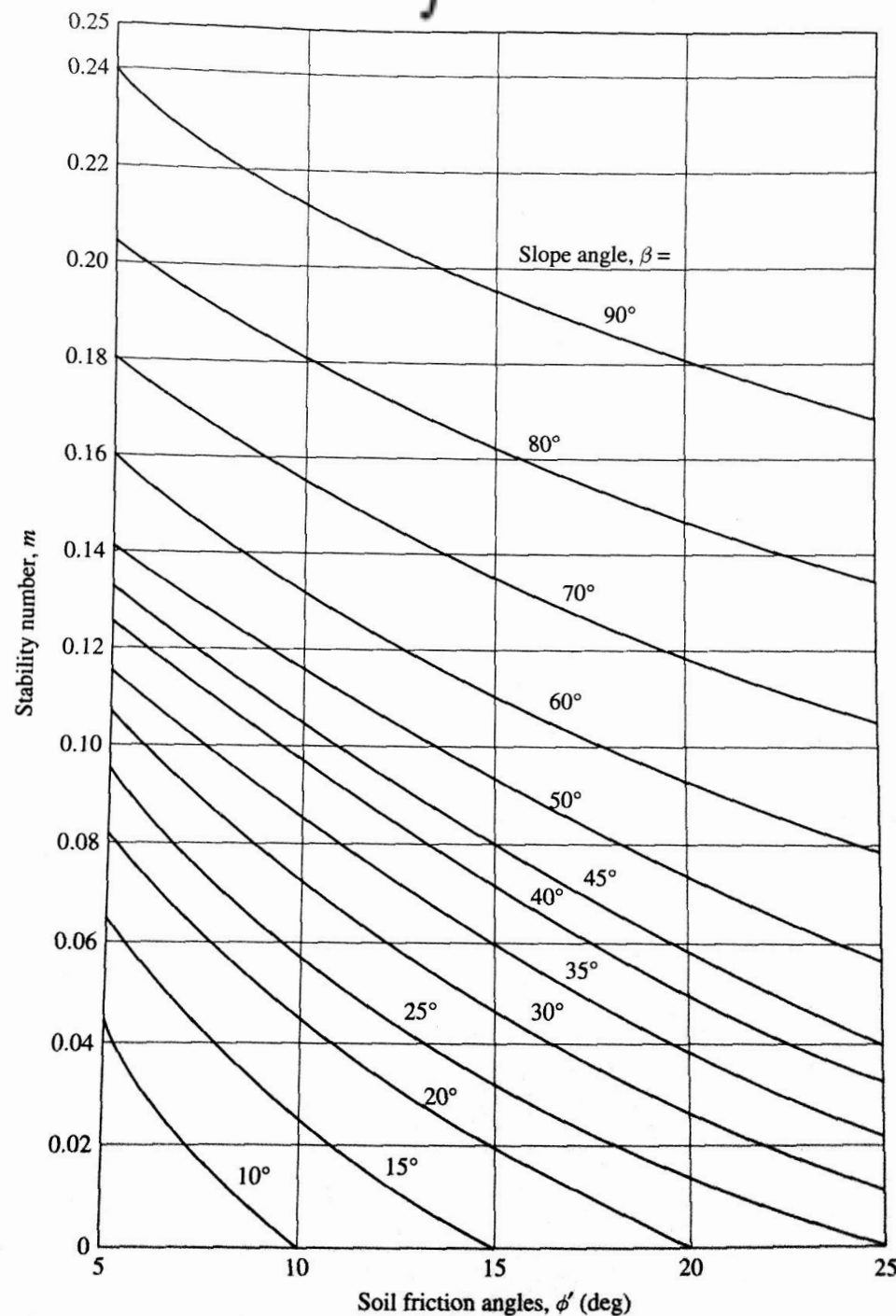
Figure 15.12 Stability analysis of slope in homogeneous saturated clay soil ($\phi = 0$)

Mass Procedure: Drained c' - ϕ' soil

As pore pressure excess falls to zero, the effective stress failure envelope returns. Critical height is again determined by the stability number m :

$$\tau_f = c' + \sigma' \tan(\phi')$$

$$H_{cr} = \frac{c'}{\gamma m}$$



The factor of safety is found by a plotting technique:

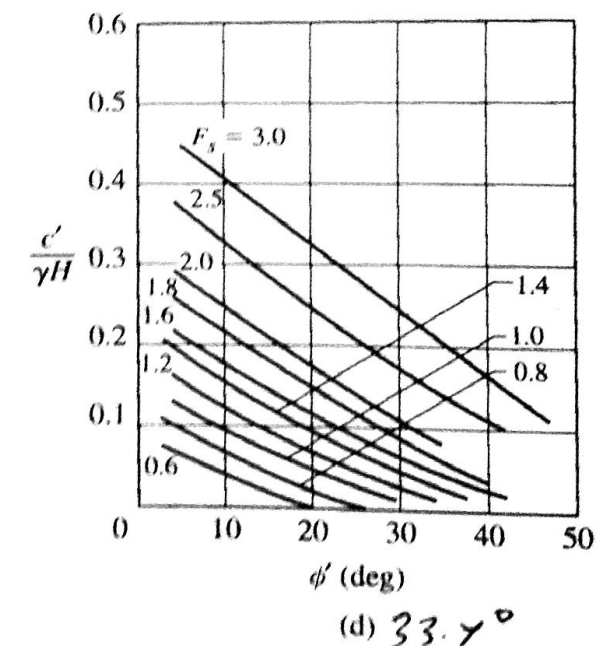
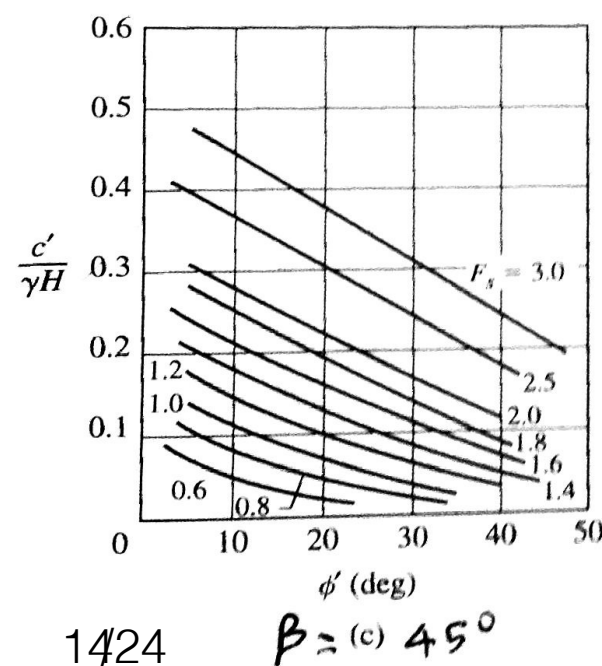
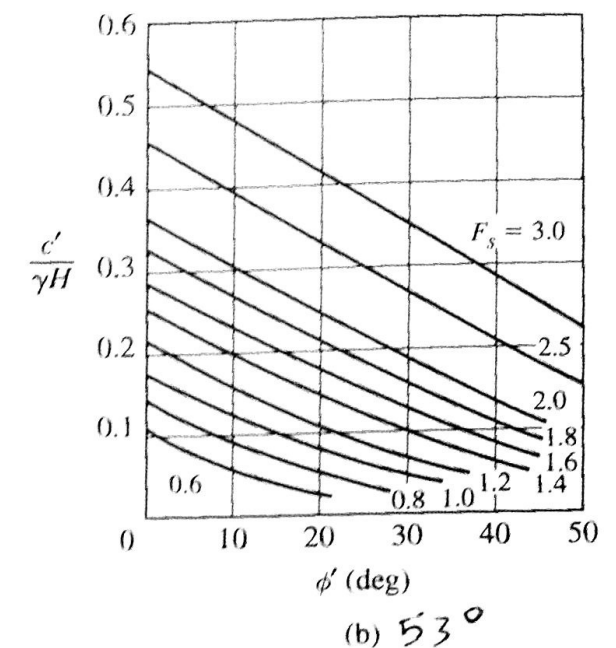
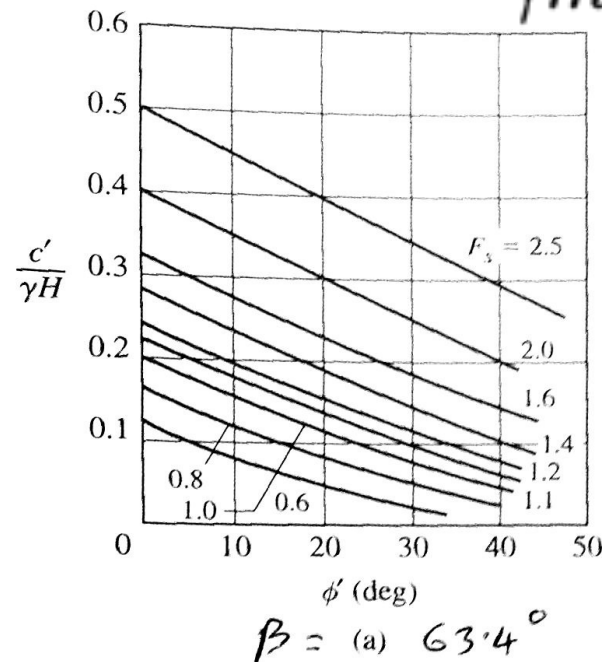


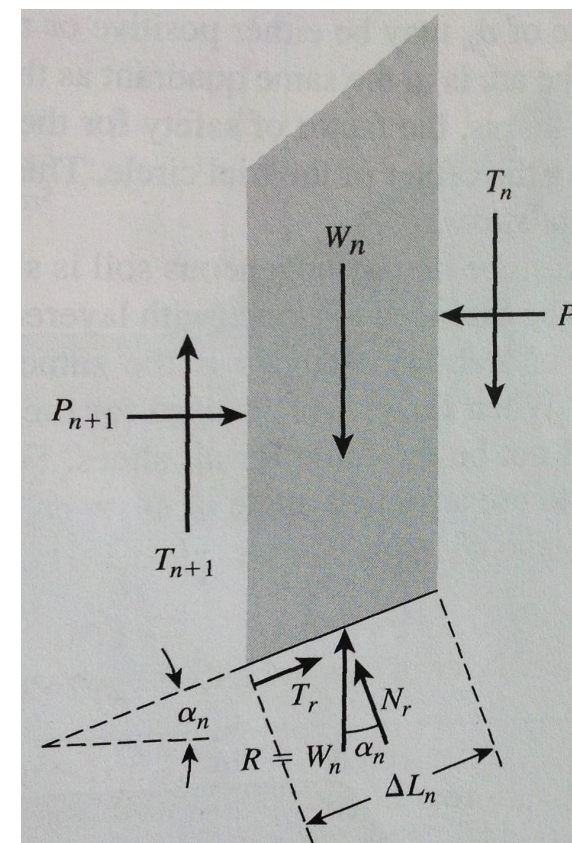
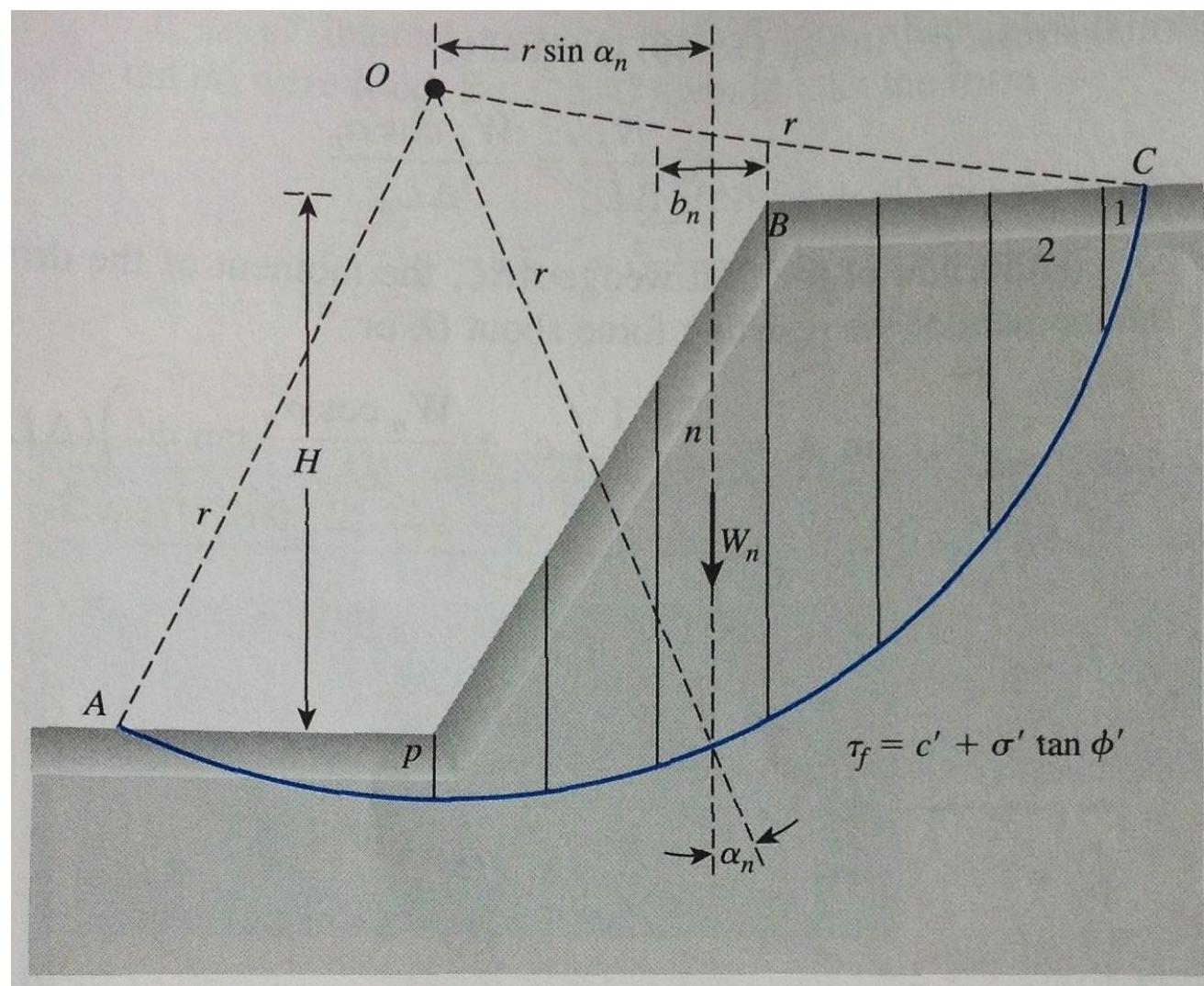
Figure 15.23
Taylor's stability number

Method of Slices

Text: Das, 2010. Chapter 15.10 - 15.16

In the ordinary or Swedish method of slices the soil above a potential failure surface is divided into vertical slices. The width of each slice need not be the same.

A typical set of slices, and the forces that act on an individual slice, are shown in the figure.



$W_n = \gamma A$ (A the slice area, W_n the slice weight)

$N_r = W_n \cos(\alpha_n)$

N_r and T_r are the normal and tangential forces acting on the slice.

T_r is determined by the developed shear stress τ_d

The shear and lateral pressure on the sides of each slice are difficult to determine, however it can be assumed that they cancel.

Method of Slices

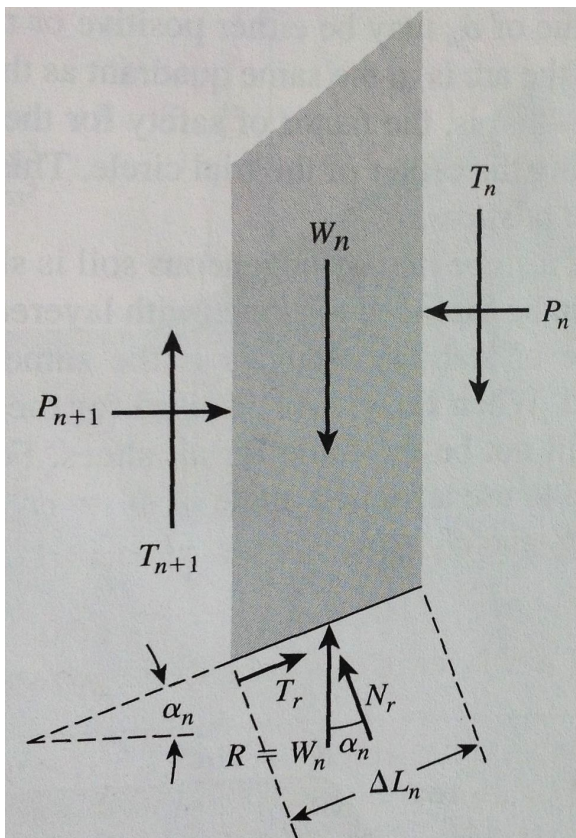
The developed shear stress is related to the failure stress envelope by a factor of safety:

$$F_s = \frac{\tau_f}{\tau_d}$$

The resisting shear force T_r can be expressed as:

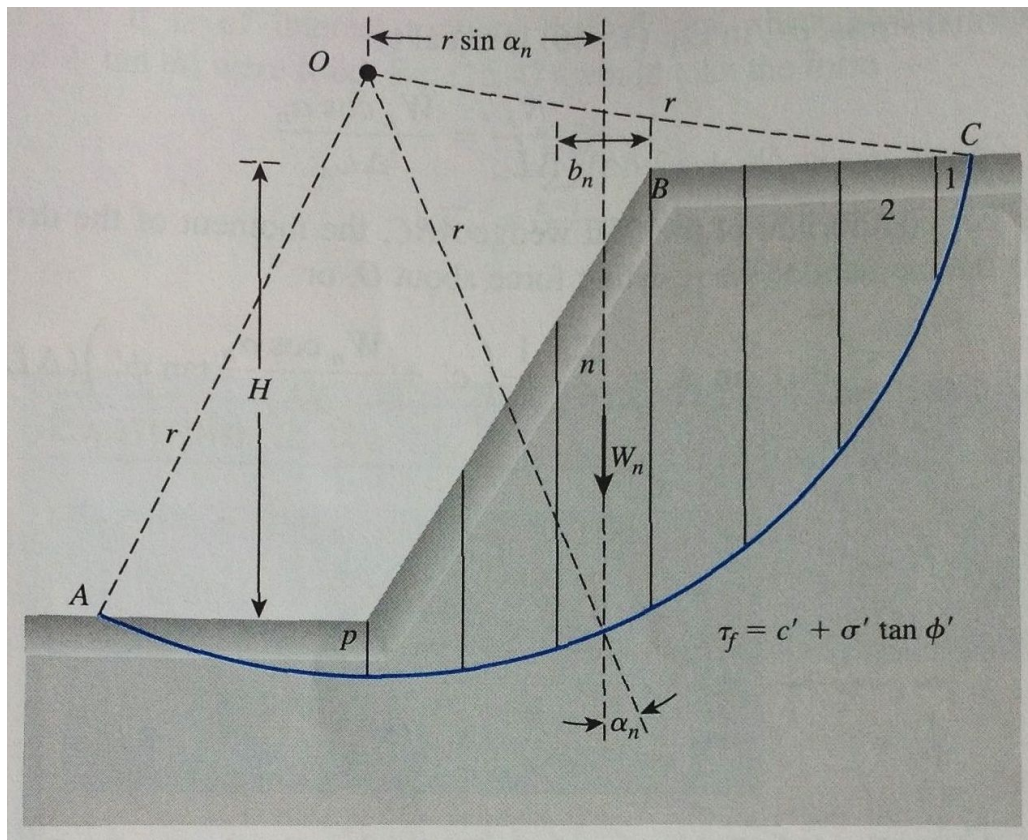
$$T_r = \tau_d(\Delta L_n) = \frac{\tau_f(\Delta L_n)}{F_s} = \frac{1}{F_s} [c' + \sigma' \tan \phi'] \Delta L_n$$

The normal stress σ' is equal to $\frac{N_r}{\Delta L_n} = \frac{W_n \cos \alpha_n}{\Delta L_n}$



Method of Slices

For equilibrium of the trial wedge the moment of the driving force and resisting force about O must be equal:



$$\sum_{n=1}^{n=p} W_n r \sin \alpha_n = \sum_{n=1}^{n=p} \frac{1}{F_s} \left(c' + \frac{W_n \cos \alpha_n}{\Delta L_n} \tan \phi' \right) (\Delta L_n) (r)$$

Each weight force acts at a leverage of $r \sin(\alpha)$. The tangential shear force acts at the radius r , while the normal force coincides with O and so has no moment.

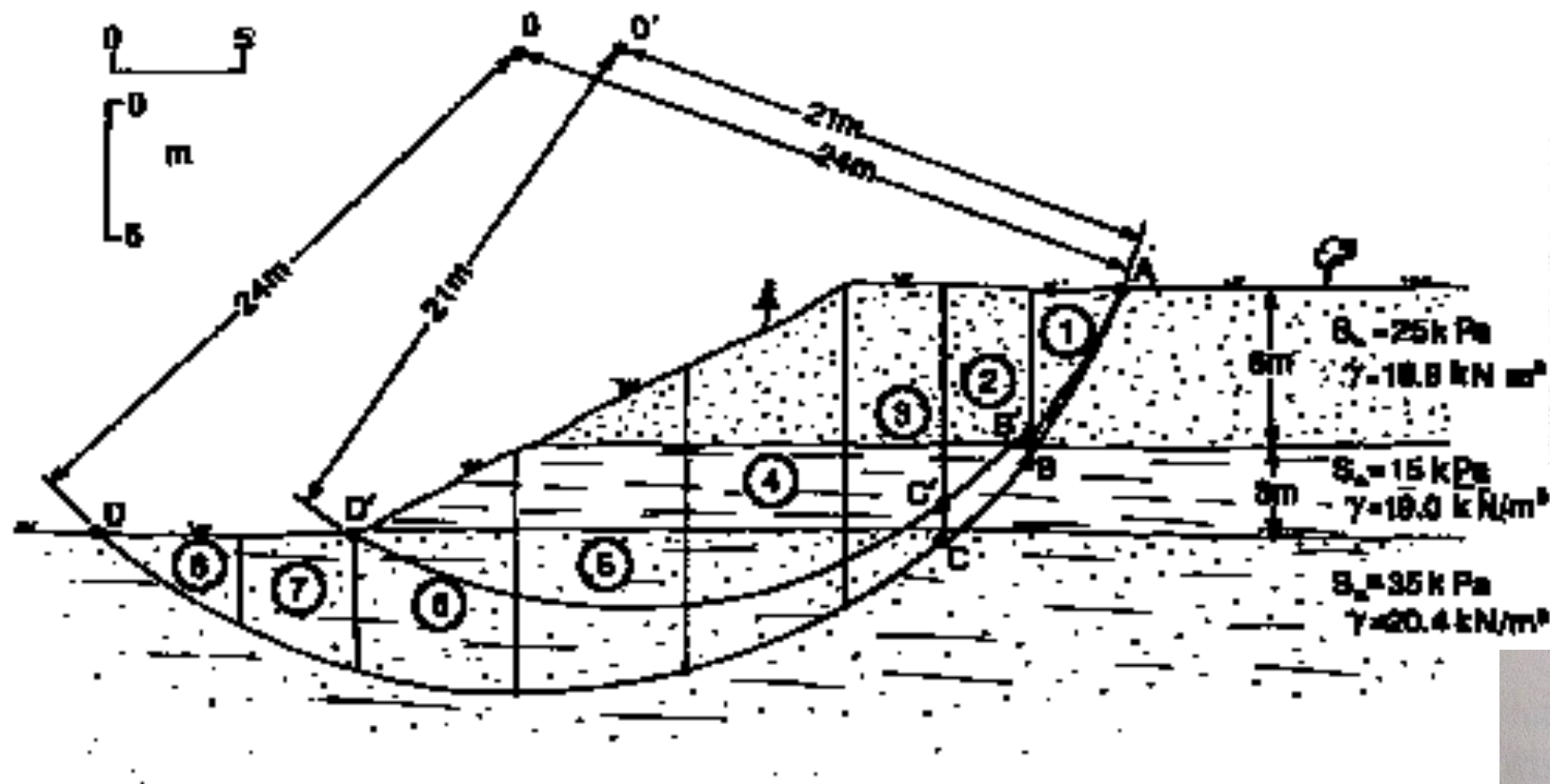
We can solve this for the factor of safety:

$$F_s = \frac{\sum_{n=1}^{n=p} (c' \Delta L_n + W_n \cos \alpha_n \tan \phi')}{\sum_{n=1}^{n=p} W_n \sin \alpha_n}$$

Note that α_n may be positive or negative. α_n is positive when the slope of the arc is in the same quadrant as the ground slope. This accounts for the positive and negative resisting moments of slices on either the left or right of O.

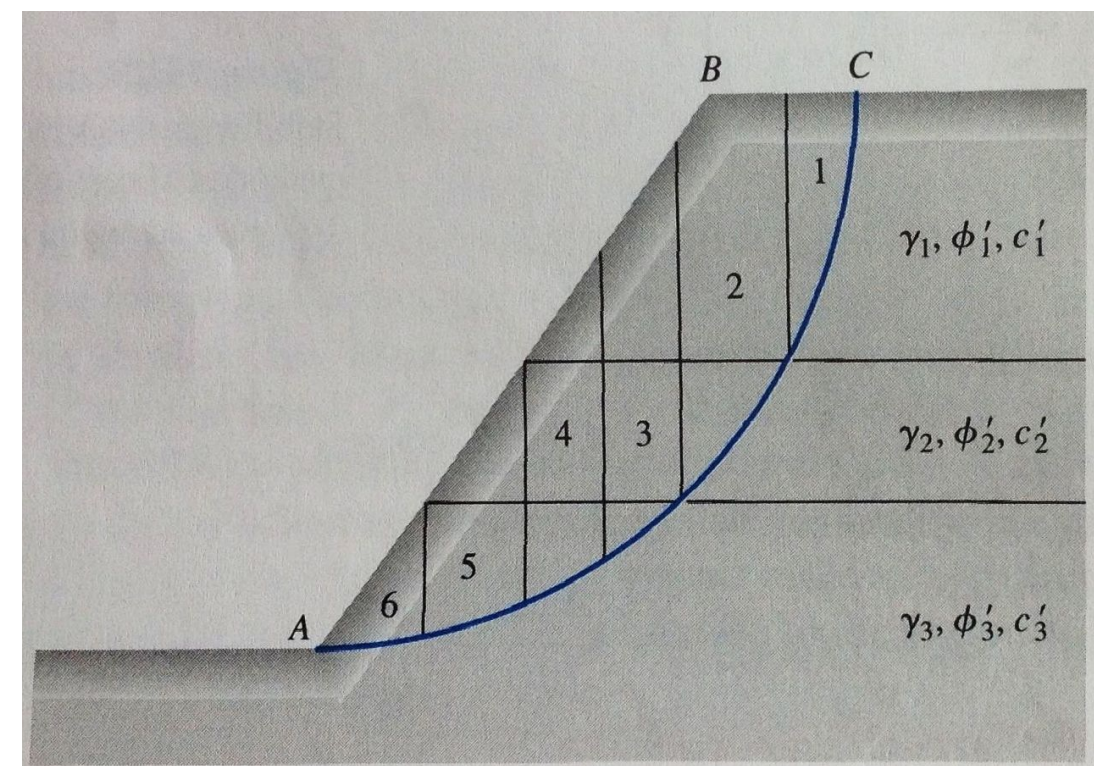
Method of Slices

To find the minimum factor of safety - that is the factor of safety for the critical circle - a number of trial circles must be solved.



$$F_s = \frac{\sum_{n=1}^{n=p} (c' \Delta L_n + W_n \cos \alpha_n \tan \phi')}{\sum_{n=1}^{n=p} W_n \sin \alpha_n}$$

The method can be extended to slopes with layered soil and variable saturation. The factors of ϕ' and c' will not be the same for all slices:



Example

Example 15.8

For the slope shown in Figure 15.31, find the factor of safety against sliding for the trial slip surface AC. Use the ordinary method of slices.

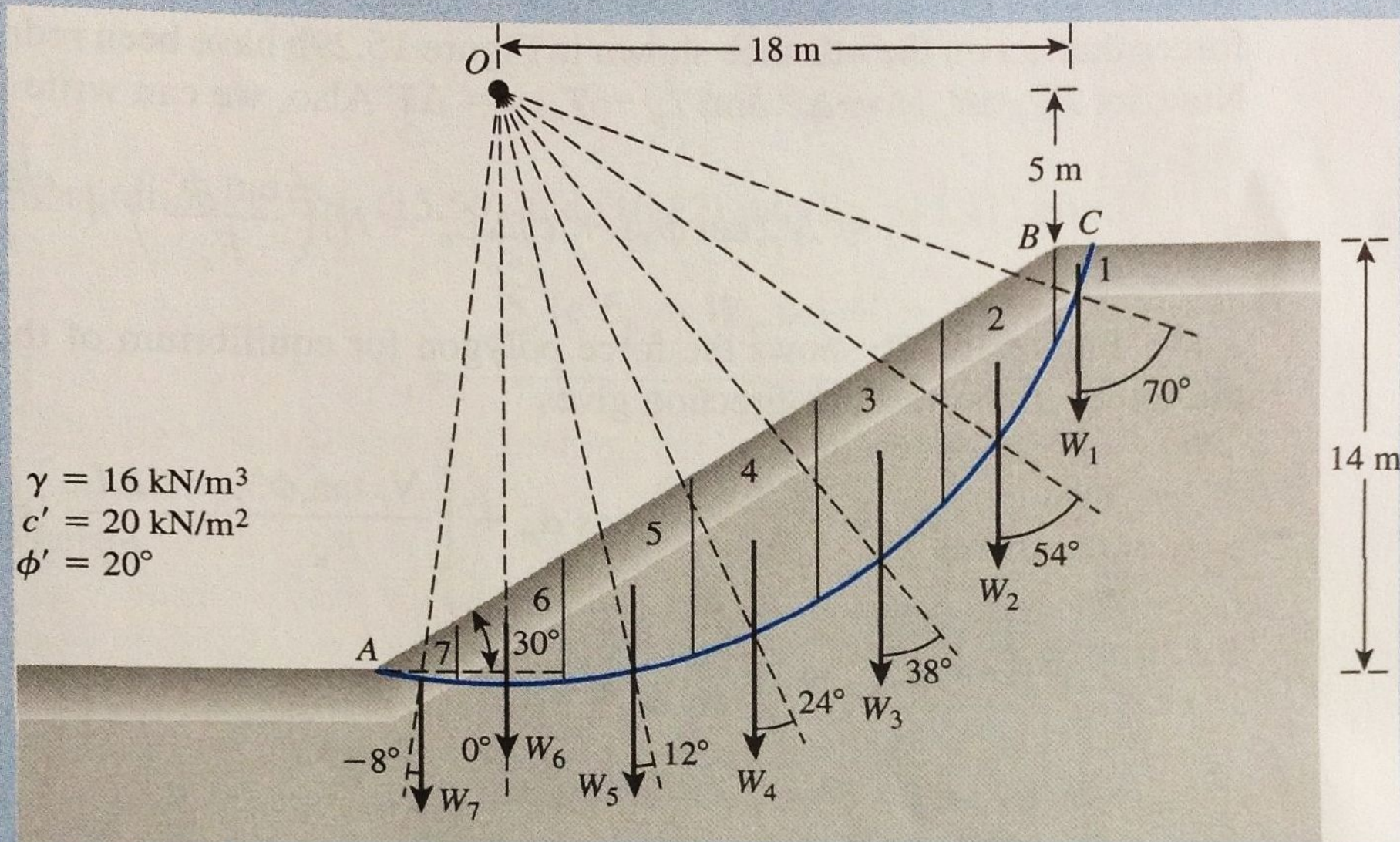


Figure 15.31 Stability analysis of a slope by ordinary method of slices

Example

Solution

The sliding wedge is divided into seven slices. Now the following table can be prepared:

Slice no. (1)	W (kN/m) (2)	α_n (deg) (3)	$\sin \alpha_n$ (4)	$\cos \alpha_n$ (5)	ΔL_n (m) (6)	$W_n \sin \alpha_n$ (kN/m) (7)	$W_n \cos \alpha_n$ (kN/m) (8)
1	22.4	70	0.94	0.342	2.924	21.1	7.66
2	294.4	54	0.81	0.588	6.803	238.5	173.1
3	435.2	38	0.616	0.788	5.076	268.1	342.94
4	435.2	24	0.407	0.914	4.376	177.1	397.8
5	390.4	12	0.208	0.978	4.09	81.2	381.8
6	268.8	0	0	1	4	0	268.8
7	66.58	-8	-0.139	0.990	3.232	-9.25	65.9
					$\Sigma \text{ Col. 6} =$ 30.501 m	$\Sigma \text{ Col. 7} =$ 776.75 kN/m	$\Sigma \text{ Col. 8} =$ 1638 kN/m

$$F_s = \frac{(\Sigma \text{ Col. 6})(c') + (\Sigma \text{ Col. 8})\tan \phi'}{\Sigma \text{ Col. 7}}$$

$$= \frac{(30.501)(20) + (1638)(\tan 20)}{776.75} = \mathbf{1.55}$$

This is repeated several times and the minimum F_s located.

Bishop's Method

Bishop's method improves on the ordinary method of slices by accounting for variable lateral earth pressure.

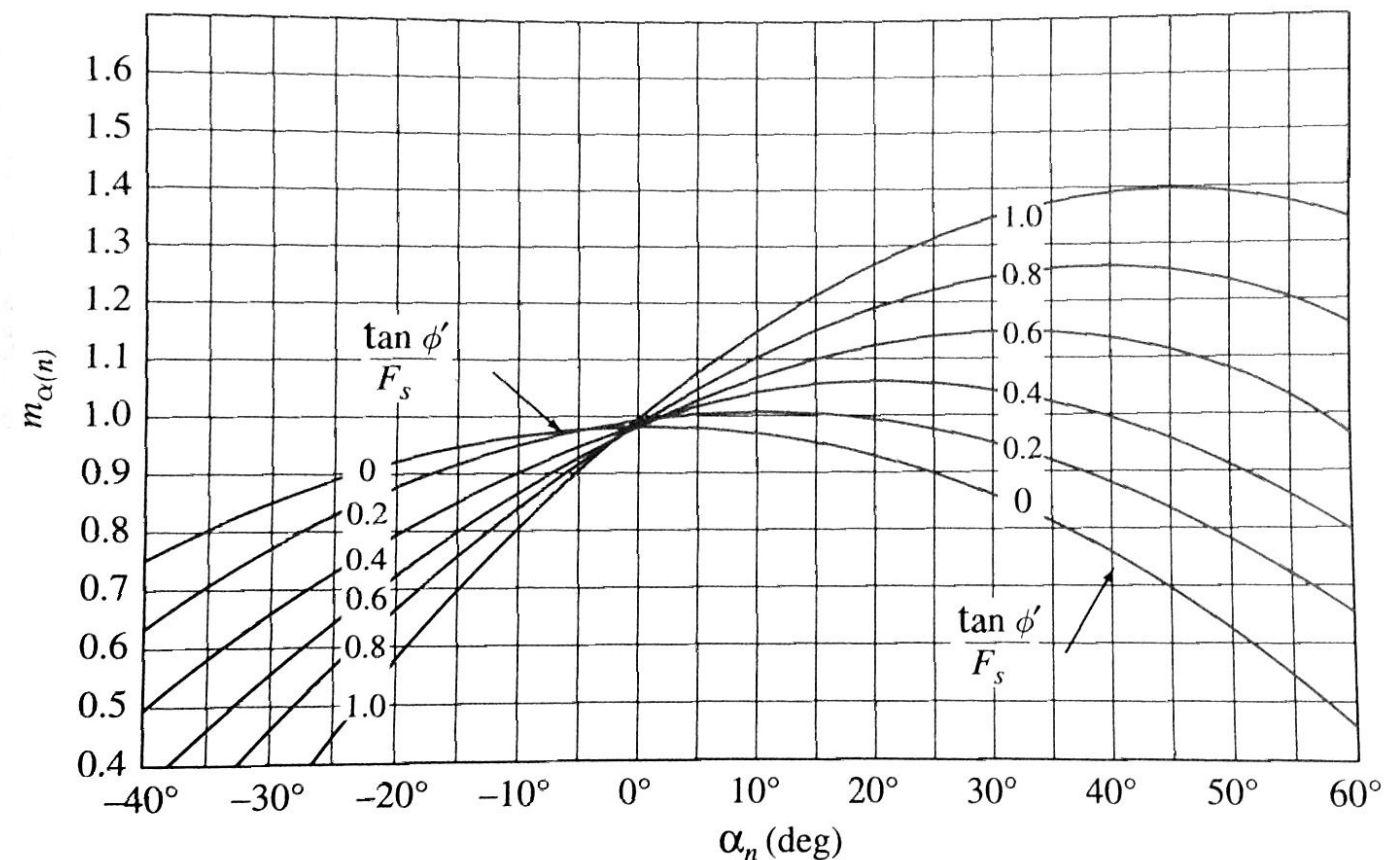
This is done by closing the force polygon with a ΔP term. The result is:

$$F_s = \frac{\sum_{n=1}^{n=p} (c' b_n + W_n \tan \phi') \frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin \alpha_n}$$

where b_n is the width of each slice and $m_{\alpha(n)}$ is a stability number:

$$m_{\alpha(n)} = \cos \alpha_n + \frac{\tan \phi' \sin \alpha_n}{F_s}$$

Note that F_s appears on both sides of the equation so an iterative method is needed, in addition to trials of multiple potential slip surfaces.



Bishop's method is less conservative than the ordinary method of slices. It is in common use in geotechnical computer programs

Stability Analysis of Seepage

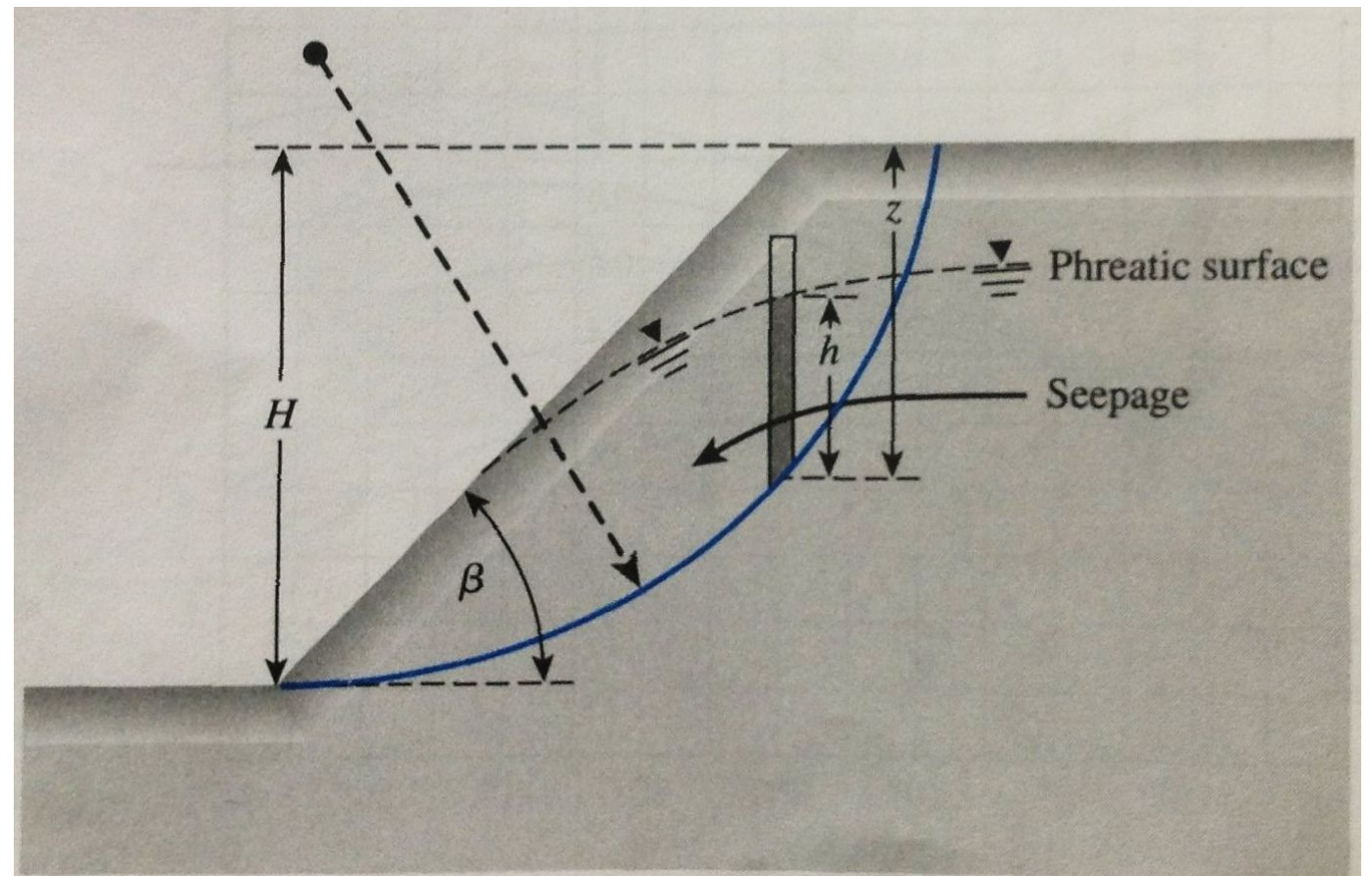
Seepage in an embankment creates additional stresses which can be solved by a method of slices.

A simple (conservative) solution is:

$$F_s = \frac{\gamma' \tan(\phi')}{\gamma_{sat} \tan(\beta)}$$

This approximation applies to the surface breakpoint where flow is parallel to and at the surface.

A more accurate solution is obtained by Morgenstern charts (see text) or a flownet analysis of pore water pressure.



Seepage may occur in groundwater baseflow into a stream or after rapid emptying of a canal.

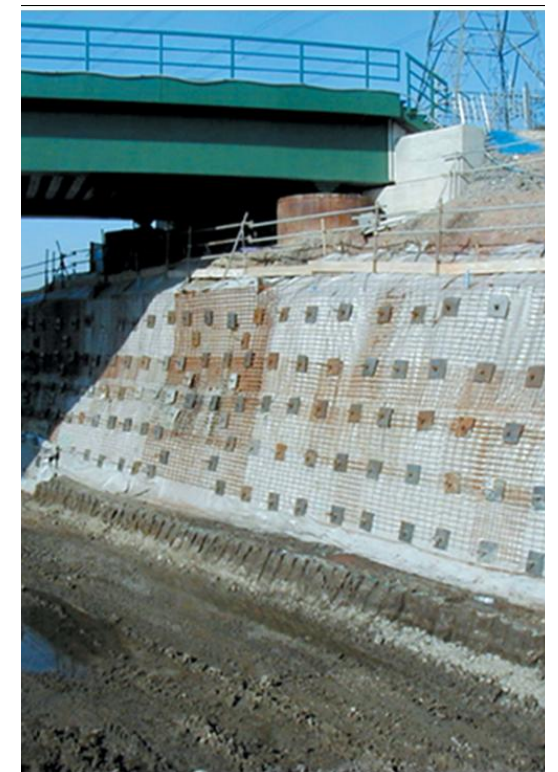
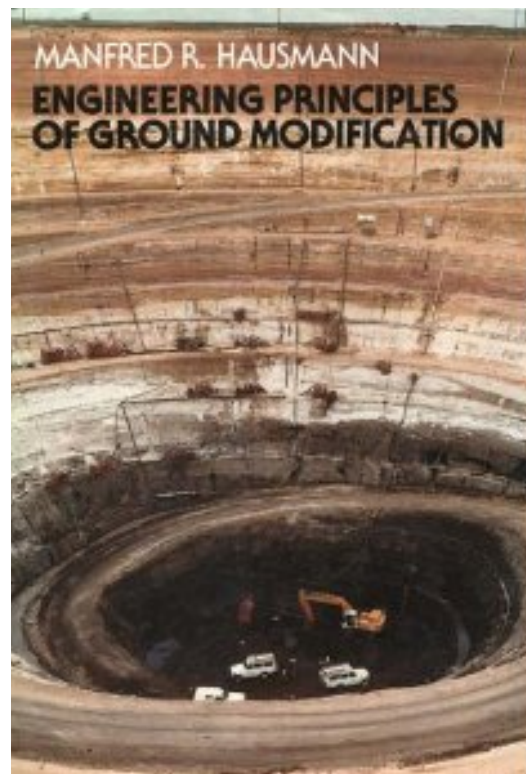
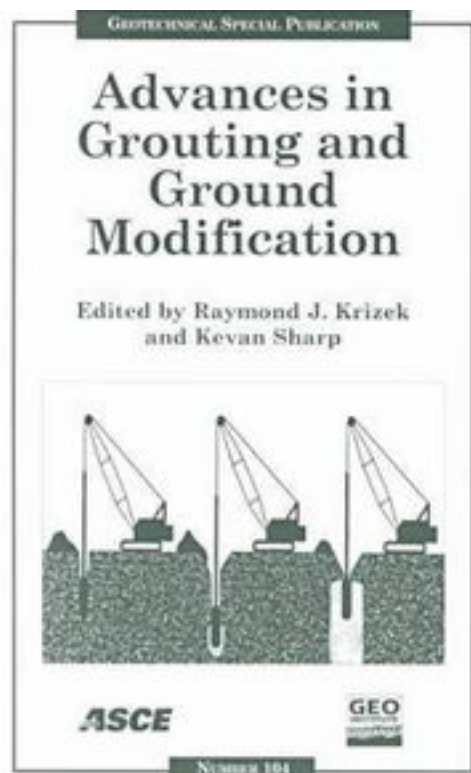
Rapid drawdown is a common failure point with low factors of safety due to saturated undrained conditions. It is common for stream banks to collapse on sudden emptying.

Slope Stabilisation

Slopes with unacceptably low safety factors can be improved by various methods.

- Soil Nailing
- Geotextile Reinforcement
- Ground Modification (mechanical or chemical)

This is a specialised area. Students are referred to recent publications.



Readings

Chapter 15 Das

Smith, D.W. 1992. Braced Excavations; Slurry Trenches; Slope Stability (Reading 10)

Smith, DW. 1992. Engineering Principles of Ground Modification (Reading 11)

