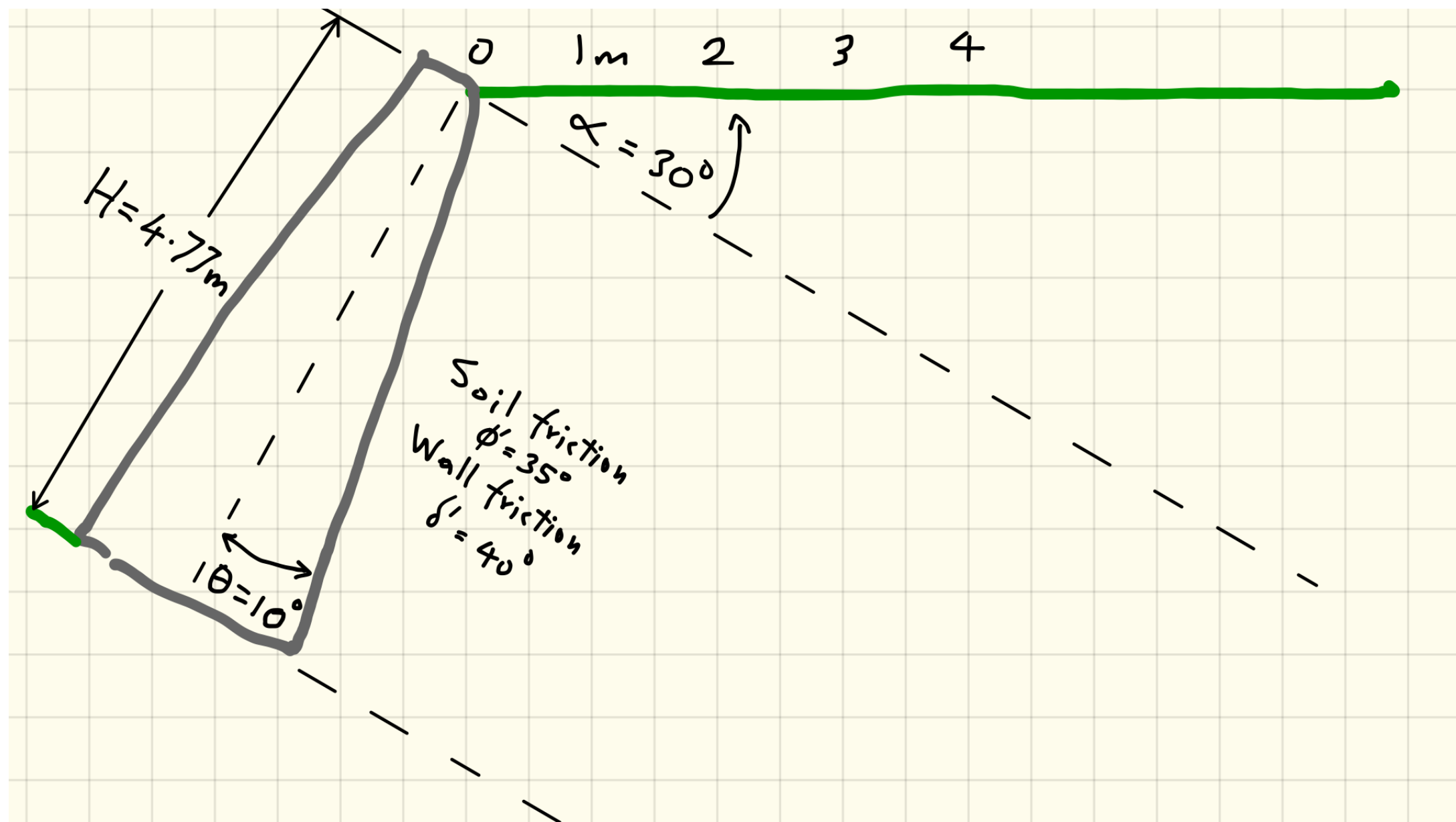


Retaining Walls Continued

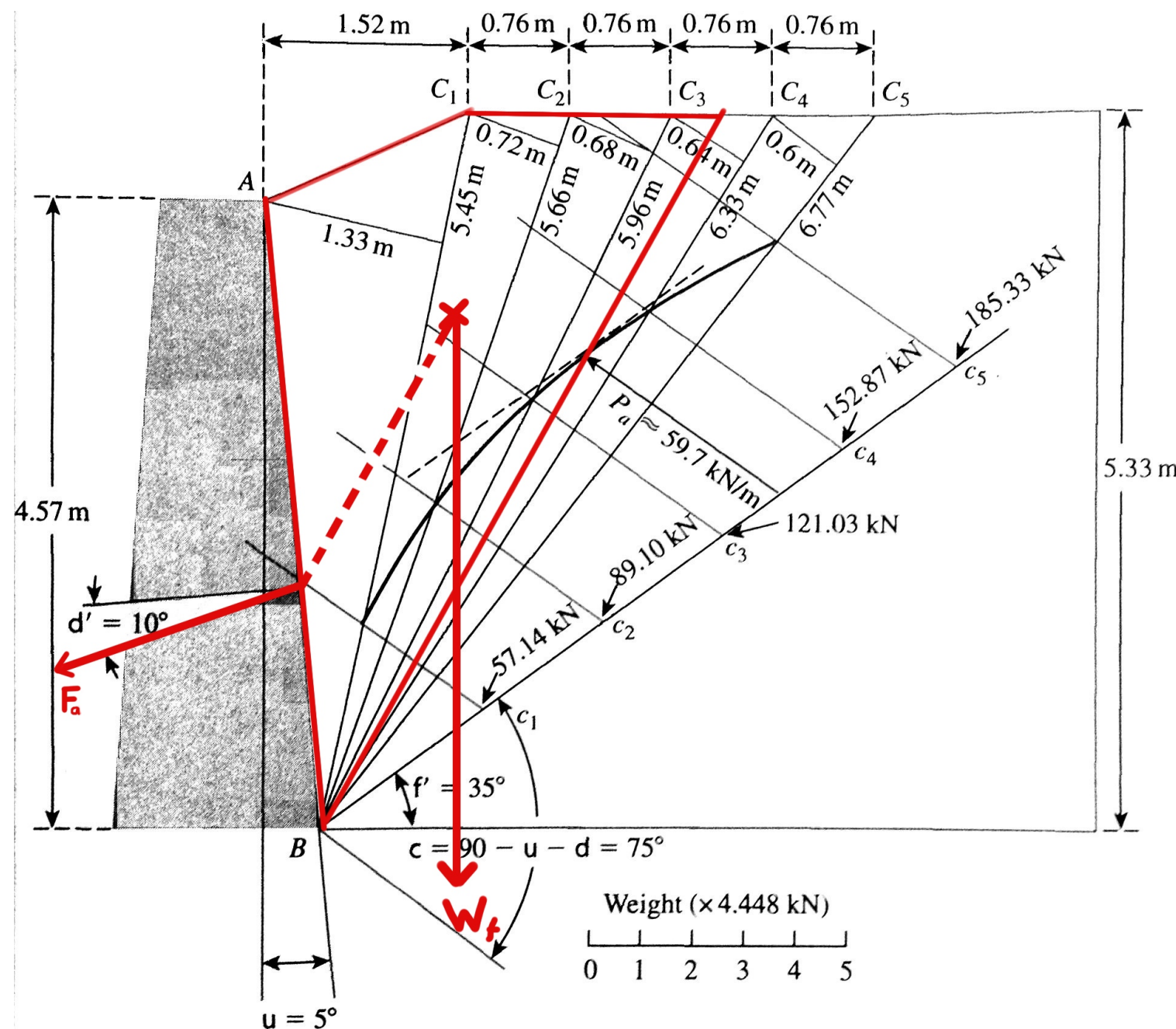
Text: Das, 2010. Chapter 13

More complex retaining wall designs, with frictional wall-soil contact and sloping backfill, can be analysed by a graphical method (Culmann, 1875)



Retaining Walls: Culmann's method

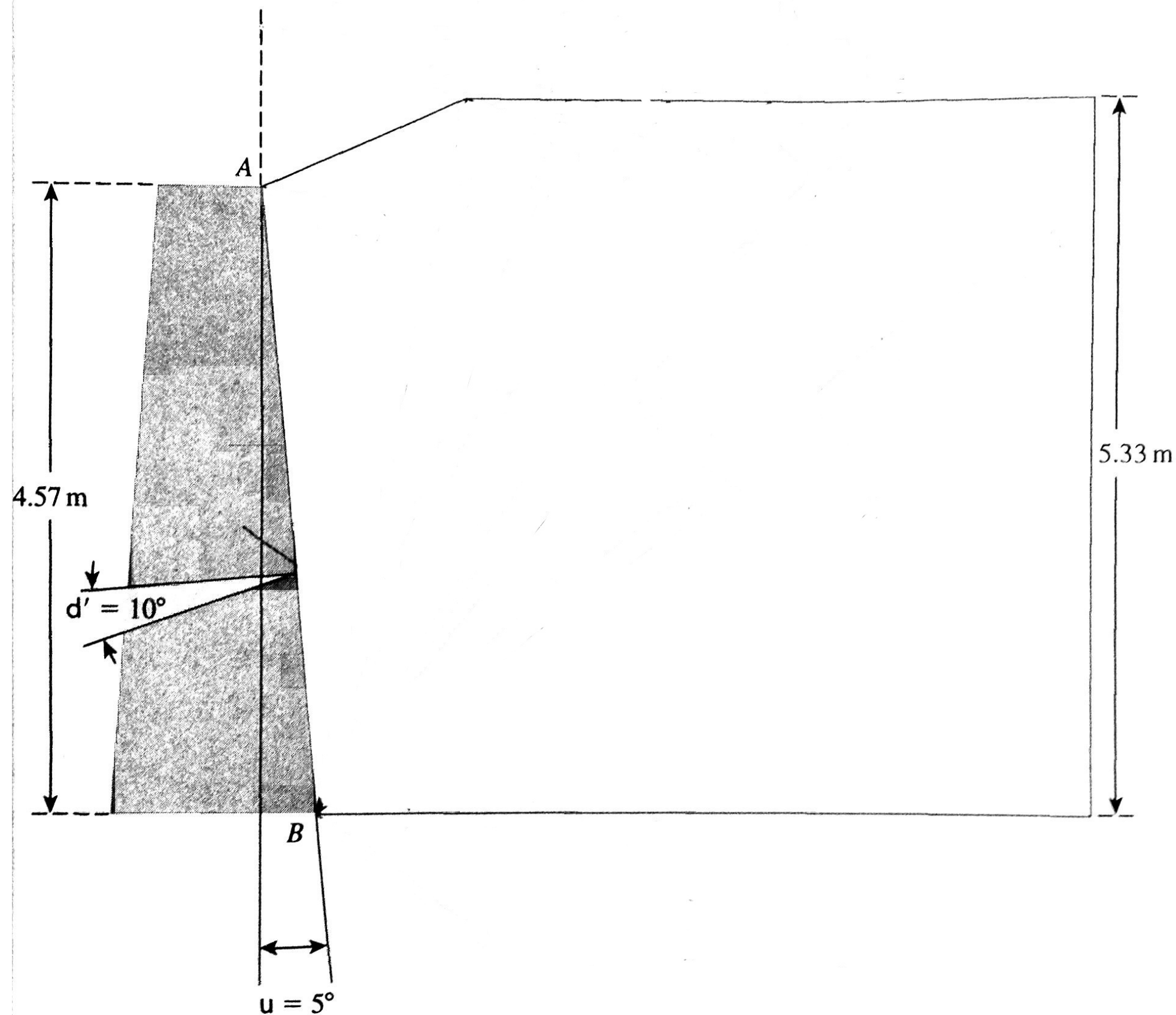
The graphical construction entails drawing force polygons for a number of trial wedges and locating the maximum value of the active force that the wall is subject to.



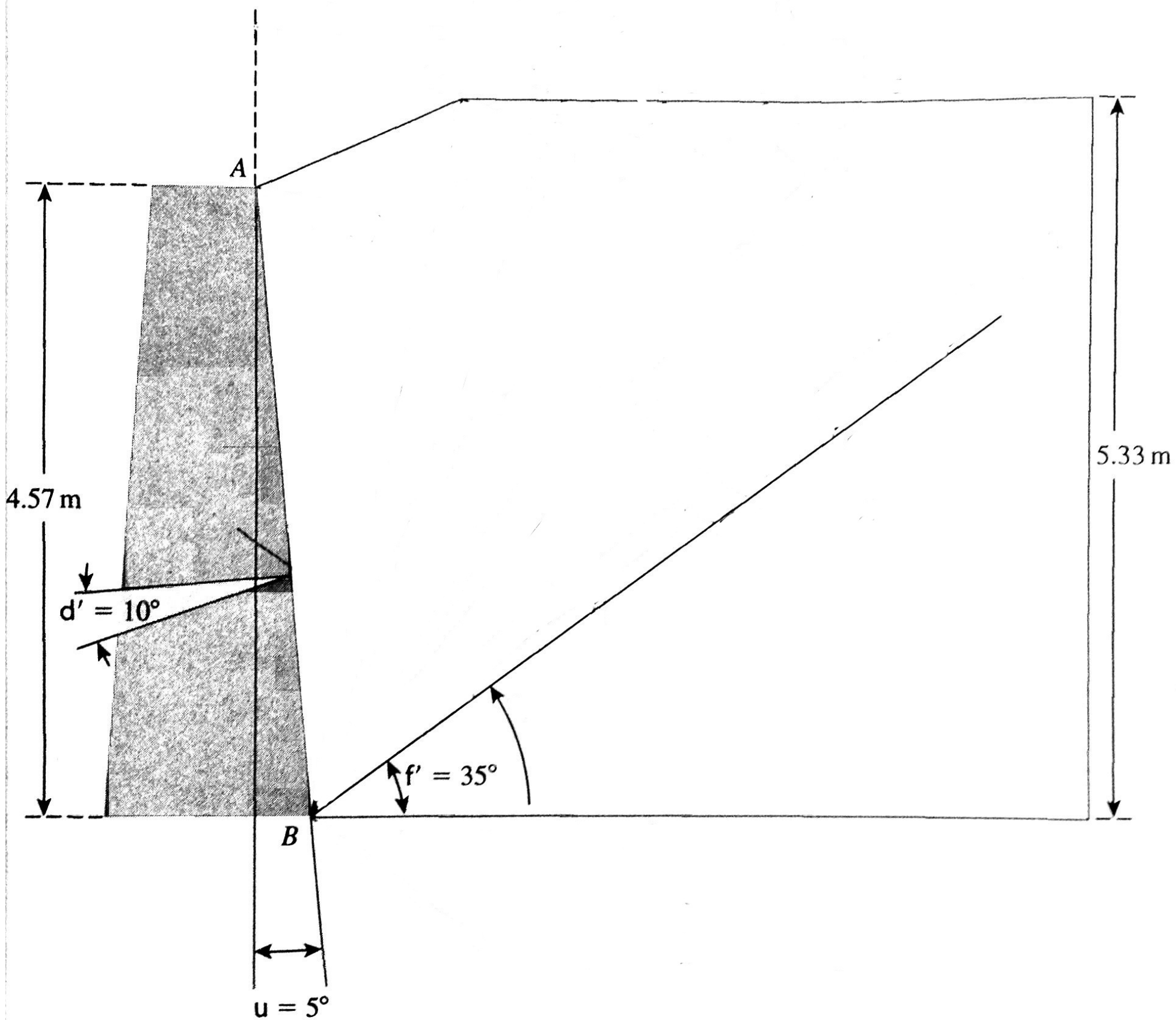
The method uses the angles of friction forces to develop a geometric construction of the unknown active force which closes the force polygon.

It is worthwhile studying this method for a general understanding of how active pressure works, and how some commercial software still calculates the active force.

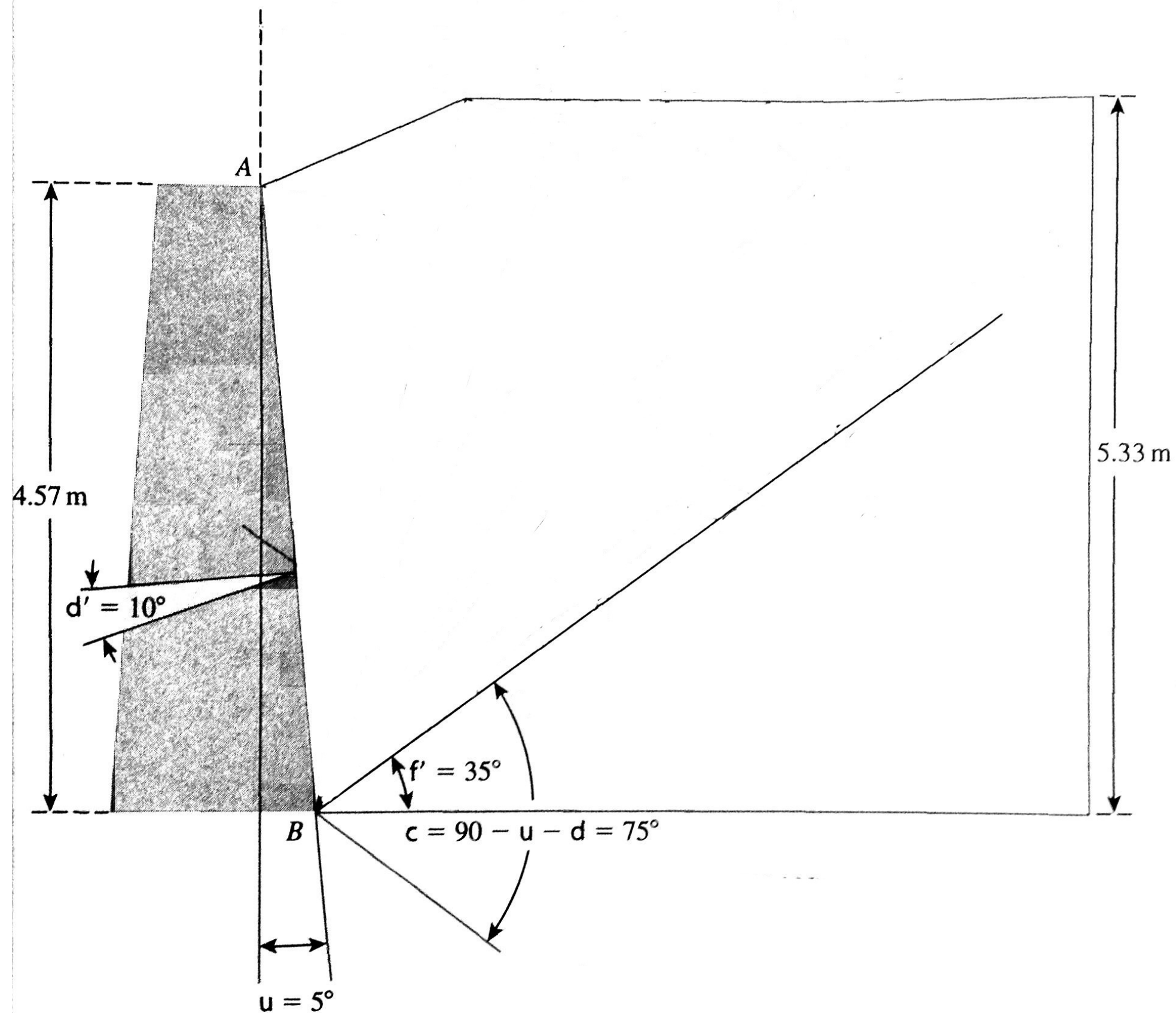
Step 1: Draw the wall and its geological / soil foundations to a suitable scale. Collate soil and wall friction angles ϕ' and δ' and wall face angle u



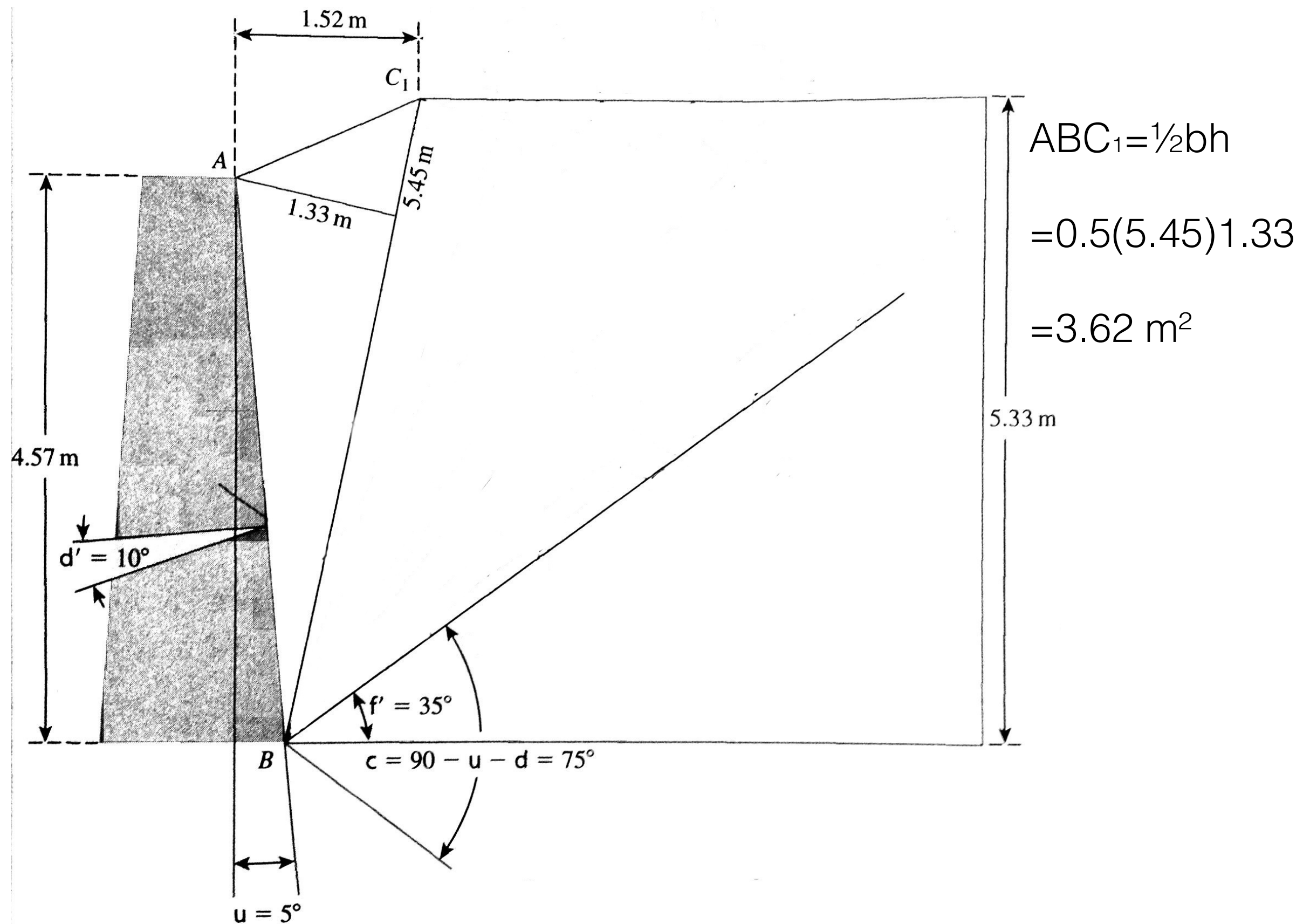
Step 2: Plot a line at the angle of repose ϕ' to the horizontal.



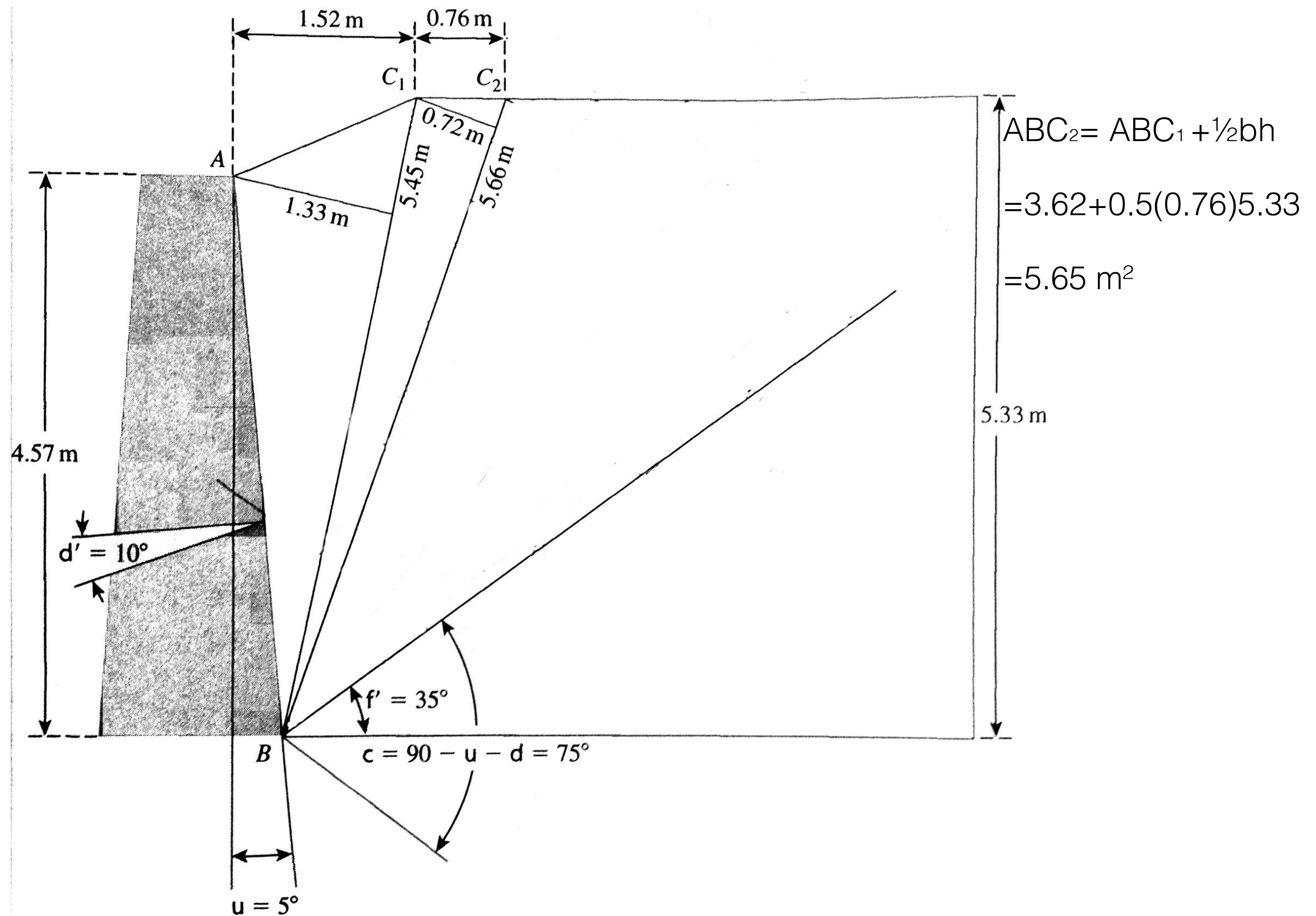
Step 3: Plot a second line at an angle $c = 90 - u - d'$ to the repose line ϕ'



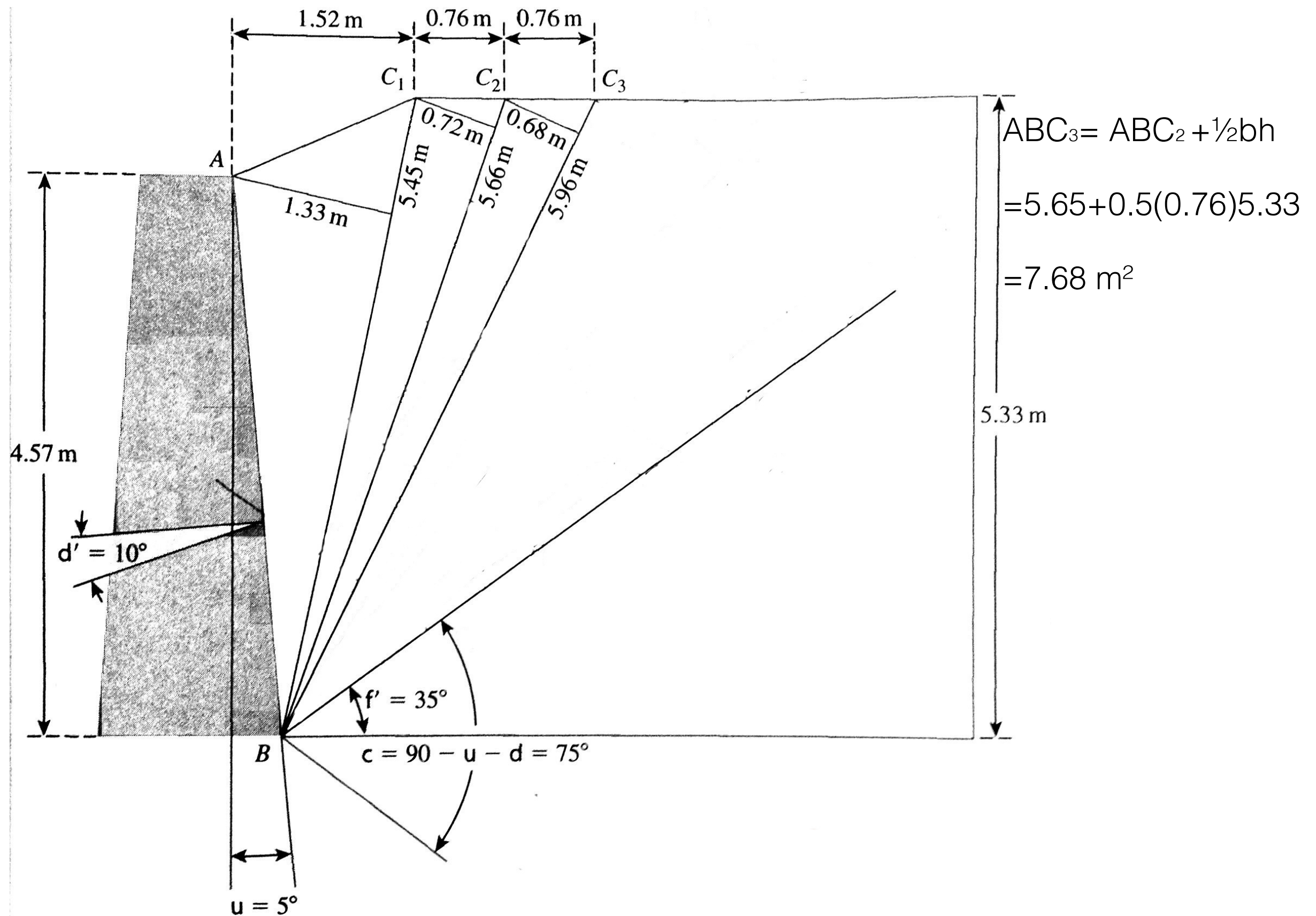
Step 4: Sketch a trial failure wedge and calculate the area of the wedge.



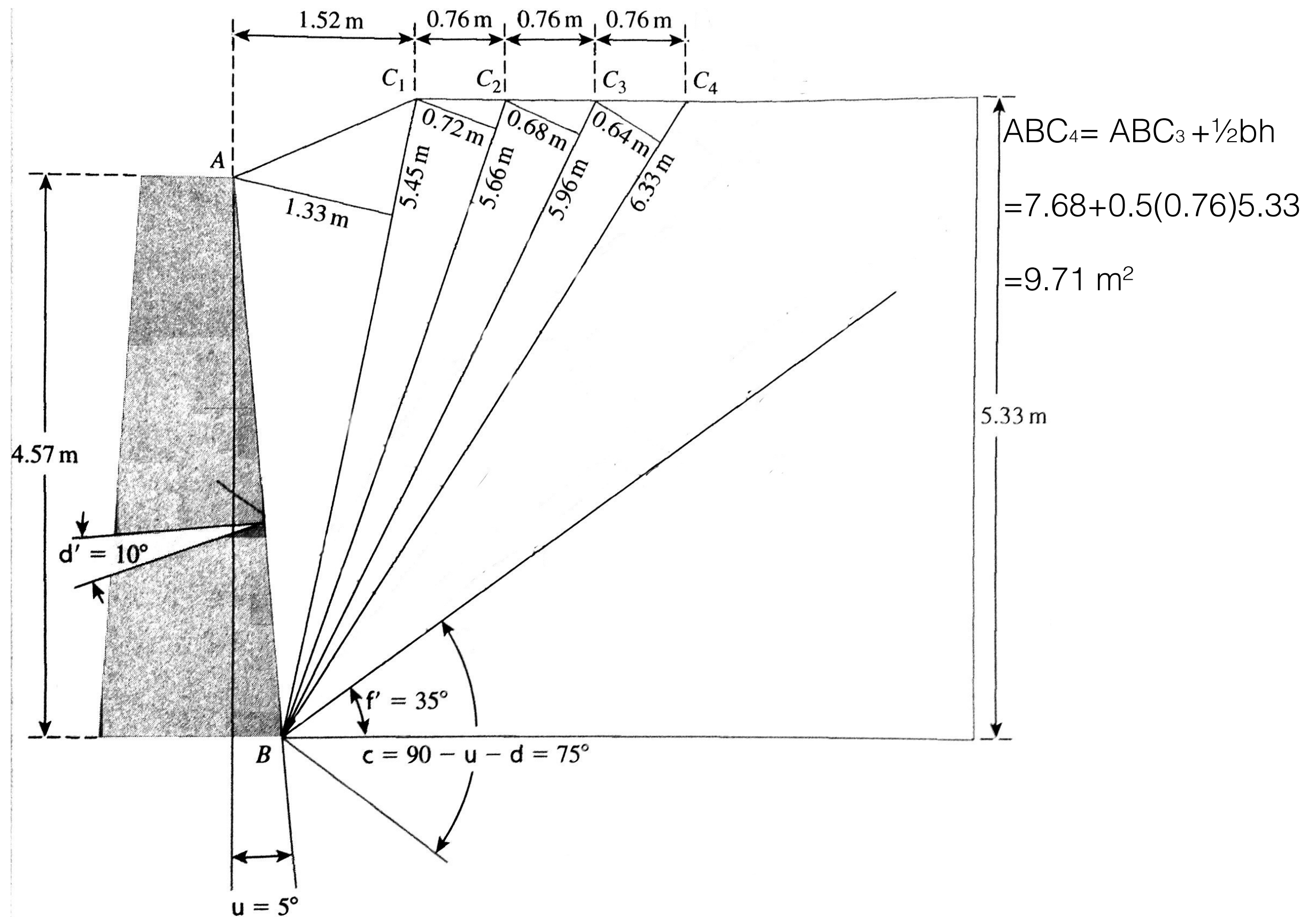
Step 4 continued: Sketch further trial failure wedges and calculate areas:



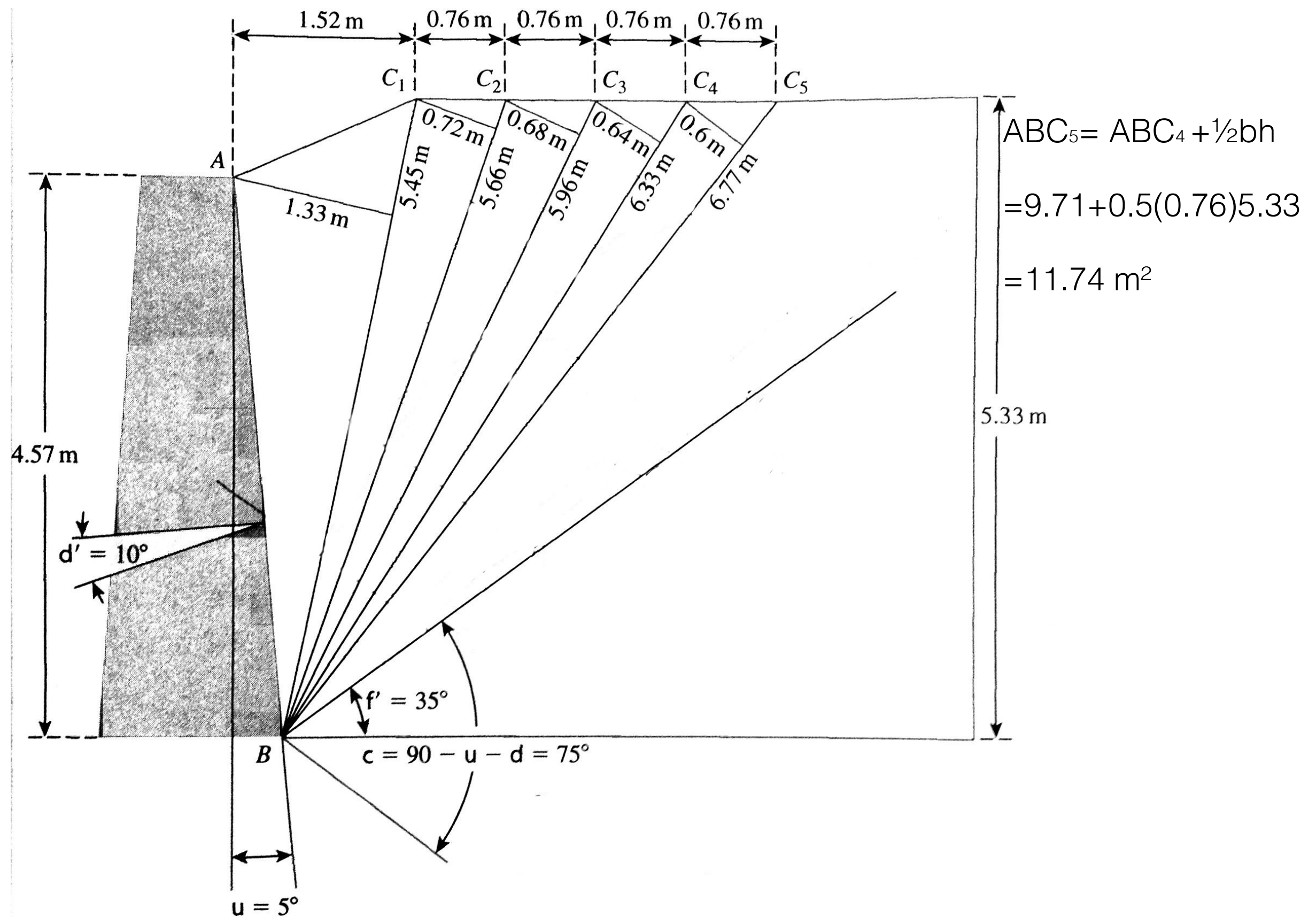
Step 4 continued: Sketch further trial failure wedges and calculate areas:



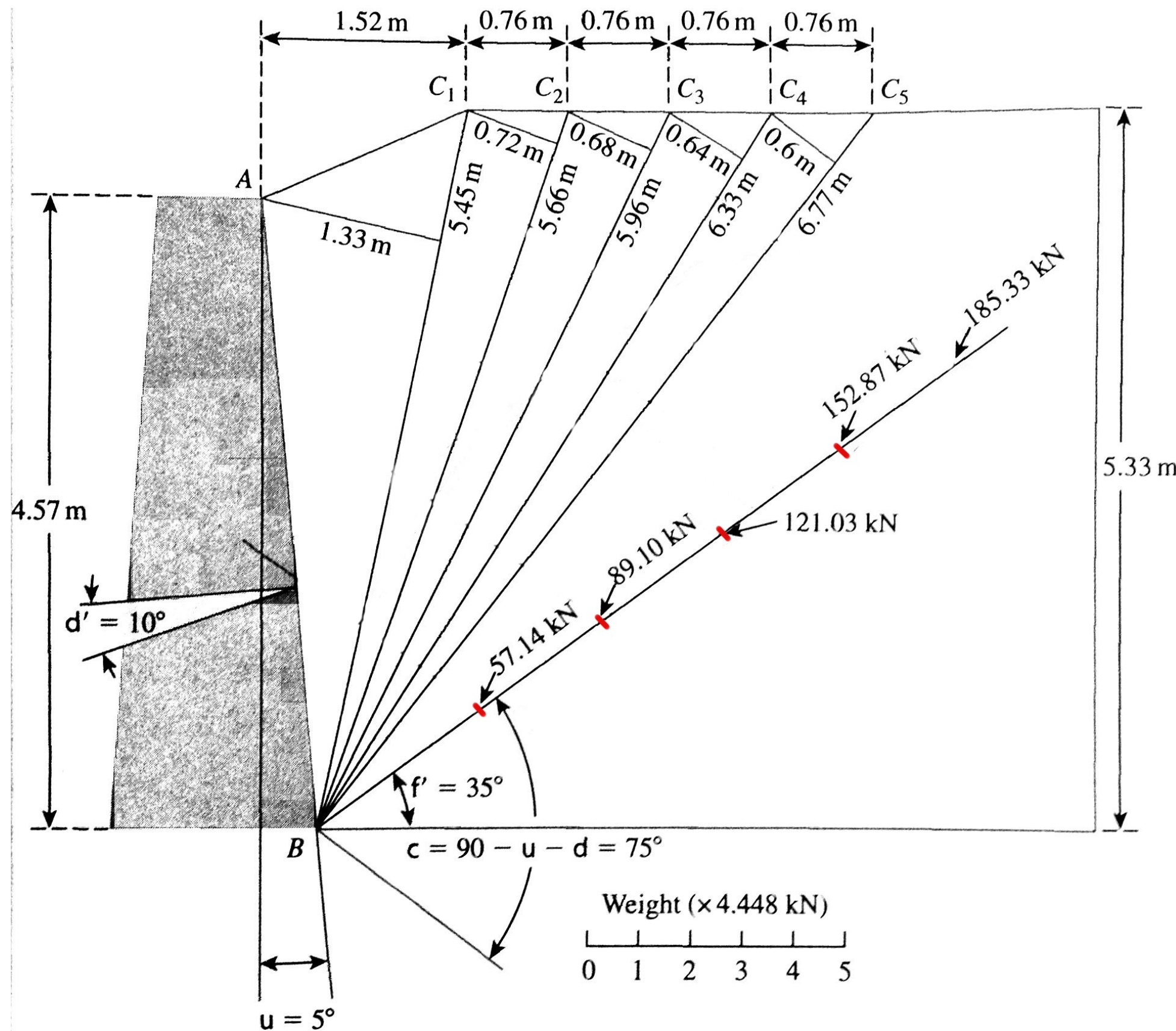
Step 4 continued: Sketch further trial failure wedges and calculate areas:



Step 4 continued: Sketch further trial failure wedges and calculate areas:



Step 5: Use saturated unit weight to calculate the weight of each wedge.
Plot weights to a suitable scale on the ϕ' line:



$$W_1 = 15.72 \text{ ABC}_1$$
$$= 57.14 \text{ kN}$$

$$W_2 = 15.72 \text{ ABC}_2$$
$$= 89.1 \text{ kN}$$

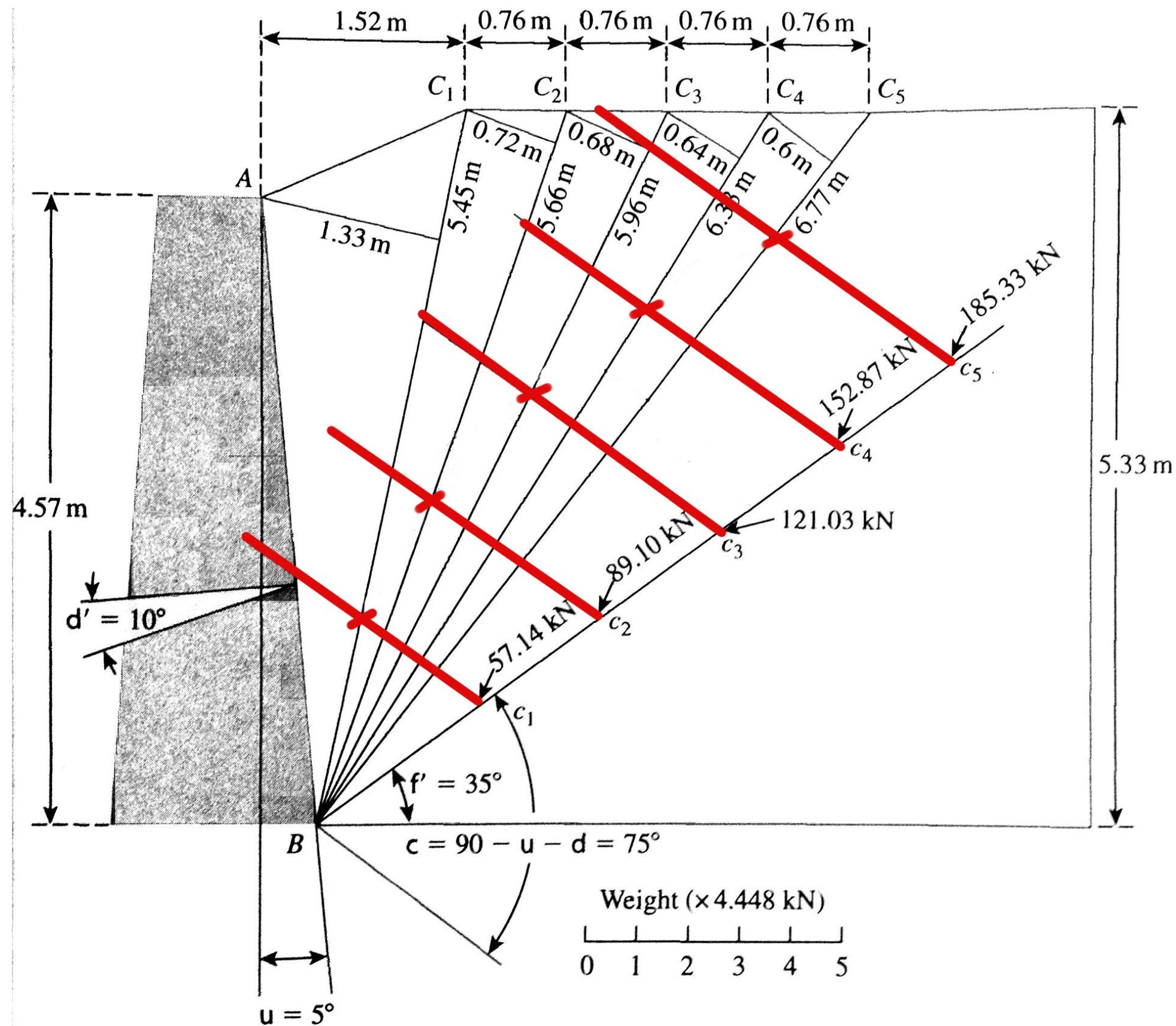
$$5.33 \text{ m } W_3 = 15.72 \text{ ABC}_3$$

$$= 123.03 \text{ kN}$$

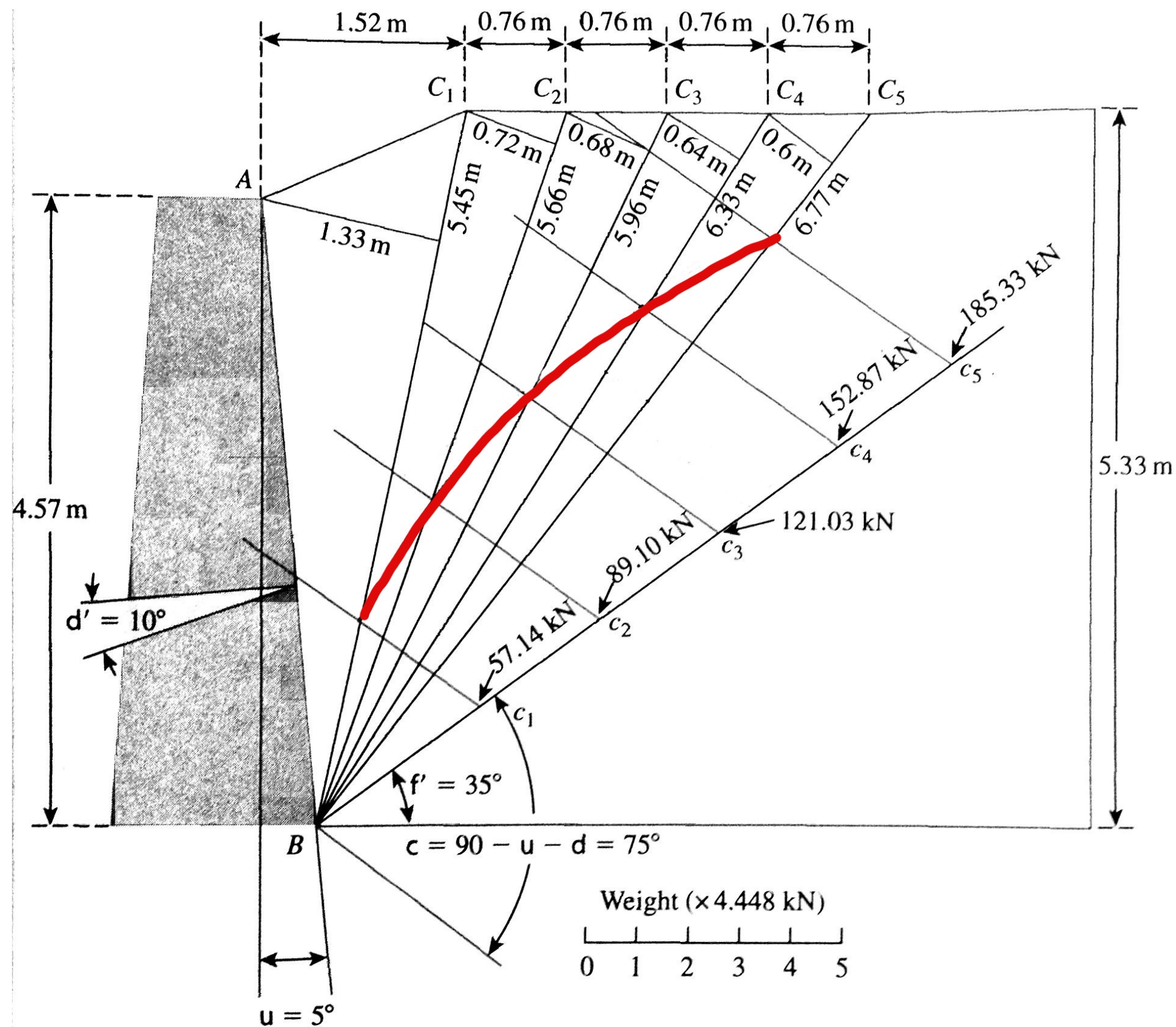
$$W_4 = 15.72 \text{ ABC}_4$$
$$= 152.87 \text{ kN}$$

$$W_5 = 15.72 \text{ ABC}_5$$
$$= 185.33 \text{ kN}$$

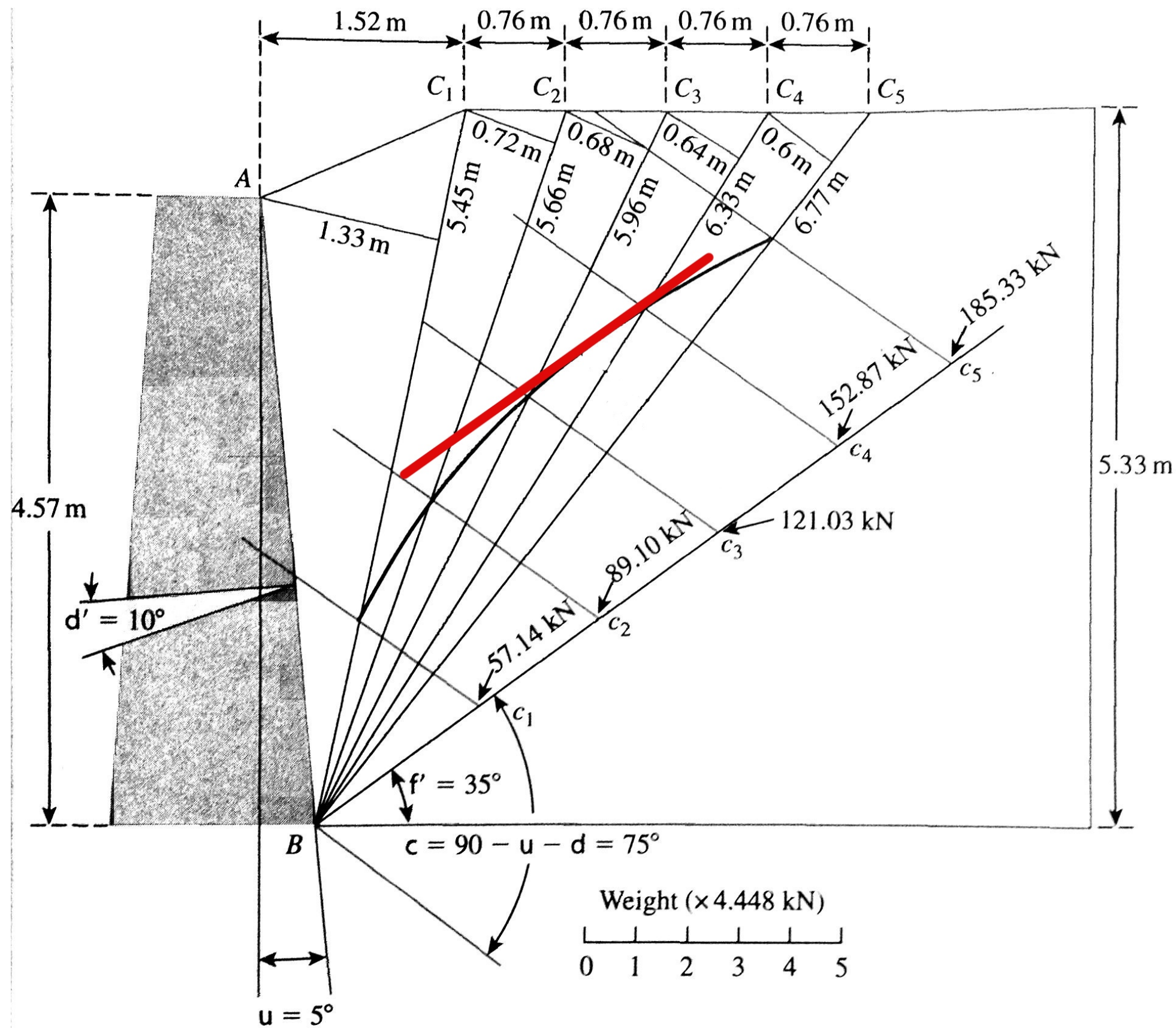
Step 6: Draw lines parallel to the c line from the load ordinates and plot their intersections with the respective failure wedges.



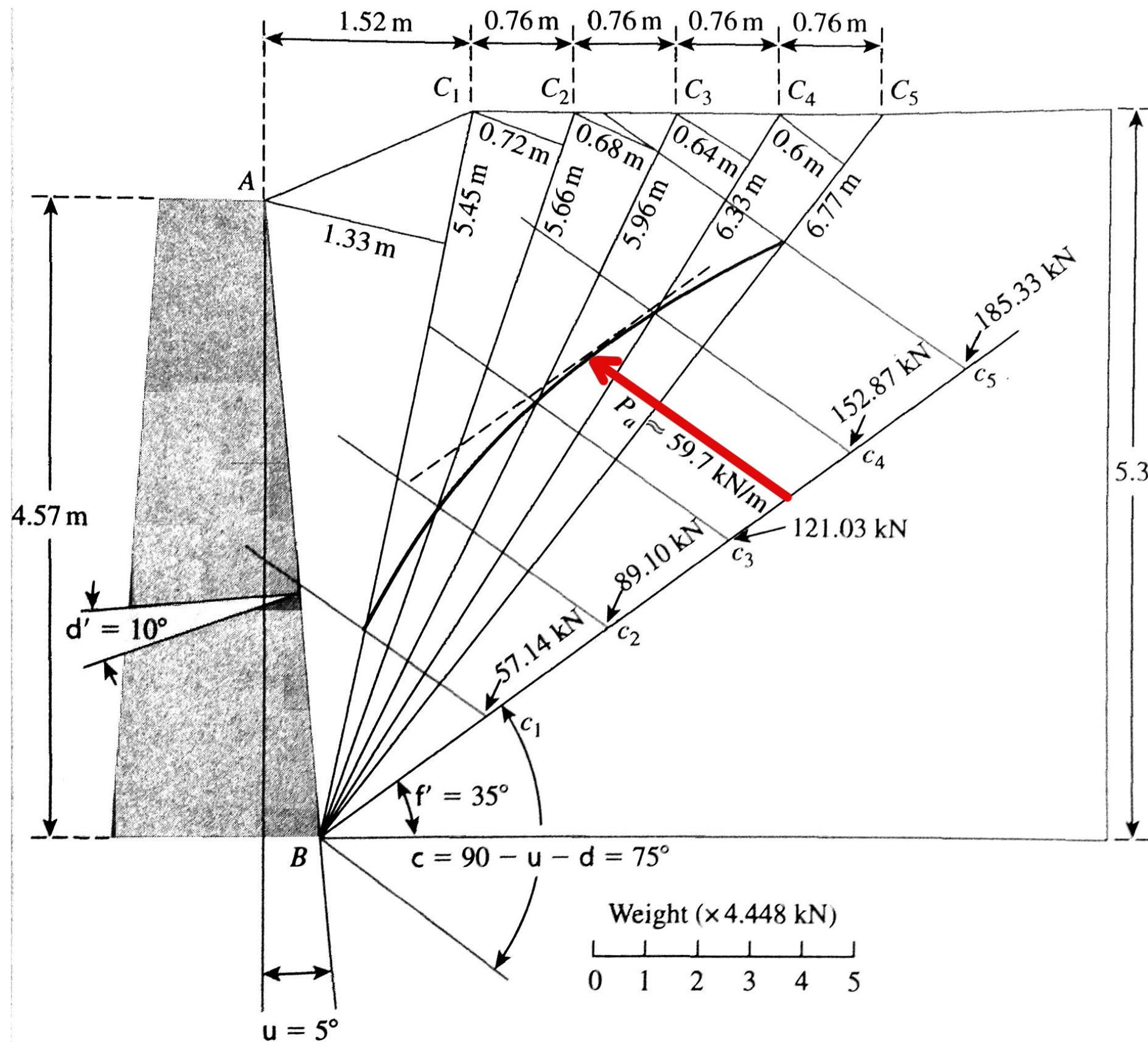
Step 7: Draw a smooth curve through the intersection points:



Step 7: Draw a line tangent to the smooth curve, parallel to the ϕ' line.



Step 8: Draw a vector parallel to the c line to meet the tangent point.

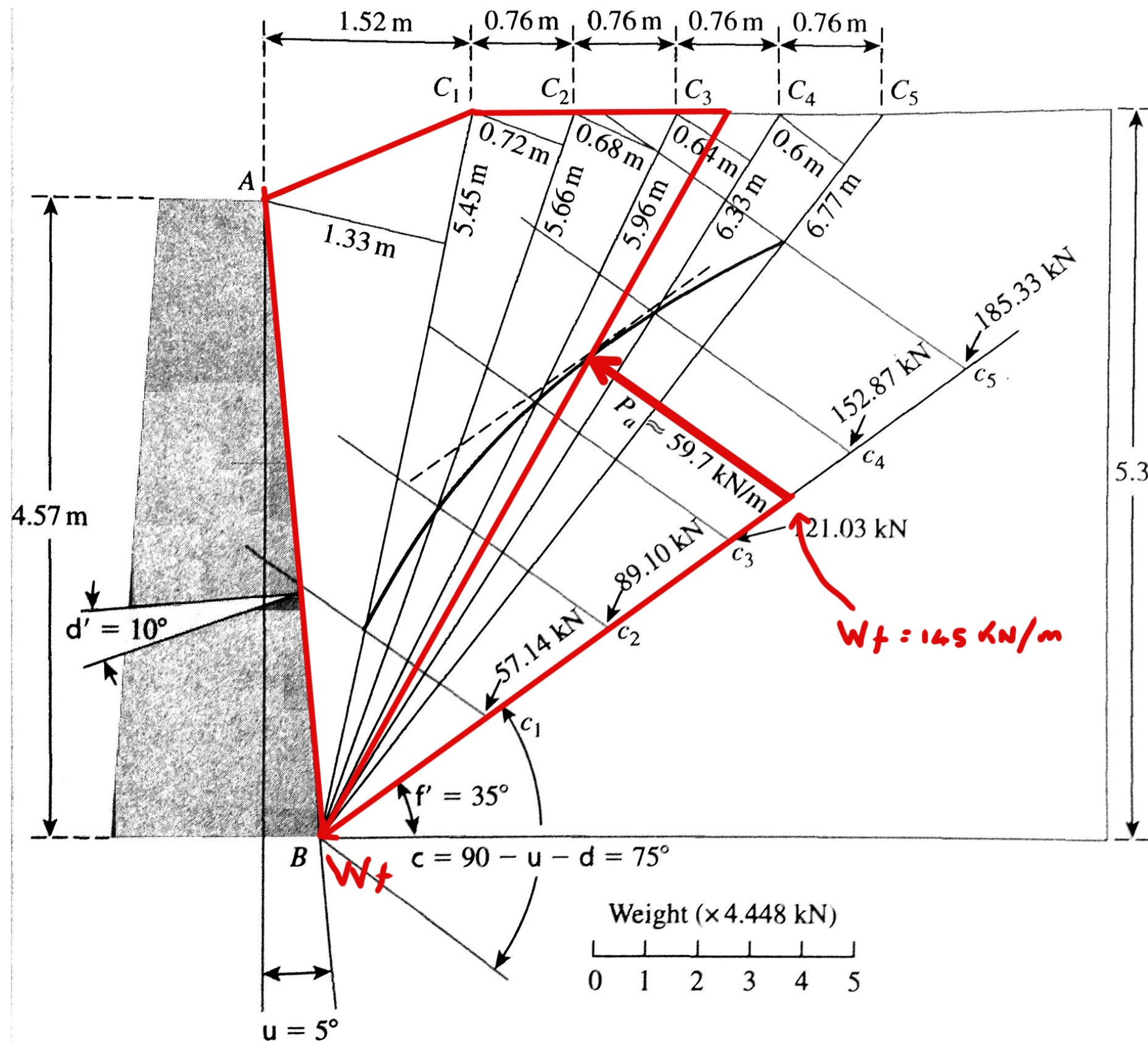


The length of this vector is the active force, at the chosen force scale.

5.33 m

Measure the length of the vector to determine the active force

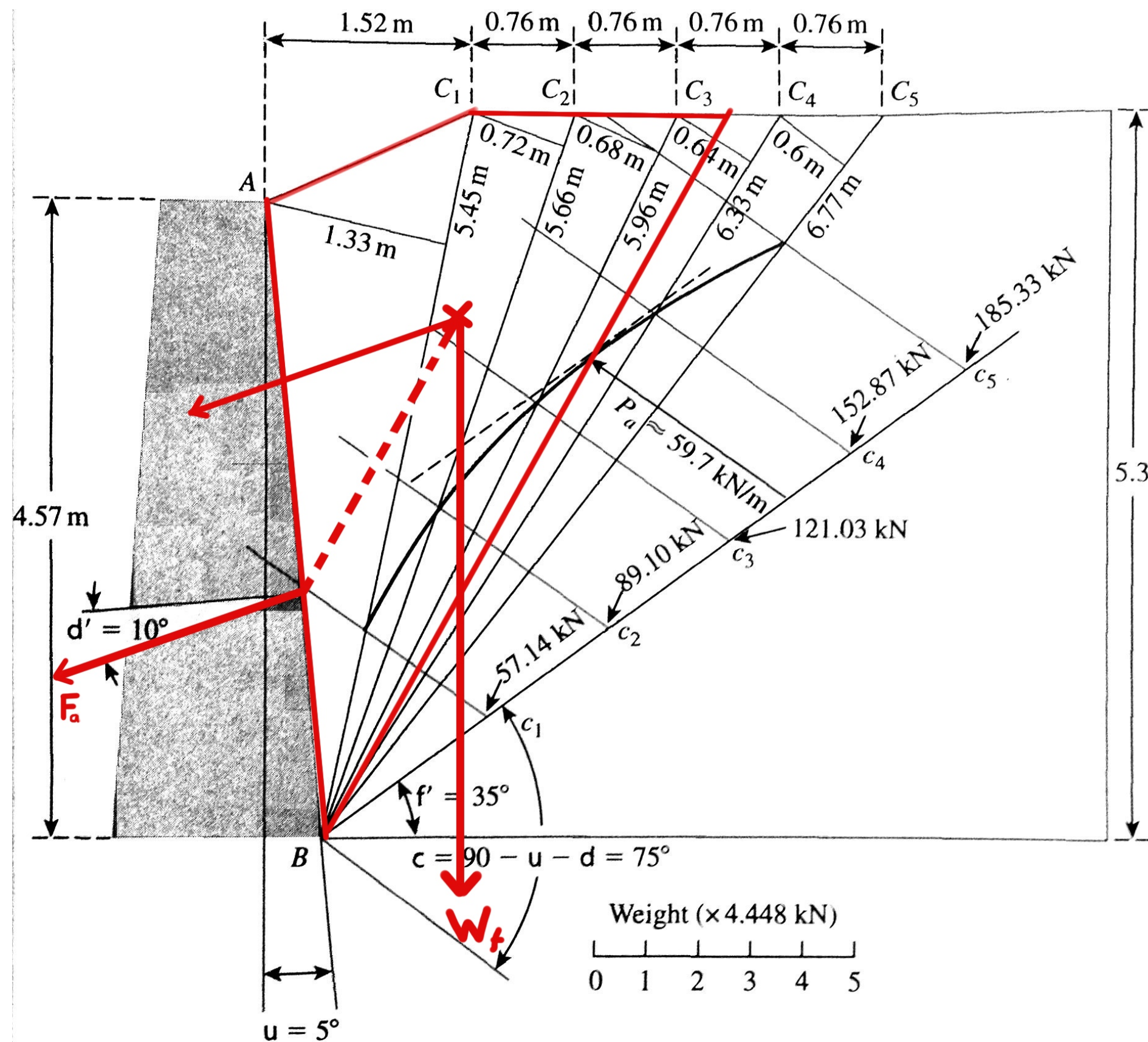
Step 8: Outline the failure wedge and draw another directed line along the ϕ' line to represent the weight of the failure wedge.



The two vectors now show a force balance between the weight of the failure wedge and the combined shear-normal forces on the slip face.

5.33 m
The vectors are rotated such that the (vertical) weight force lies on the ϕ' line. To resolve the vectors as forces, we only need to rotate and position the vectors at the centroid of the failure wedge.

Step 9: Rotate and translate the force balance vectors such that the weight force is vertical and located at the centroid.



Lastly the active force vector is translated along a line parallel to the failure surface.

The point of intersection with the wall is approximately the line of application of the active force.

Theoretical Check

Coulomb's theoretical analysis of sloping frictional retaining walls can be used as a check on graphical or computational methods.

Active Pressure:

$$K_a = \frac{\cos^2(\phi' - \theta)}{\cos^2 \theta \cos(\delta' + \theta) \left[1 + \sqrt{\frac{\sin(\delta' + \phi') \sin(\phi' - \alpha)}{\cos(\delta' + \theta) \cos(\theta - \alpha)}} \right]^2}$$

Passive Pressure:

$$K_p = \frac{\cos^2(\phi' + \theta)}{\cos^2 \theta \cos(\delta' - \theta) \left[1 - \sqrt{\frac{\sin(\phi' + \delta') \sin(\phi' + \alpha)}{\cos(\delta' - \theta) \cos(\alpha - \theta)}} \right]^2}$$

Where θ is the wall angle to the vertical, α is the backfill angle to the horizontal, and δ' is the wall friction. Note that this reduces to the simpler case (previous lecture) when $\alpha = \delta' = \theta = 0$. Passive and active forces are then, for homogenous backfill:

$$F_a = \frac{1}{2} K_a \gamma_b H^2 \quad F_p = \frac{1}{2} K_p \gamma_b H^2$$

Theoretical Check: Example

For the graphical example shown, $H = 4.57$ m, $\phi' = 35$, $\delta' = 10$, $\theta = 5$, $\alpha \sim 5$

Active Pressure:

$$K_a = \frac{\cos^2(\phi' - \theta)}{\cos^2 \theta \cos(\delta' + \theta) \left[1 + \sqrt{\frac{\sin(\delta' + \phi') \sin(\phi' - \alpha)}{\cos(\delta' + \theta) \cos(\theta - \alpha)}} \right]^2}$$

$$\frac{\text{Cos}(30)^2}{\text{Cos}(5)^2 \text{Cos}(15) \left(1 + \sqrt{\frac{\text{Sin}(45) \text{Sin}(30)}{\text{Cos}(15) \text{Cos}(0)}}} \right)^2} = 0.303$$

$$F_a = \frac{1}{2} K_a \gamma_b H^2 \quad \frac{1}{2} 0.303 (15.72) 4.57^2 = 49.7 \text{ kN/m}$$

cf 59.7 m ...

Culmann's Method: Granular Soils Only

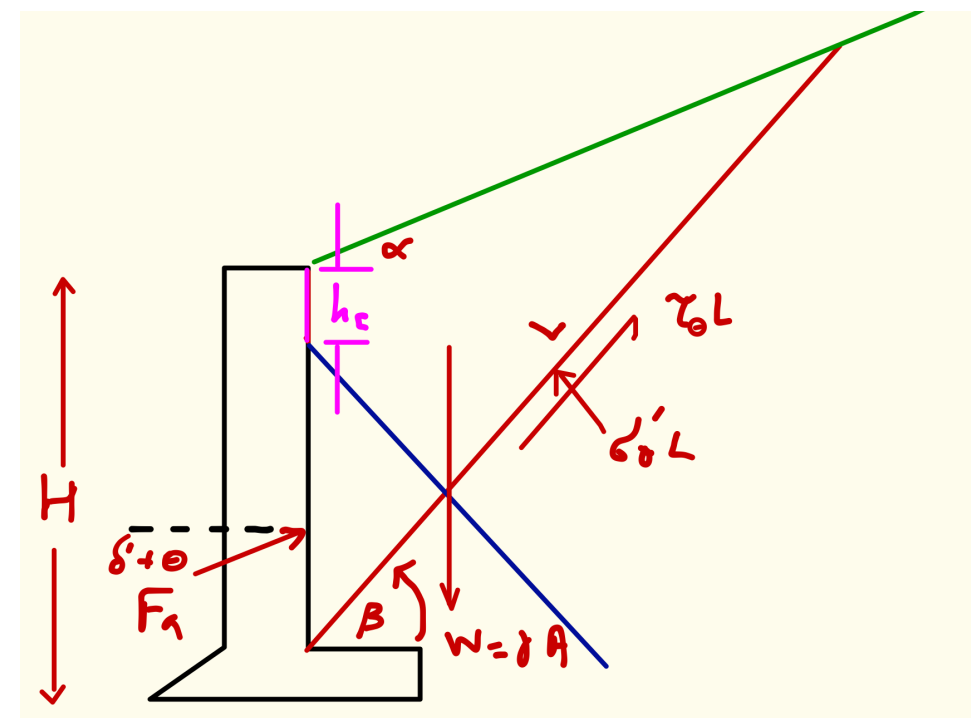
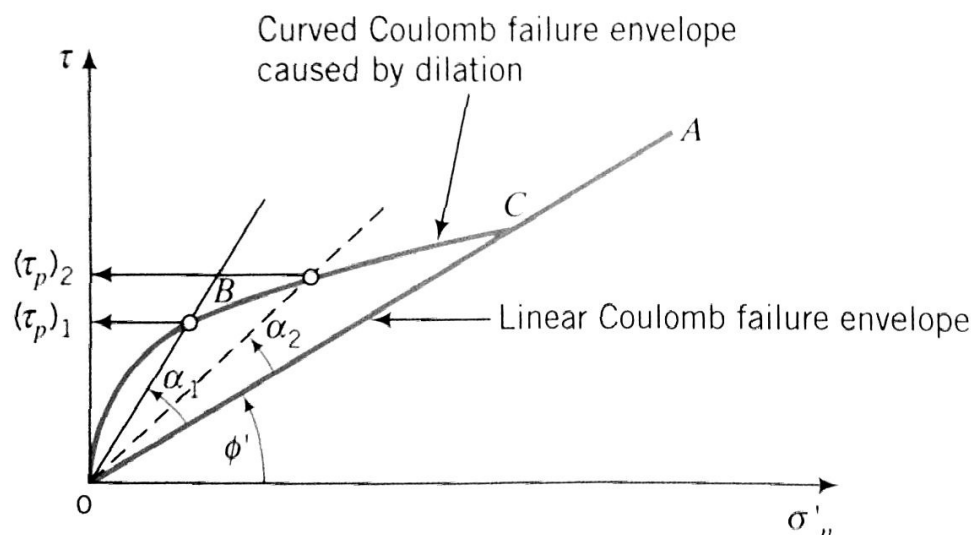
You may have noticed that this graphical method, powerful though it is, applies only to Type I soils with $c' = 0$.

Fortunately granular soils with $c' = 0$ are more suitable for retaining wall backfill than clay soils for a number of reasons:

- Granular soils are more easily drained, minimising the risk of a wall becoming saturated
- Granular soils are easier to grade for design and construction.

In many cases a dilating cohesive soil can be conservatively modelled as non-cohesive ($c' = 0$) by assuming a diagonal shear failure graph.

An advanced Culmann technique models a cohesive failure wedge with a tensile crack interrupting contact with the wall.



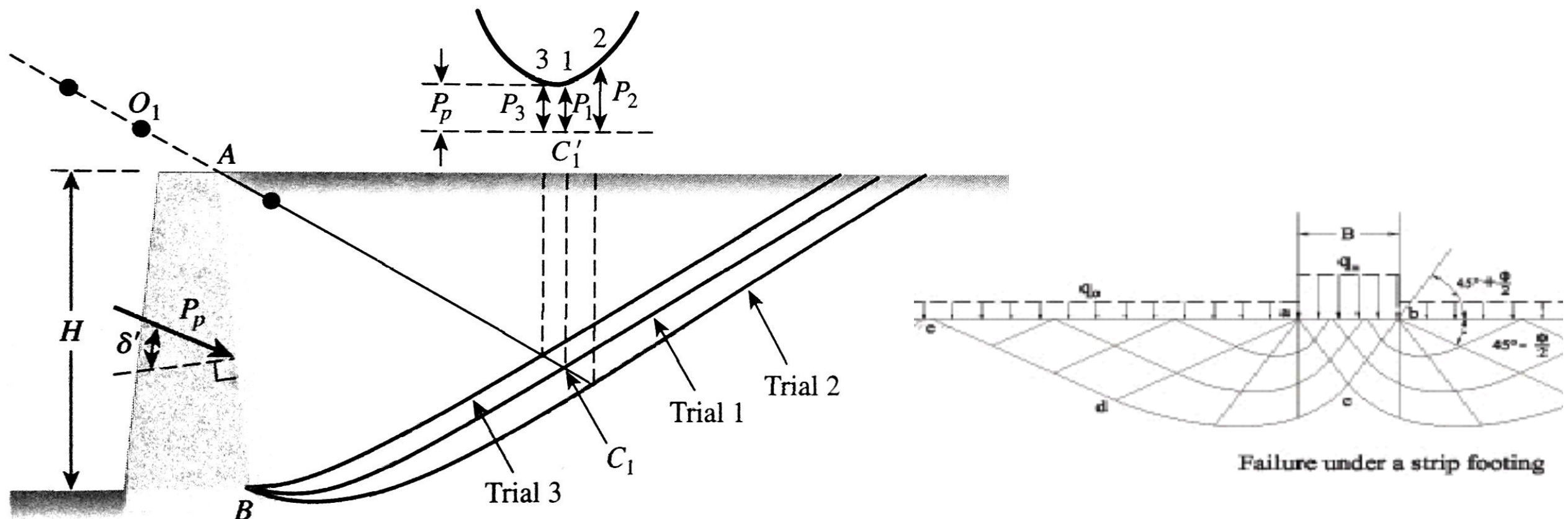
Culmann's Method: Active Pressure Only

You would also have noticed that this graphical method applies only to active pressure.

Passive pressure conditions are often found to have a curved failure surface, often approximated as a logarithmic spiral (see Das chapter 14)

The method for a curved passive retaining wall failure is beyond the scope of this unit, and indeed is not well represented even in the Australian Standard for earth retaining structures.

A simplified version of a numerical method (Zhu and Qian, 2000) is provided here...



Passive Pressure: Curved Failure Surface

Zhu and Qian's (2000) method uses the *Method of Triangular Slices*. We'll cover this in more detail for slope stability in the next section.

The method uses a modifier equation with numerically derived values that are easily represented as tables:

$$K_p = K_{p(\delta=0)} R$$

Table 14.1 Variation of $K_{p(\delta'=0)}$ [see Eq. (14.12) and Figure 14.3a]*

ϕ' (deg)	θ (deg)						
	30	25	20	15	10	5	0
20	1.70	1.69	1.72	1.77	1.83	1.92	2.04
21	1.74	1.73	1.76	1.81	1.89	1.99	2.12
22	1.77	1.77	1.80	1.87	1.95	2.06	2.20
23	1.81	1.81	1.85	1.92	2.01	2.13	2.28
24	1.84	1.85	1.90	1.97	2.07	2.21	2.37
25	1.88	1.89	1.95	2.03	2.14	2.28	2.46
26	1.91	1.93	1.99	2.09	2.21	2.36	2.56
27	1.95	1.98	2.05	2.15	2.28	2.45	2.66
28	1.99	2.02	2.10	2.21	2.35	2.54	2.77
29	2.03	2.07	2.15	2.27	2.43	2.63	2.88
30	2.07	2.11	2.21	2.34	2.51	2.73	3.00
31	2.11	2.16	2.27	2.41	2.60	2.83	3.12
32	2.15	2.21	2.33	2.48	2.68	2.93	3.25
33	2.20	2.26	2.39	2.56	2.77	3.04	3.39
34	2.24	2.32	2.45	2.64	2.87	3.16	3.53
35	2.29	2.37	2.52	2.72	2.97	3.28	3.68
36	2.33	2.43	2.59	2.80	3.07	3.41	3.84
37	2.38	2.49	2.66	2.89	3.18	3.55	4.01
38	2.43	2.55	2.73	2.98	3.29	3.69	4.19
39	2.48	2.61	2.81	3.07	3.41	3.84	4.38
40	2.53	2.67	2.89	3.17	3.53	4.00	4.59
41	2.59	2.74	2.97	3.27	3.66	4.16	4.80
42	2.64	2.80	3.05	3.38	3.80	4.34	5.03
43	2.70	2.88	3.14	3.49	3.94	4.52	5.27
44	2.76	2.94	3.23	3.61	4.09	4.72	5.53
45	2.82	3.02	3.32	3.73	4.25	4.92	5.80

*Based on Zhu and Qian, 2000

Table 14.2 Variation of R [Eq. (14.12)]

θ (deg)	δ'/ϕ'	R for ϕ' (deg)			
		30	35	40	45
0	0.2	1.2	1.28	1.35	1.45
	0.4	1.4	1.6	1.8	2.2
	0.6	1.65	1.95	2.4	3.2
	0.8	1.95	2.4	3.15	4.45
	1.0	2.2	2.85	3.95	6.1
5	0.2	1.2	1.25	1.32	1.4
	0.4	1.4	1.6	1.8	2.1
	0.6	1.6	1.9	2.35	3.0
	0.8	1.9	2.35	3.05	4.3
	1.0	2.15	2.8	3.8	5.7
10	0.2	1.15	1.2	1.3	1.4
	0.4	1.35	1.5	1.7	2.0
	0.6	1.6	1.85	2.25	2.9
	0.8	1.8	2.25	2.9	4.0
	1.0	2.05	2.65	3.6	5.3
15	0.2	1.15	1.2	1.3	1.35
	0.4	1.35	1.5	1.65	1.95
	0.6	1.55	1.8	2.2	2.7
	0.8	1.8	2.2	2.8	3.8
	1.0	2.0	2.6	3.4	4.95
20	0.2	1.15	1.2	1.3	1.35
	0.4	1.35	1.45	1.65	1.9
	0.6	1.5	1.8	2.1	2.6
	0.8	1.8	2.1	2.6	3.55
	1.0	1.9	2.4	3.2	4.8

θ is the wall angle to the vertical and δ' is the wall friction. More complex cases with sloping backfill are not required in passive failure models

Readings

Chapter 13 Das;

Geotechnical Information and Resources:

http://www.geotechnicalinfo.com/retaining_walls_technical_guidance.html

