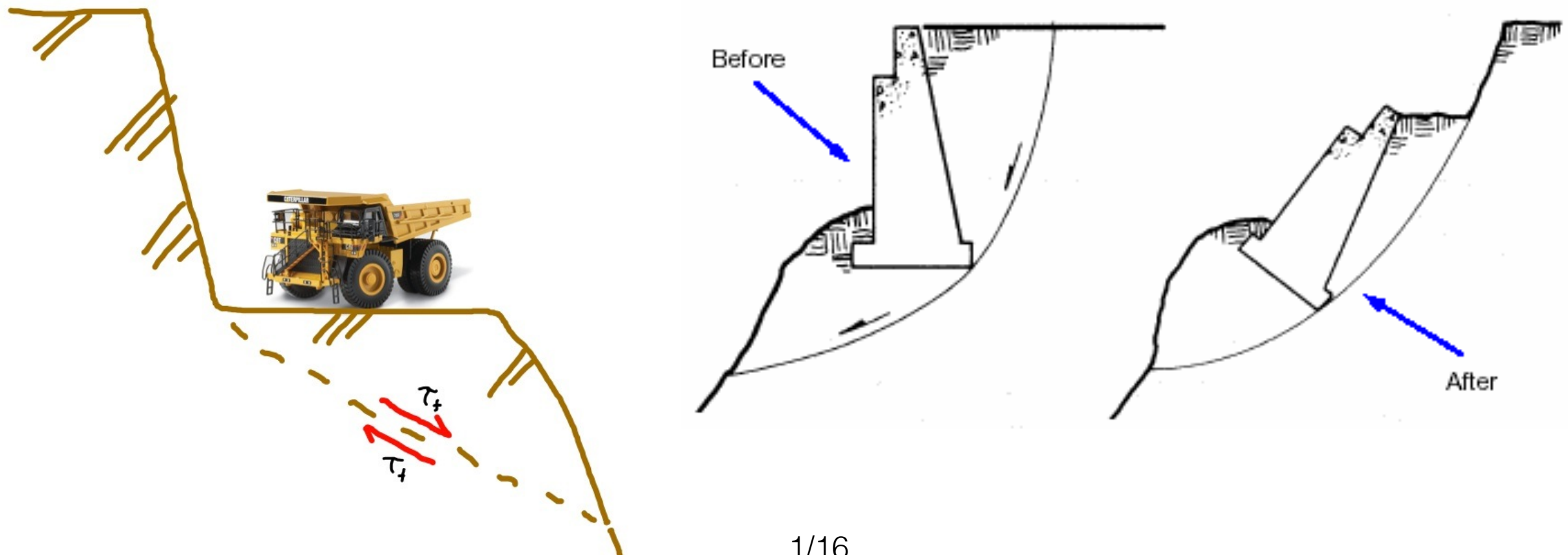


Shear Strength of Soil

Text: Das, 2010. Chapter 12

While excess load can cause serviceability problems due to soil settlement, more serious structural soil failures generally occur due to shear stress and *sliding* failure rather than direct compression.

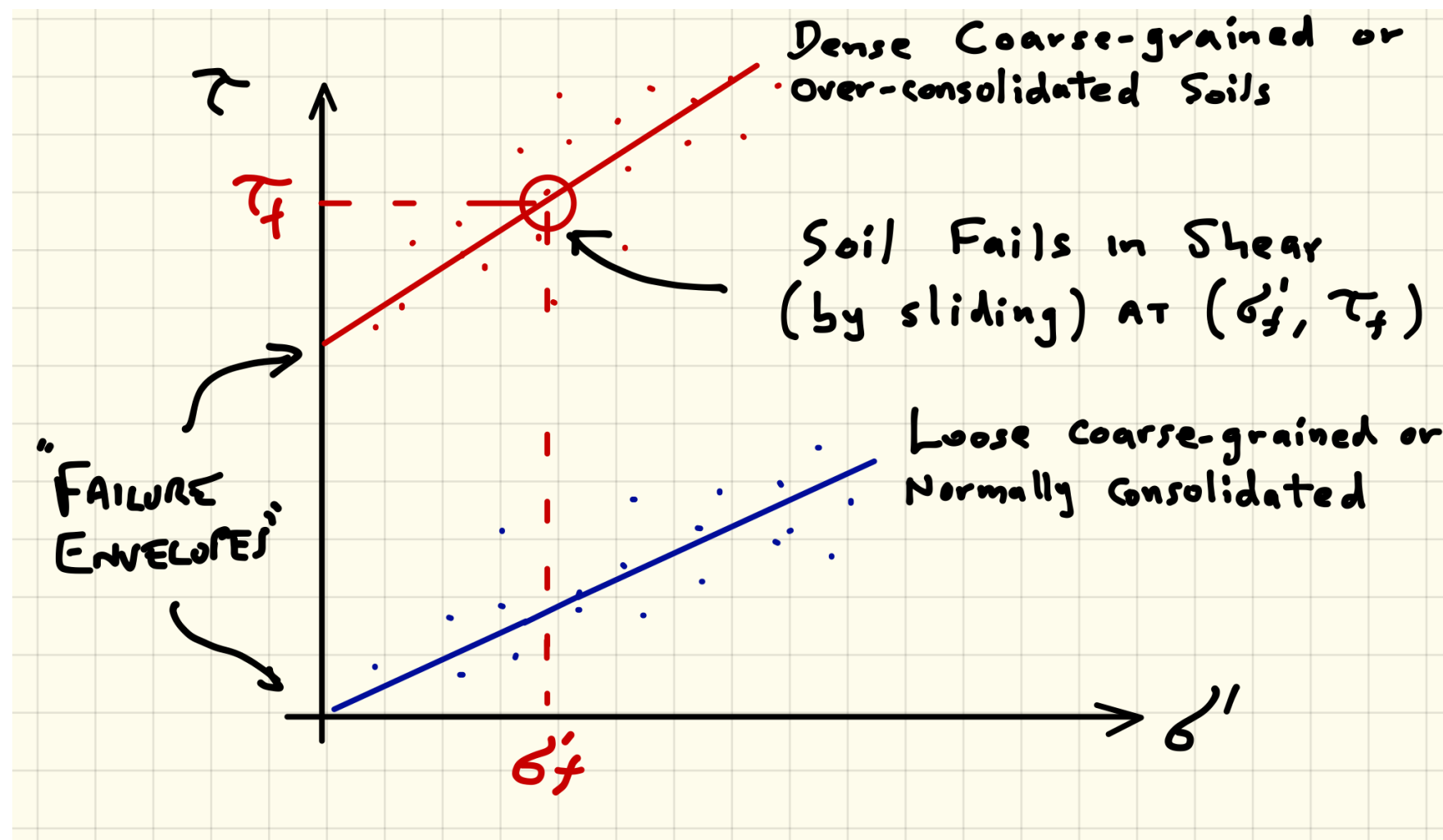
Assessments of soil strength are therefore mainly concerned with shear strength due to internal friction and cohesion, that resist shear stress through interlocking of grains and development of plastic shear strength.



Mohr-Coulomb Failure Criterion

Coulomb (1776) developed a theory from observations of shear failure under different combinations of normal and shear stresses.

Experiments showed that the shear strength of a soil increases with increasing normal stress:

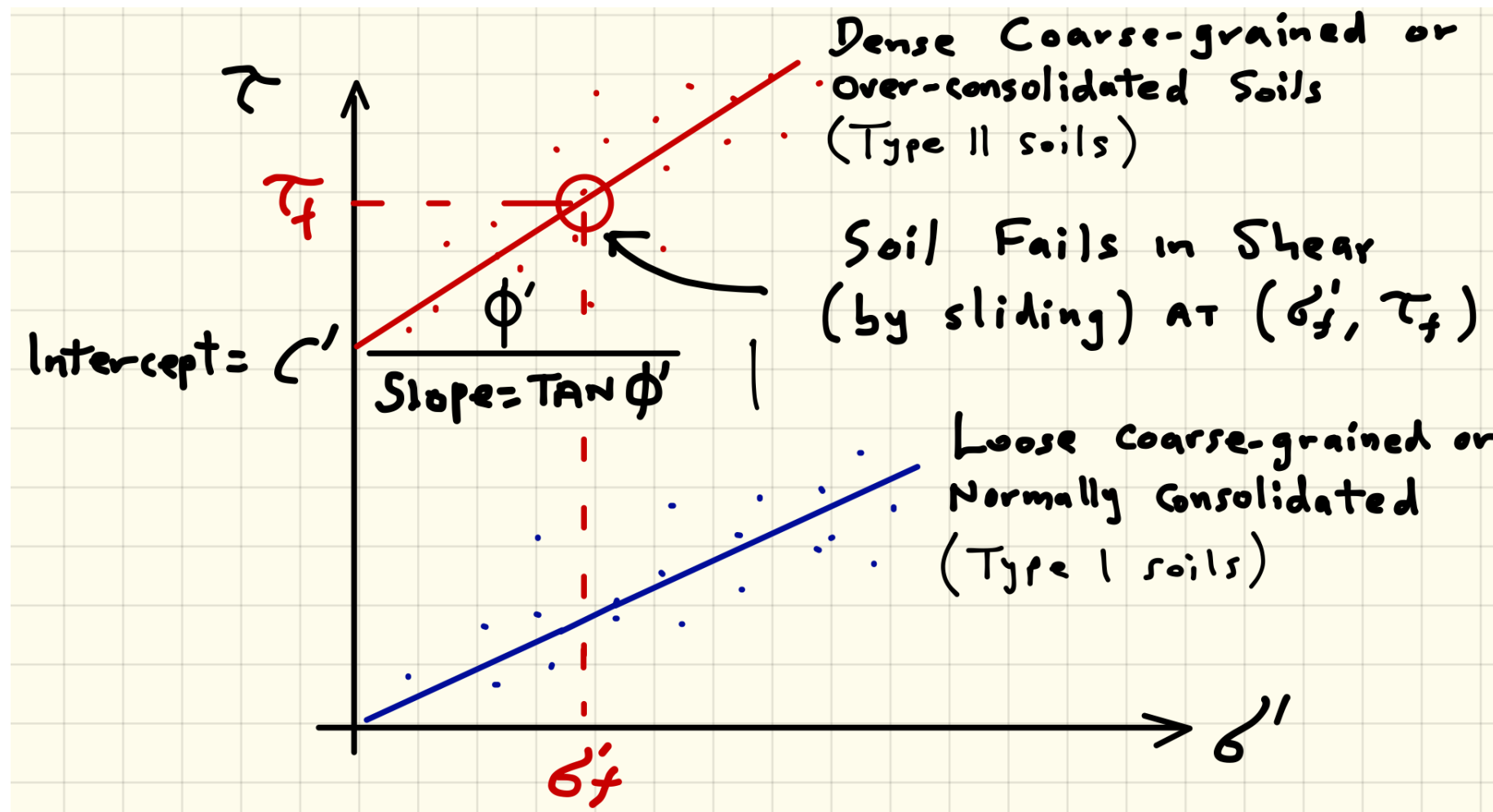


All we need is the equation of a straight line:

$$\tau_f = c' + \sigma'_f \tan(\phi')$$

where c' and ϕ' are constant parameters of particular soils

Mohr-Coulomb Failure Criterion



$$\tau_f = c' + \sigma'_f \tan(\phi')$$

c' is known as the *cohesion* of the soil.

ϕ' is the *friction angle*.

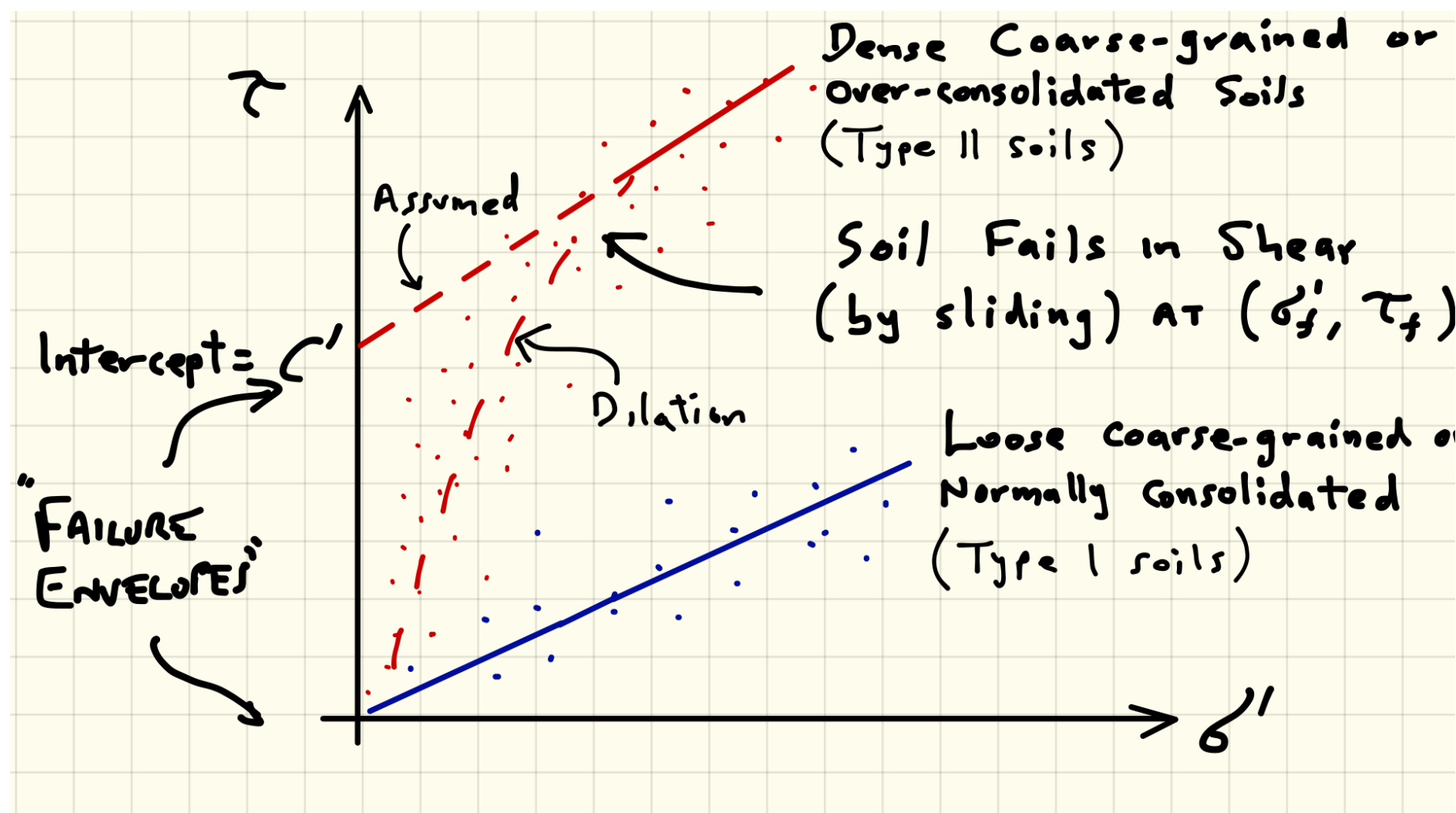
Loosely packed non-cohesive soils and normally consolidated clays (**Type I soils**) have effectively zero shear strength unless a normal stress is applied. Normal stress causes interlocking of grains and confinement of pore water, which tend to increase the friction between particles.

Densely packed coarse-grained (nominally cohesionless) soils and overconsolidated clays (**Type II soils**) have non-zero shear strength even when no normal stress is applied. This is because densely packed grains have a degree of interlocking even under zero normal stress, and overconsolidated clays are in a state of confined residual stress roughly equivalent to grain interlocking.

Mohr-Coulomb Failure Criterion

More recent evidence shows that in many if most soils the cohesiveness c' is an artefact of the data.

An initial rapid increase in shear strength under increasing normal load indicates *dilation* (covered later in the unit) rather than cohesion: this looks like a constant intercept and can be modelled as such, but the "true value" of the intercept cannot be measured as it often does not exist.



$$\tau_f = c' + \sigma'_f \tan(\phi')$$

Mohr-Coulomb Failure Criterion

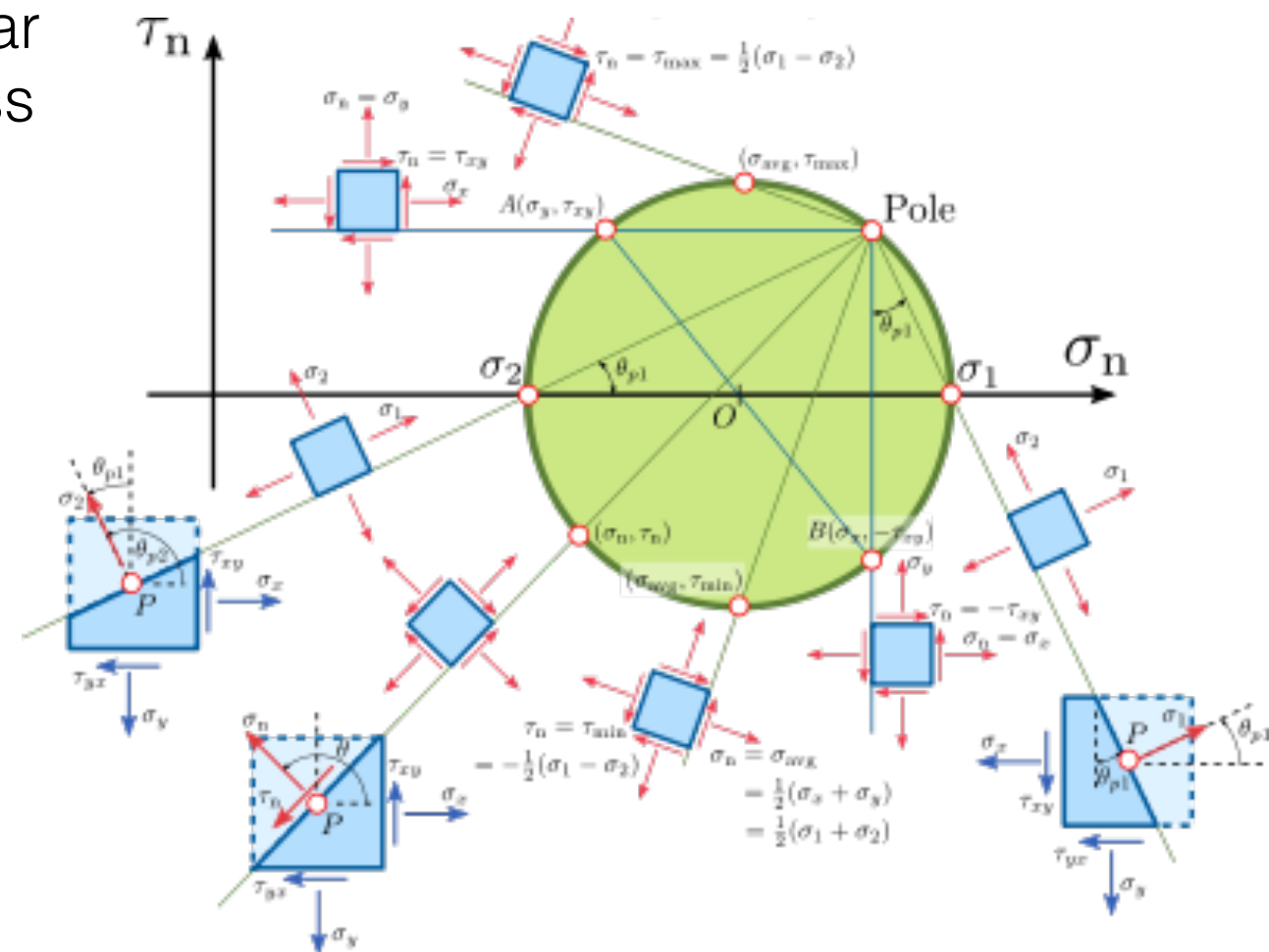
Mohr (1900) generalised the idea that stress failure is due to combined normal and shear stresses, in all types of structural materials, not just soil.

Mohr identified combinations of stresses on planes oriented at arbitrary angles within the body of a solid under stress. Normal and shear stresses depend not only on the applied stress but also on the direction of a plane surface under consideration.

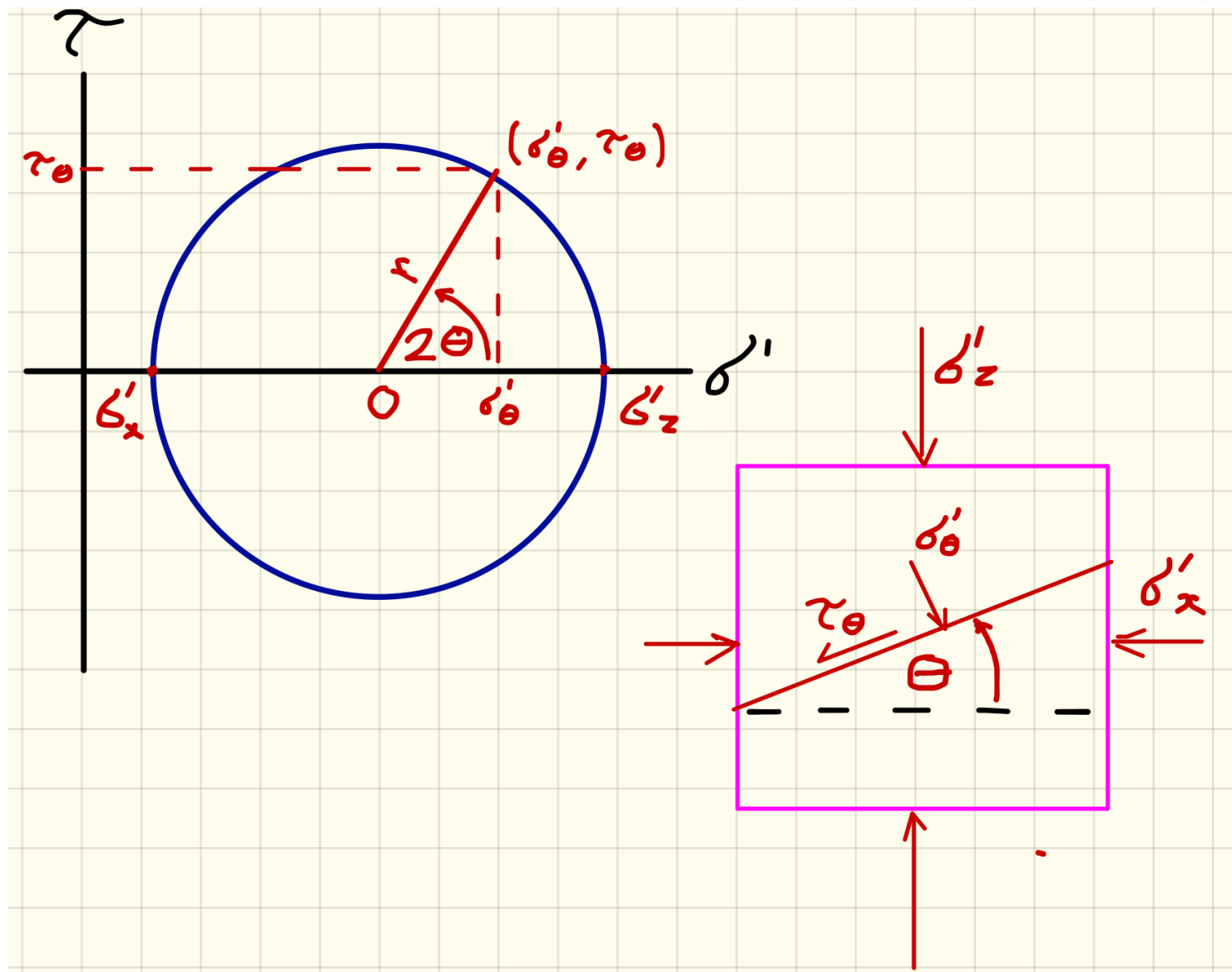
A specimen will fail along the plane where stresses are largest and hence first to exceed the specimen's strength.

The result (derived in structural mechanics) provides a circular geometry which is easy to work with so long as you stick to certain conventions.

MOHR'S CIRCLE:



Mohr's Circle



A soil element (pink square) is subjected to vertical and horizontal effective stresses, σ'_z and σ'_x

By convention σ'_z and σ'_x are positive in compression, and $\sigma'_z > \sigma'_x$ ie vertical compression is dominant

At any angle θ to the horizontal, equilibrium requires combinations of shear and normal stress that balance the vertical and horizontal applied forces.

It can be shown (see structural mechanics) that the result is a circular graph with radius r and centre o as illustrated.

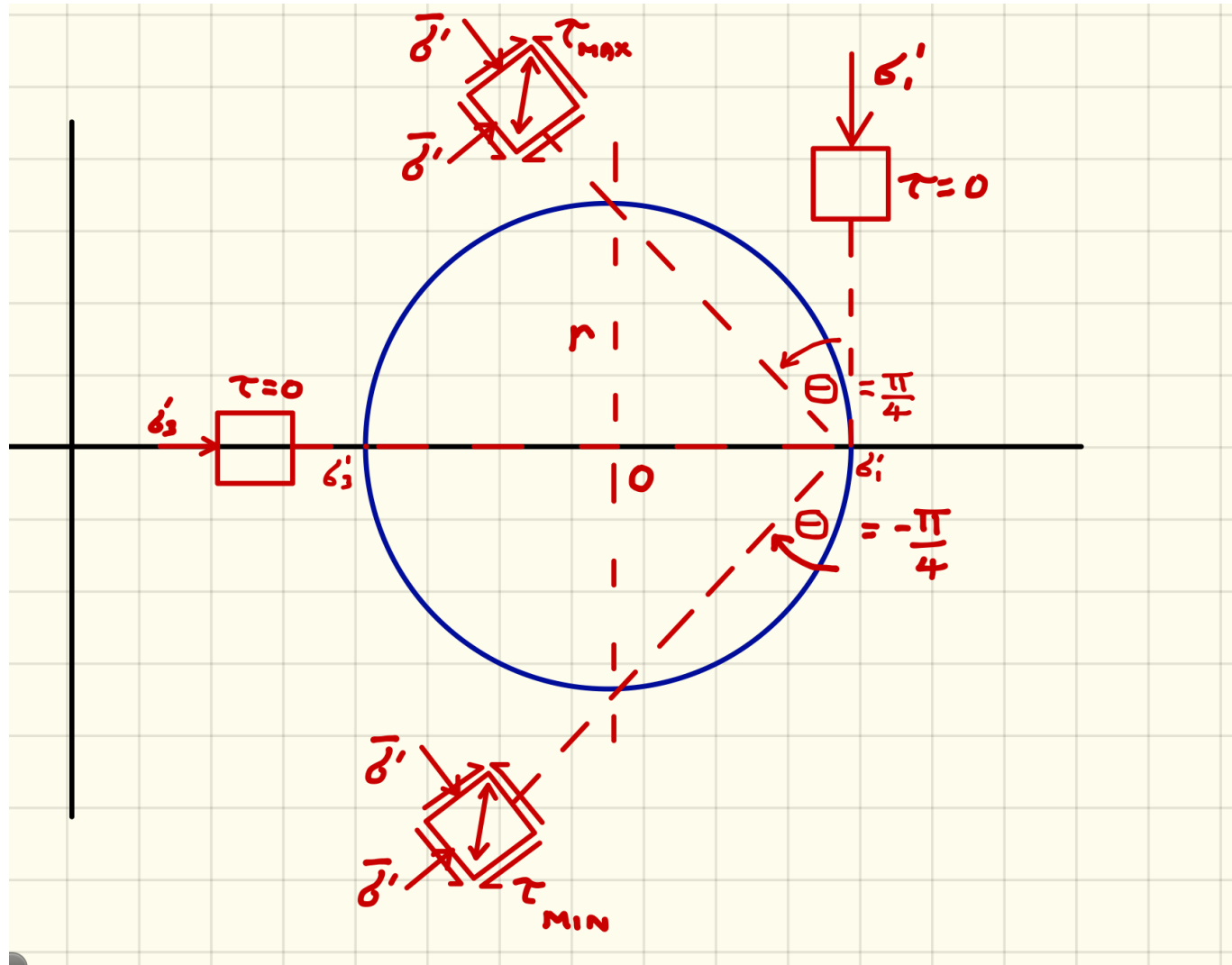
$$r = \frac{\sigma'_z - \sigma'_x}{2} \quad o = \frac{\sigma'_z + \sigma'_x}{2}$$

$$\sigma'_\theta = o + r \cos(2\theta) \quad \tau_\theta = r \sin(2\theta)$$

The resulting construction allows us to calculate normal and shear stresses at any angle in a stressed solid.

Note that the maximum shear stress is $\tau_{\max} = r$. In a soil specimen, this is *not* the failure stress

Mohr's Circle



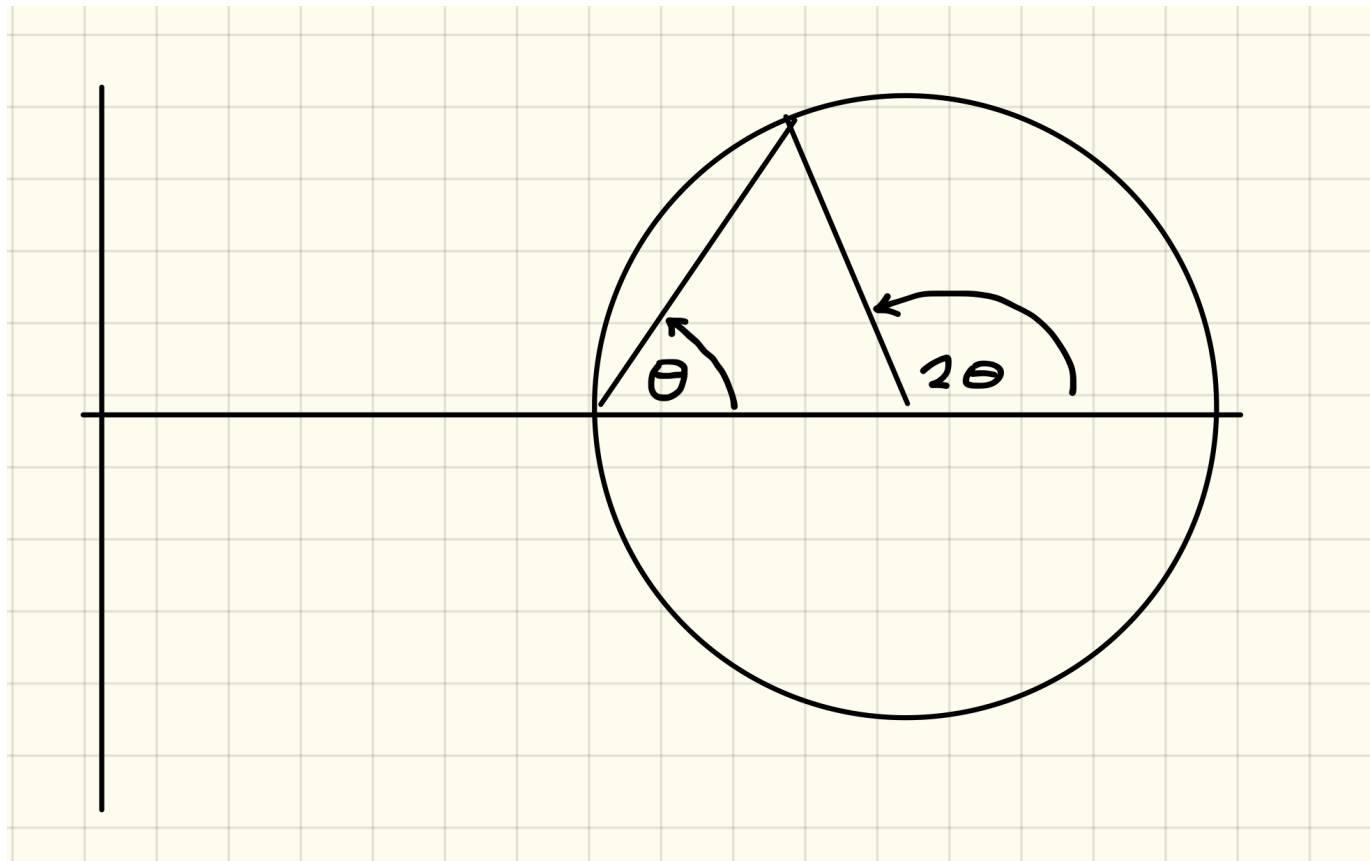
An alternative construction shows the orientation of soil elements at angles associated with particular levels of normal and shear stress.

Assuming zero shear stress on the horizontal-vertical axes the result gives maximum +ve and -ve shear stresses on the 45 and -45 degree diagonal planes.

The same planes have an equal mean compression on adjacent faces. The mean stress is as defined, the centre of the circle at o.

Again, in the case of soil, this is not the failure stress...

Mohr's Circle



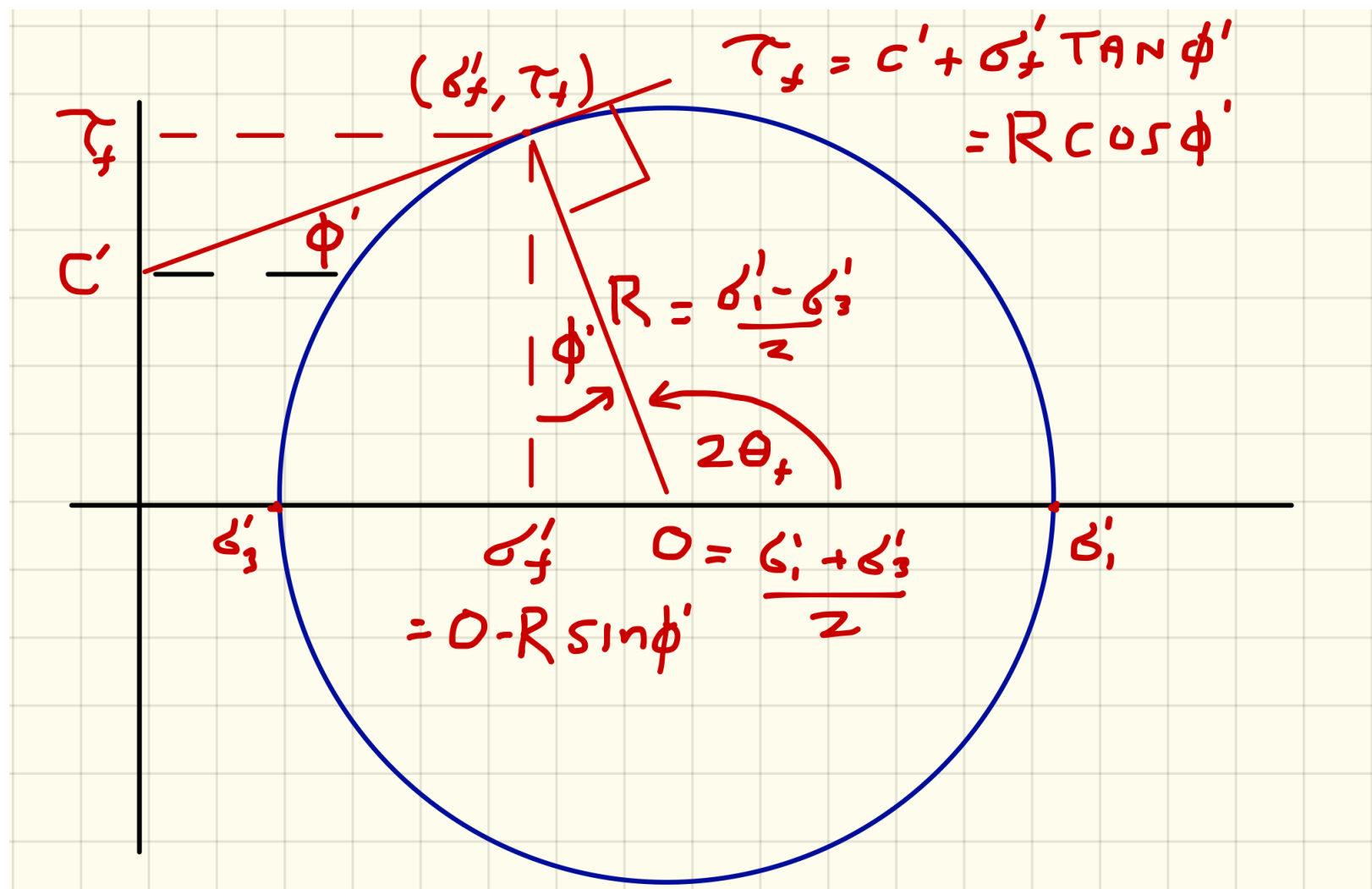
An even simpler construction shows the angle θ directly : measured from the minor principal stress rather than the centre, the angle to a given plane is geometrically to scale.

Mohr-Coulomb Failure Criterion

The combination of Mohr and Coulomb theories arises when we vary the horizontal and vertical stresses to the point of soil failure.

Mohr's circle expands outwards as σ'_z is increased or σ'_x is decreased to failure at vertical and horizontal stresses σ'_1 and σ'_3 . Soils must remain in compression (cannot support tensile forces).

When the circle intersects the failure envelope, soil failure occurs on the *shear failure plane* or *slip plane* at an angle θ_f . The geometry of this situation is surprisingly simple:



The circle intersects the line at a tangent; the radius is thus at a right angle to the envelope, with the failure angle defined by:

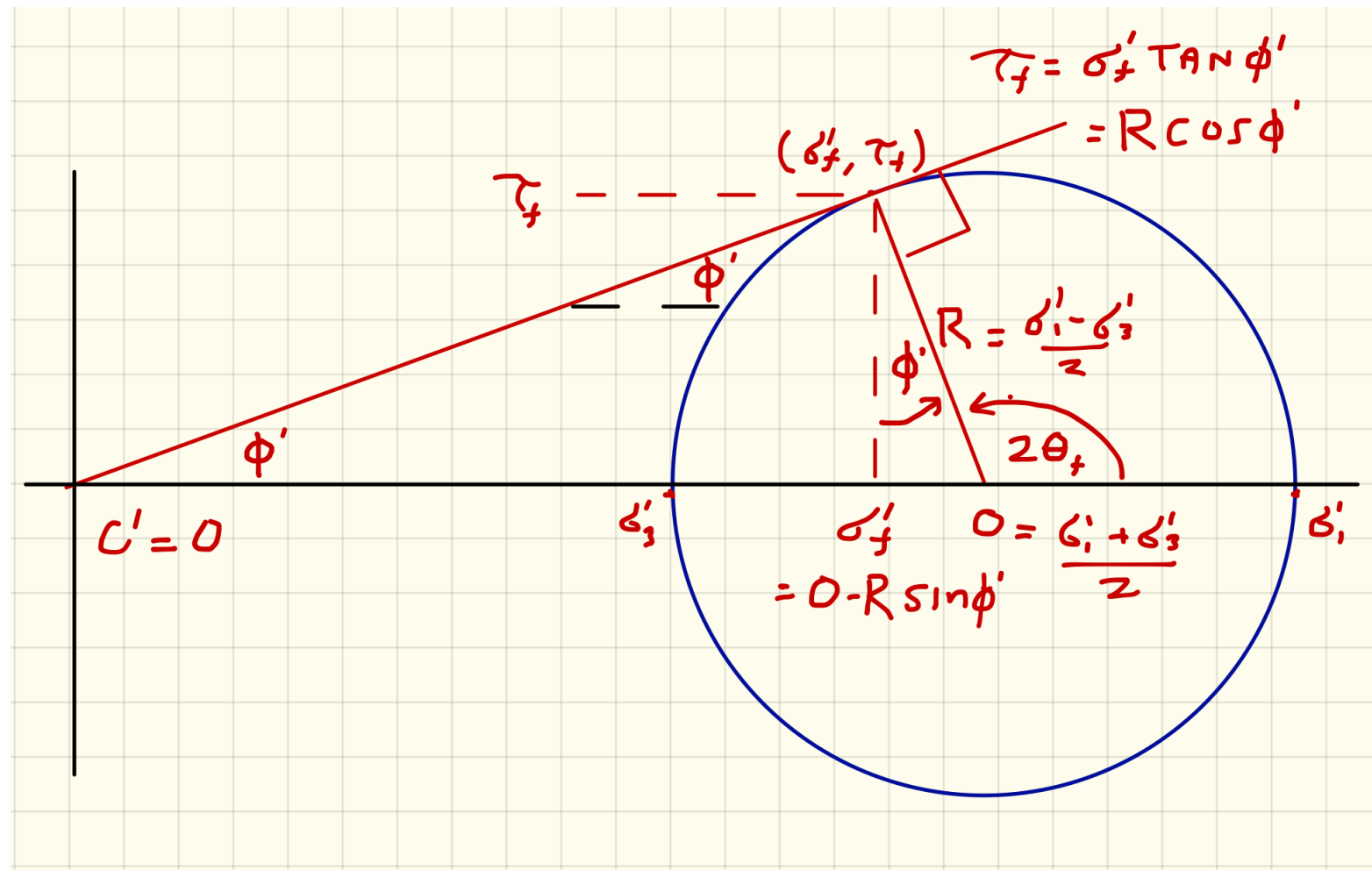
$$2\theta_f = \frac{\pi}{2} + \phi'$$

$$\theta_f = \frac{\pi}{4} + \frac{\phi'}{2}$$

In general, soil failure occurs on a *slip plane* with $\theta_f > 45$ degrees..

Mohr-Coulomb Failure Criterion

Cohesionless soils have a similar solution, but with limits on the range of vertical and horizontal stresses.



Cohesionless soils require some level of lateral stress to bind against shear stress, ie they require lateral restraint: $\sigma'_3 > 0$ (Type I soils)

In either type I or type II soils the failure mode depends on specific combinations of stress, cohesion and friction:

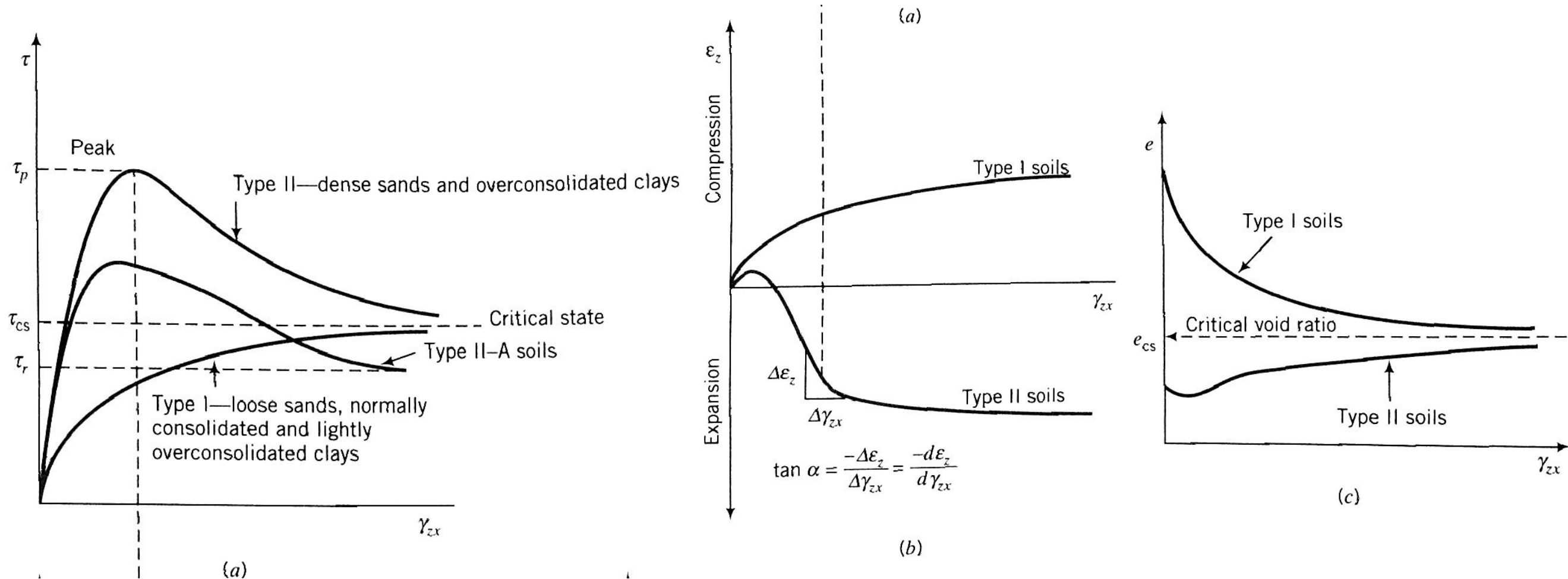
$$\tau_f = c' + \sigma'_f \tan(\phi') = R \cos(\phi')$$

$$c' + \left(\frac{\sigma'_1 + \sigma'_3}{2} - \frac{\sigma'_1 - \sigma'_3}{2} \sin(\phi') \right) \tan(\phi') = \frac{\sigma'_1 - \sigma'_3}{2} \cos(\phi')$$

For a given soil this provides a unique relationship between vertical and horizontal failure stresses σ'_1 and σ'_3 . Vertical stress cannot exceed a certain maximum, and lateral stress cannot be less than a minimum. For Type II soils the minimum is zero, but in type I soils the minimum is a positive compression or restraining stress. From the previous section on soil stresses we saw a relationship between vertical and lateral stresses depending on the type of load, which provides a further control on the failure point.

Soil Failure: Stress-Strain Diagrams

Soil shear failure can be represented by a) stress strain diagram, b) volume expansion diagram and c) void ratio-strain diagram.

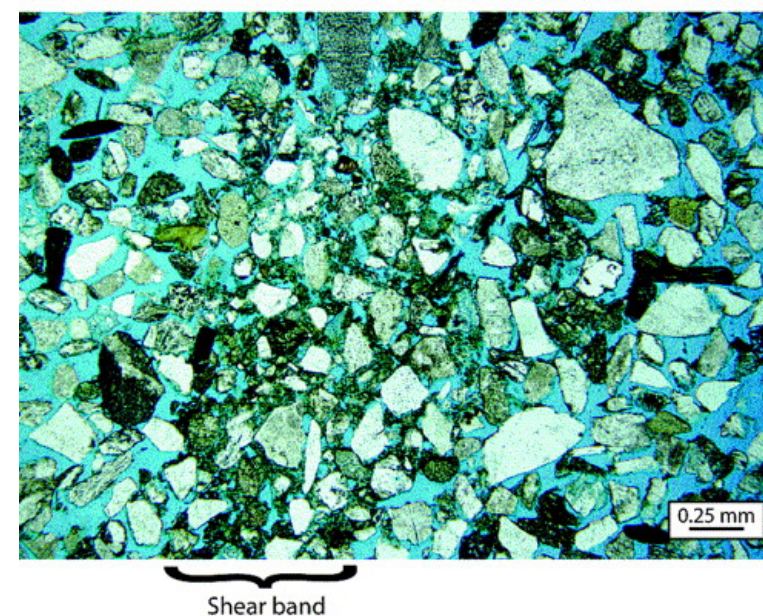
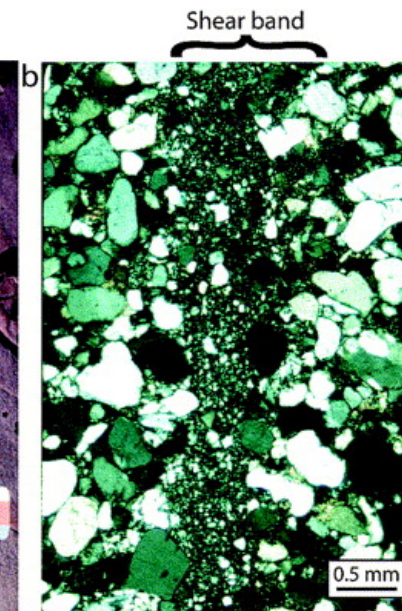
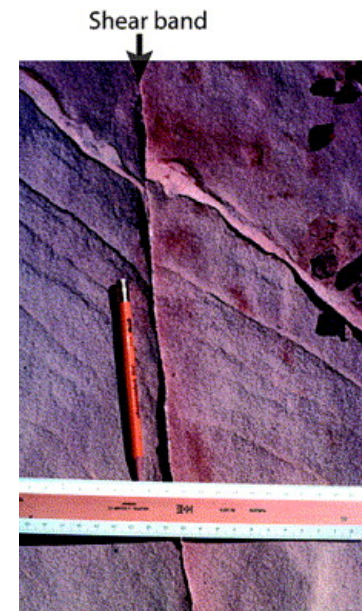
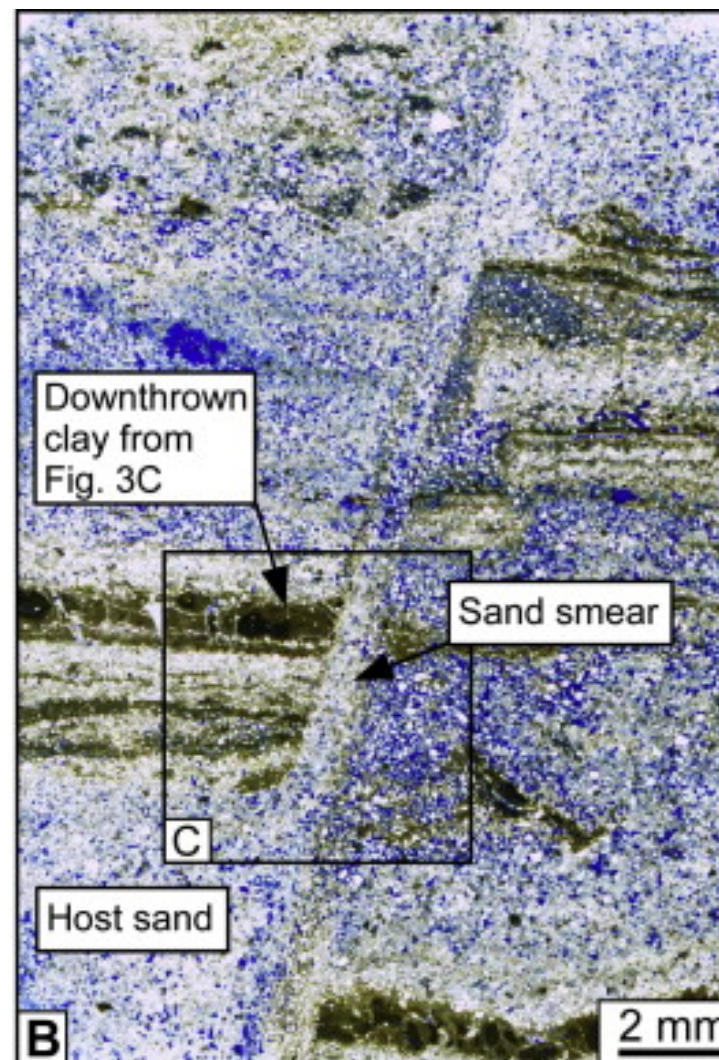
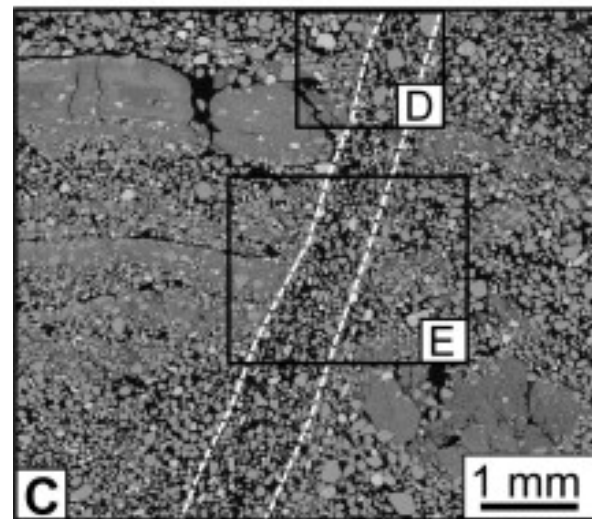
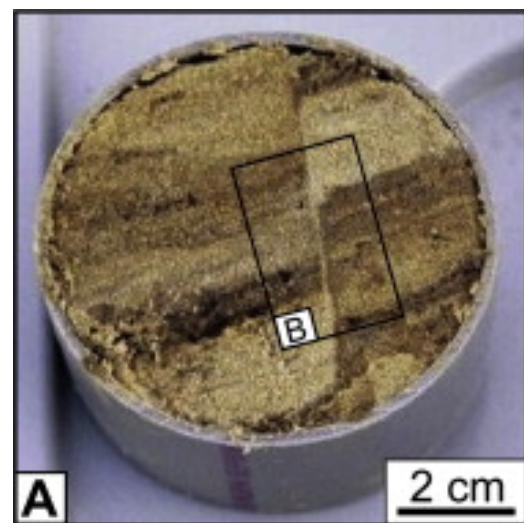


Type I soils: Shear produces compression, a decreasing void ratio and strain-hardening up to critical state failure.

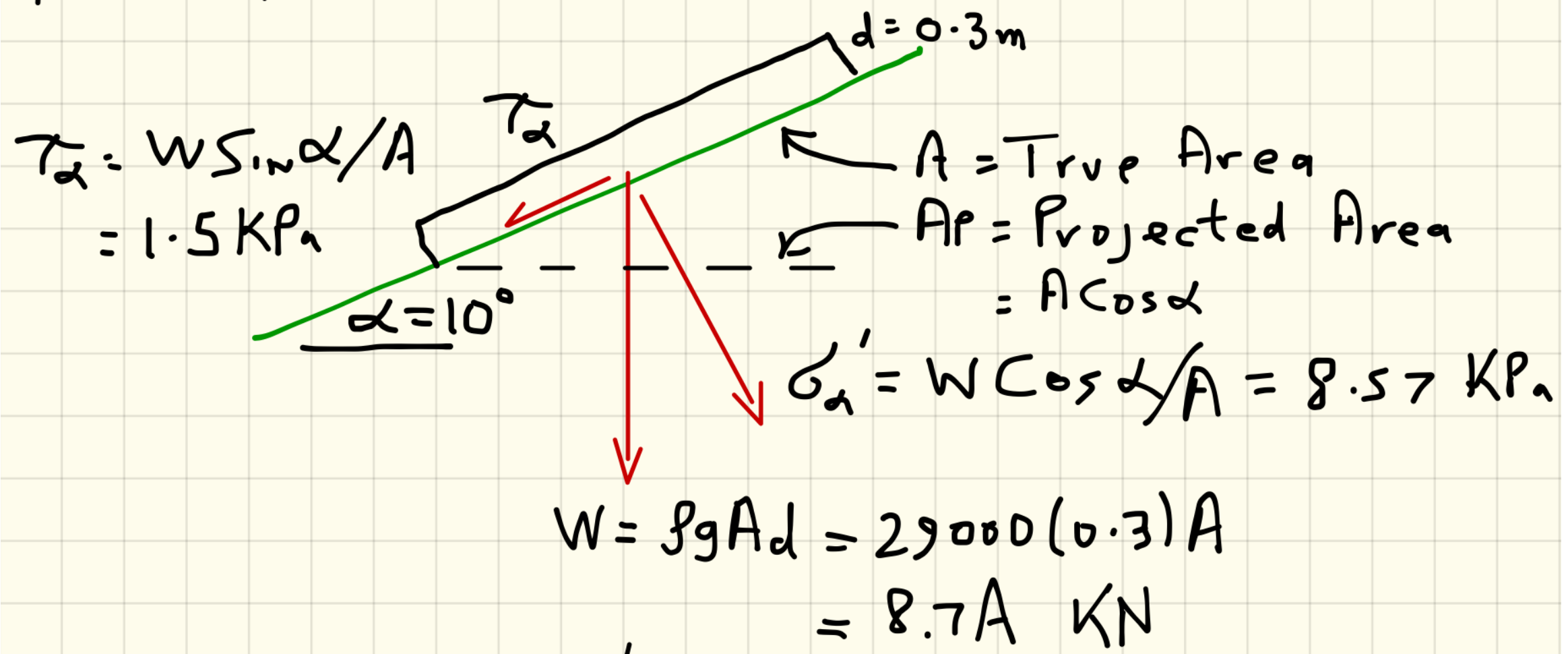
Type II soils : Shear results in volume expansion (after an initial small compression), with a peak shrear stress before failure (with shear softening) at critical state.

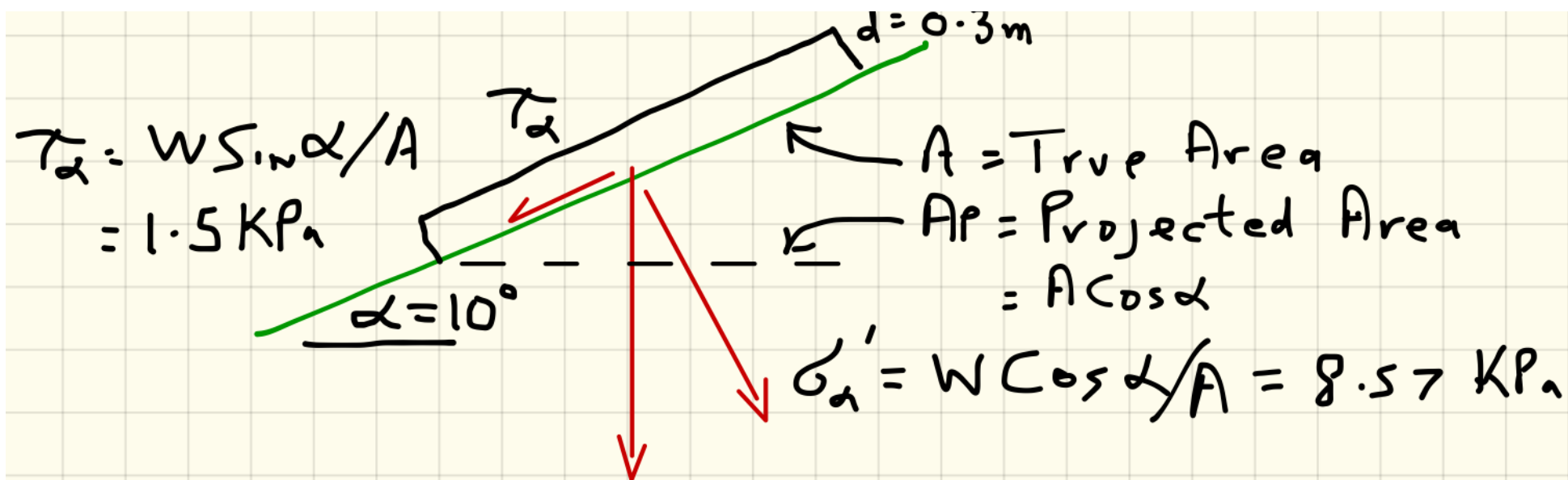
Soil Failure: Shear Bands

Soil shear failure is often accompanied by the formation of shear bands: layers of shear-failed plastic soil between layers of elastic soil.



Example: A concrete slab of density 2900 Kg/m^3 is constructed to 300 mm thickness on ground with a 10° slope. What is the peak shear stress? Given a cohesion $c' = 4 \text{ KPa}$ and friction angle $\phi' = 17^\circ$, will the soil fail?

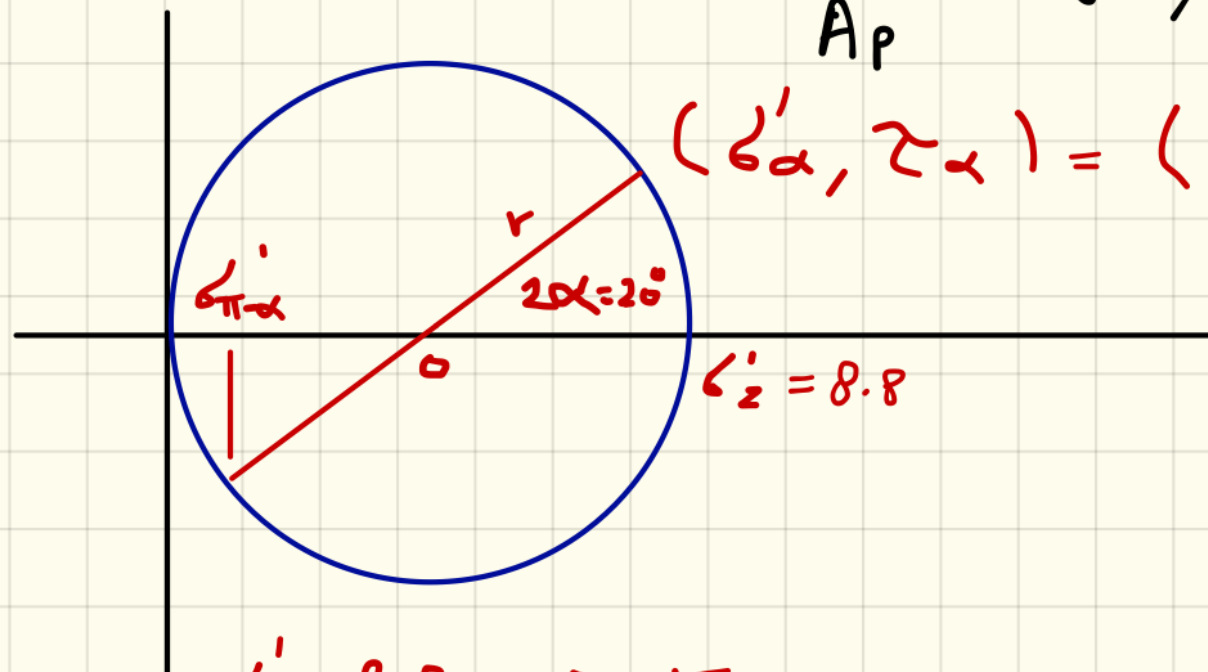




$$W = \rho g A d = 29000 (0.3) A$$

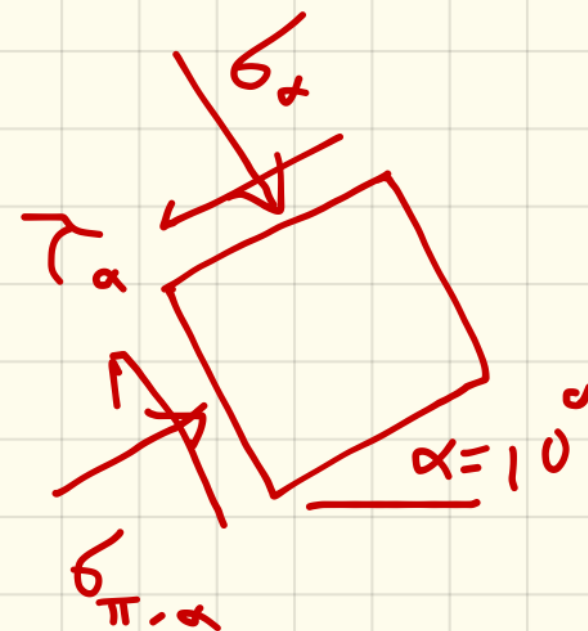
$$= 8.7 A \text{ KN}$$

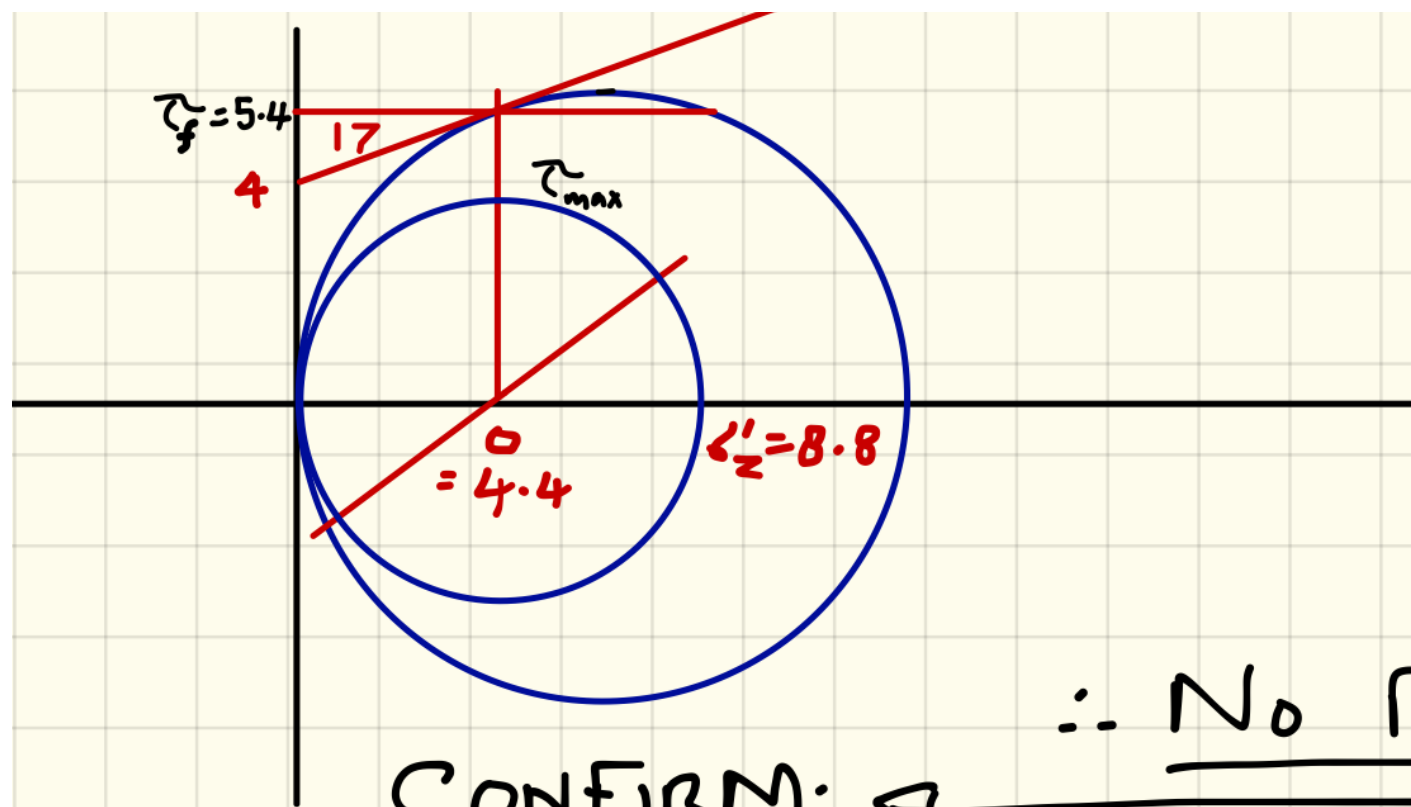
$$\sigma'_z = \frac{W}{A_p} = 8.7 / \cos \alpha = 8.8 \text{ KPa}$$



$$\sigma'_z = 8.8 = 0 + r$$

$$\sigma'_\alpha = 8.57 = 0 + r \cos 2\alpha$$





$$\begin{aligned}
 \tau_f &= c' + \sigma \tan 17 \\
 &= 4 + 4.4 \tan 17 \\
 &= \underline{\underline{5.4 \text{ kPa}}} \\
 &> \tau_{\max}
 \end{aligned}$$

∴ NO RISK OF FAILURE

CONFIRM: ←

$$: c' + \left(\frac{\sigma'_1 + \sigma'_3}{2} - \frac{\sigma'_1 - \sigma'_3}{2} \sin \phi' \right) \tan \phi' = \frac{\sigma'_1 - \sigma'_3}{2} \cos \phi'$$

$$c' + \frac{\sigma'_1}{2} (\tan \phi' - \sin \phi' \tan \phi' - \cos \phi') = \frac{\sigma'_3}{2} (\dots) = 0 \quad (\sigma'_3 = 0)$$

$$\sigma'_1 = \frac{2(-4)}{\tan 17 - \sin 17 \tan 17 - \cos 17} = \underline{\underline{10.8 \text{ kPa}}} > \sigma'_z$$

Readings

Chapter 12.1 -12.2 Das

