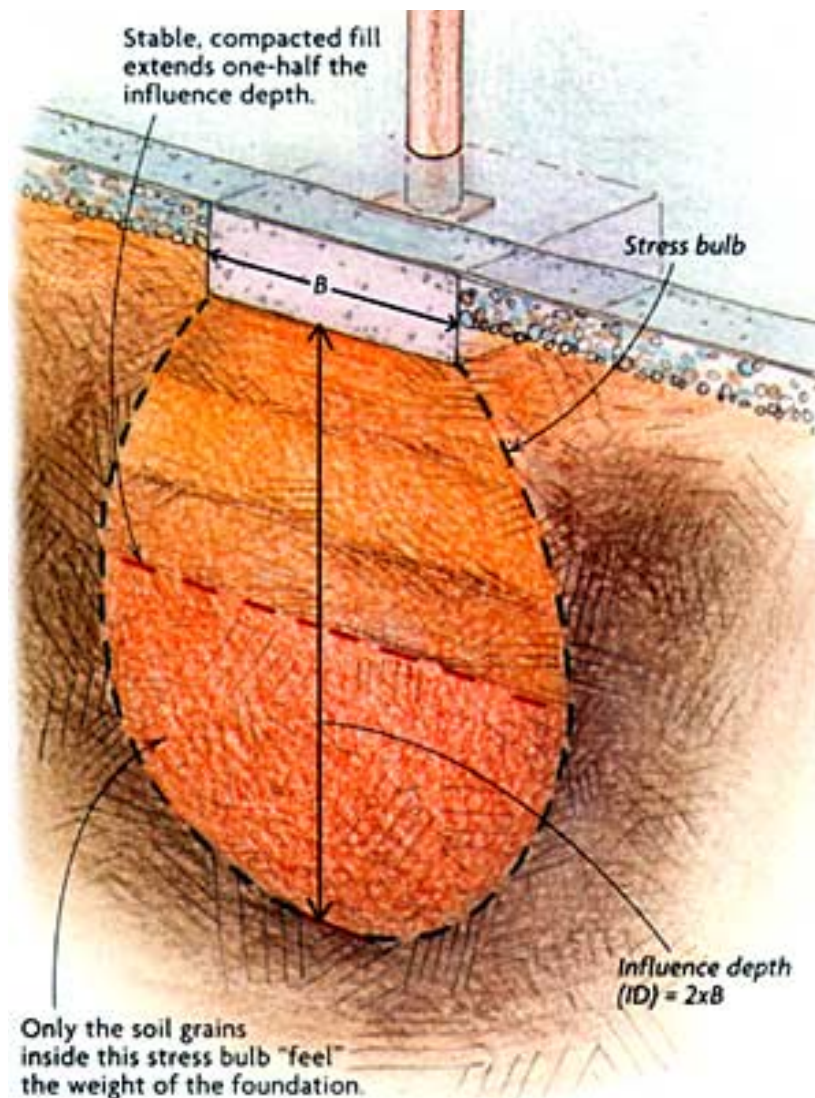


Soil Stresses

Text: Das, 2010. Chapters 9 and 10

We now have sufficient information to define stresses in soil due to self-weight in various states of saturation.

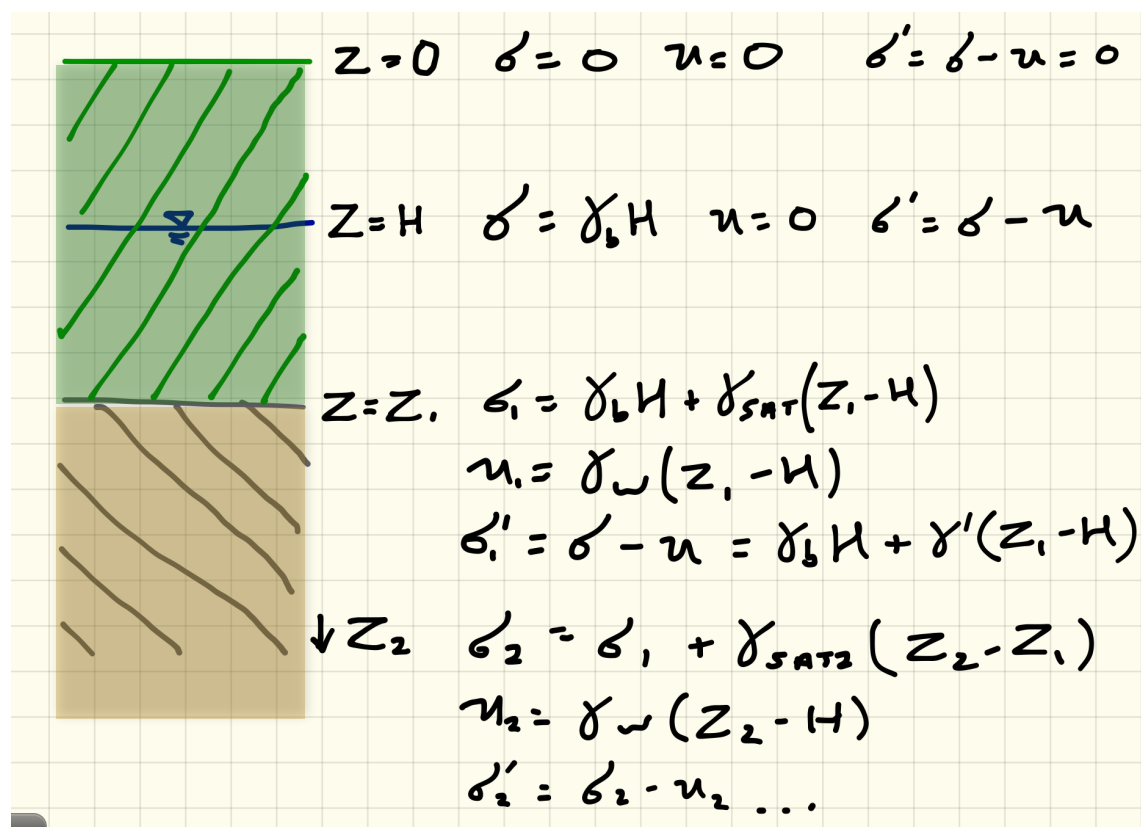
We can build on that to include stresses due to applied loads under structural foundations as well as overburden of imported or excavated soil.



Self-weight, Buoyancy and Seepage

A quick recap on soil stresses due to soil and water forces:

1. Hydrostatic: Assuming zero groundwater flow, pore water stresses are hydrostatic and no seepage forces are present:



Soil layers of varying bulk density γ_b and saturation density γ_{sat} overlay to produce total and effective stresses σ and σ' related by the pore water pressure u .

The effective stress σ' represents the mean stress between soil particles as a measure of the effect of self-weight on the bearing capacity of soil particles.

As with hydrostatic pressure the depth and hence pressure (stress) increase in the downward direction.

Each soil layer of distinctly different bulk density must be treated as a separate integration step. Only the pore pressure increases at a uniform rate.

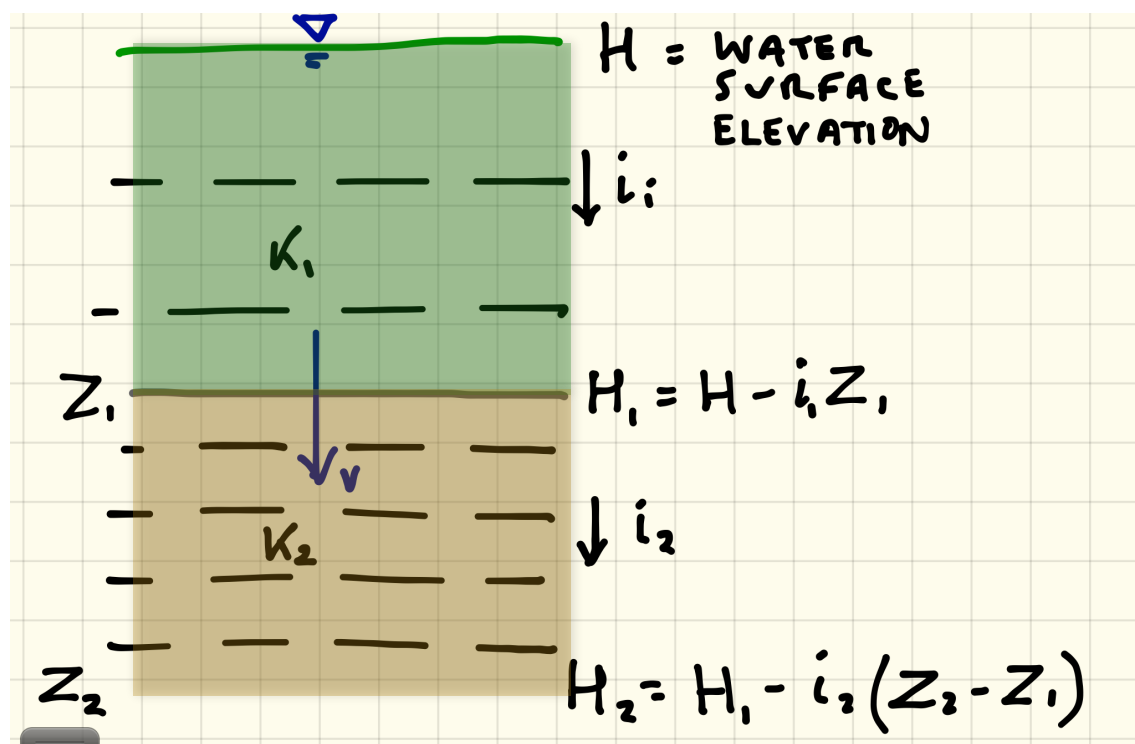
$$\gamma_{sat} = \gamma_w \frac{G + e}{1 + e}$$

$$\gamma' = \gamma_{sat} - \gamma_w = \gamma_w \left(\frac{G - 1}{1 + e} \right)$$

Self-weight, Buoyancy and Seepage

A quick recap on soil stresses due to soil and water forces:

2. Seepage: With downward or upward seepage the effective stress increases or decreases due to a hydraulic gradient imposing drag forces on particles.



Soil layers of varying hydraulic conductivity overlay to produce a sequence of hydraulic gradients.

The effective stress σ' is modified by converting the head loss to pressure loss and adding (or subtracting) this to the hydrostatic case:

$$\frac{dP}{dz} = \gamma_w \frac{dH}{dz} = \gamma_w i$$

$$dP = \gamma_w i dz \quad \text{or} \quad \Delta P = \gamma_w i \Delta z$$

$$\sigma' = \sigma - u \pm \Delta P$$

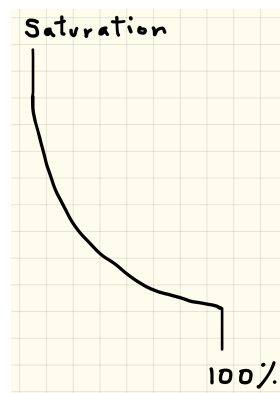
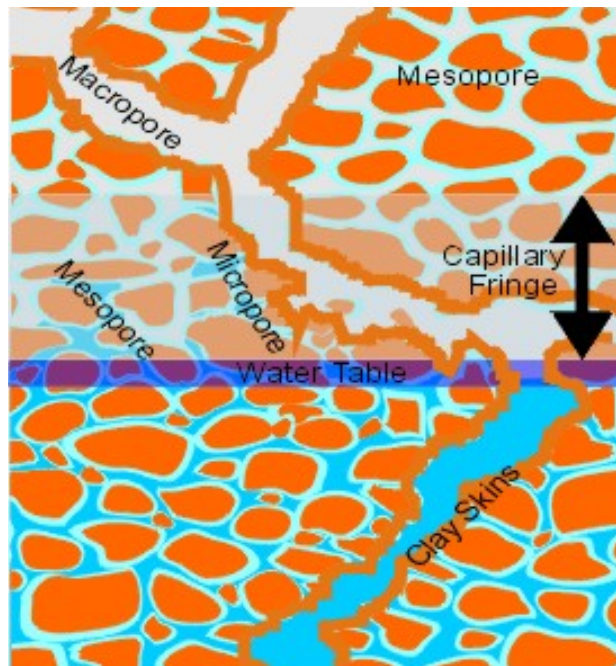
$$\sigma'_1 = (\gamma_{sat} - \gamma_w)_1 Z_1 + \gamma_w i_1 Z_1$$

$$\sigma'_2 = \sigma'_1 + (\gamma_{sat} - \gamma_w)_2 (Z_1 - Z_2) + \gamma_w i_2 (Z_1 - Z_2)$$

ΔP is added in the case of downward flow, or subtracted for upward flow.

Effective Stress in the Capillary Zone

In the capillary zone, bulk density and the effect of capillary tension are both functions of variable saturation.



Bulk density $\rho_b = \rho_s \frac{1 + W}{1 + e}$

Porosity and moisture content both vary with depth in the capillary zone. An estimate of the average bulk density uses a mean saturation (>50%) and $S = WG/e$

Pore pressures are negative in the capillary zone. This has the effect of *increasing* the effective stress:

$$u = -S\gamma_w h$$

$$\sigma' = \sigma - u = \sigma + S\gamma_w h$$

Where h is the height above the water table. Saturation does not fall to zero, so the top of the capillary fringe is the point where S becomes more or less constant.

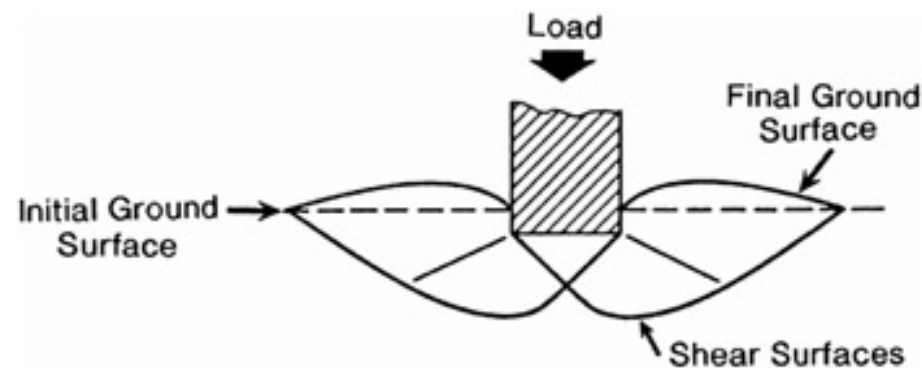
The net effective stress includes the effect of saturation on bulk density and pore pressure:

$$\sigma' = \sigma - u = \rho_b g Z + S\gamma_w h$$

Where Z is the depth below the ground surface.

Soil Stresses from Surface Loads

The effect of a surface load on soil stress at a given depth and offset from the load depends mainly on the load geometry.

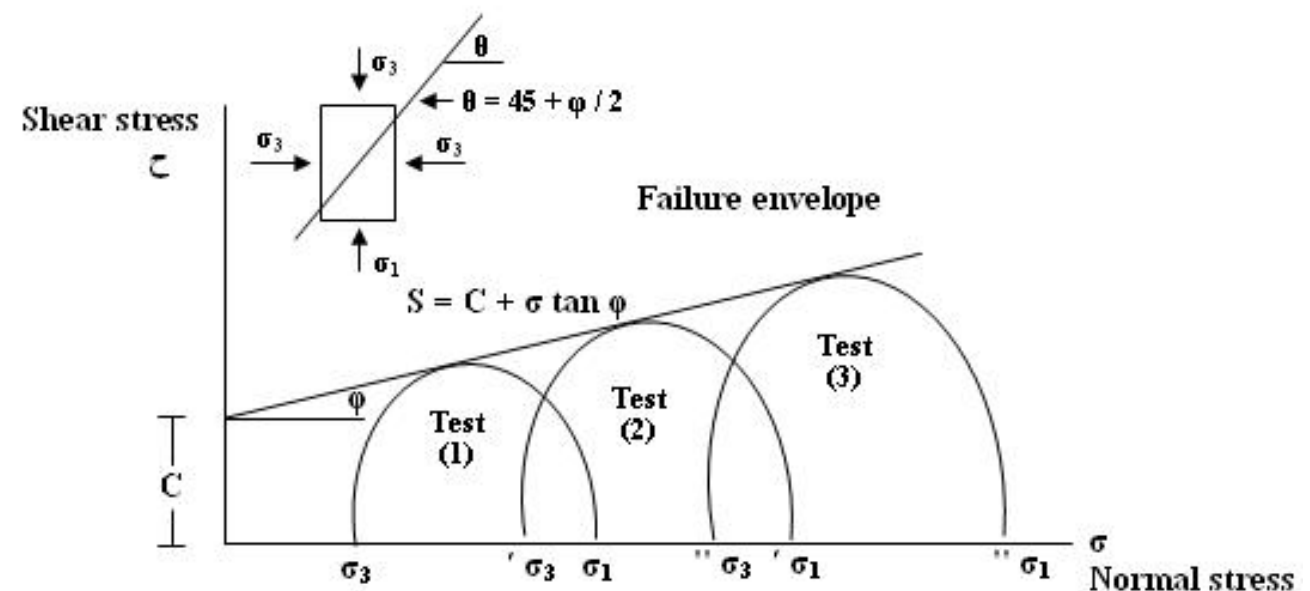
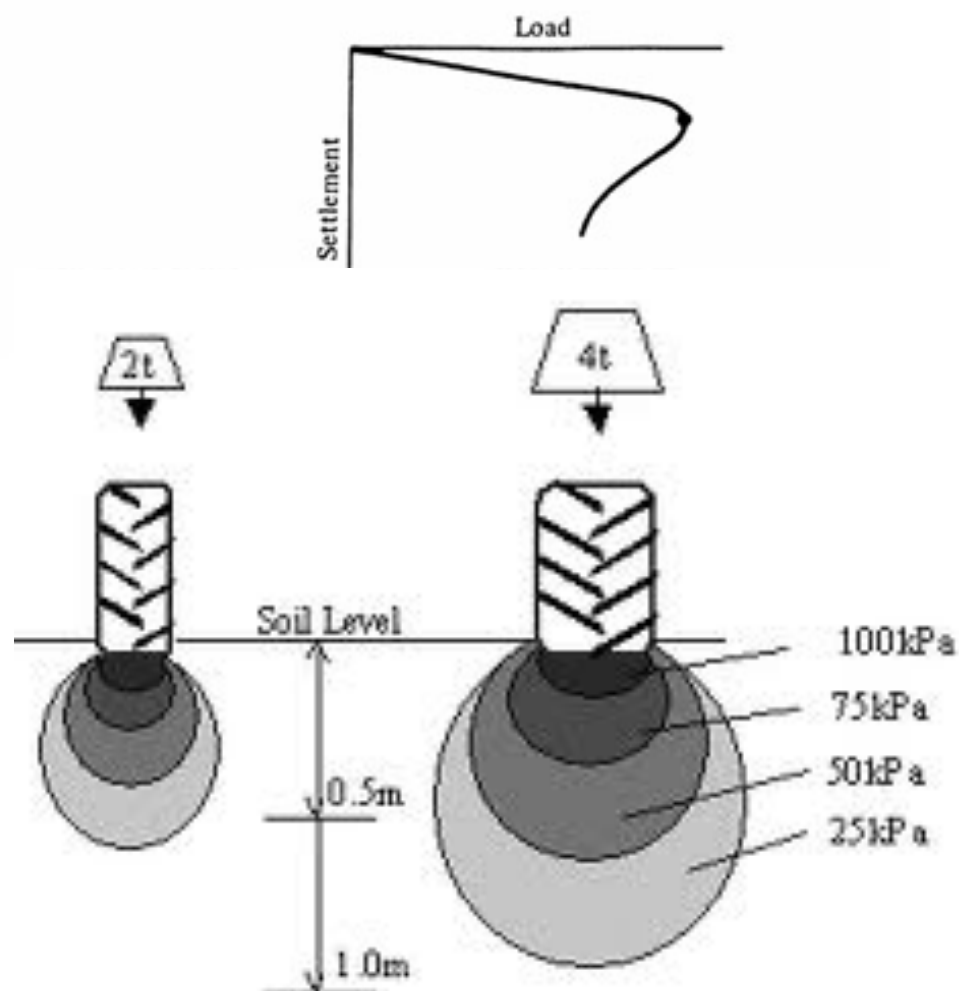


We often need to know the distribution of stress in soil to predict settlement and failure.

Loads may be applied at a point, such as a pier or vehicle load, or on a line or strip such as a wall footing, or over regular/irregular-shaped areas such as column footings and building foundations.

Stresses include normal and shear stresses induced by the applied load. Failure of soil is generally due to shear on a *maximum shear plane* which depends on the combination of horizontal and vertical loads.

Mohr's circle provides a means to predict shear stresses and angles of slip failure planes.



Lateral Stress

Lateral stresses arise because a soil mass in compression is confined by the surrounding soil. Soil's tendency to expand laterally under pressure (Poisson's ratio) is constrained, which results in a stress in that direction.

Poisson's ratio can be included in Hooke's law to estimate this effect:

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = 0$$

ϵ_x is the constrained x-direction strain given Young's modulus and Poisson's ratio for horizontal and vertical stresses σ_x , σ_y and σ_z

$$\sigma_x = \nu(\sigma_y + \sigma_z) = \sigma_y$$

The constraint allows us to solve for zero. The same result applies in the other lateral direction, so $\sigma_x = \sigma_y$.

$$\sigma_x = \nu(\sigma_x + \sigma_z) = \frac{\nu\sigma_z}{1 - \nu}$$

The result is that lateral stress in either the x or y direction is a simple function of vertical stress and Poisson's ratio.

This form of *lateral earth pressure* is unfortunately not accurate, mainly because soil is not ideally elastic. Other forms of lateral earth pressure will be developed when we cover the topic of retaining walls.

Total and Effective Stress

Surface stress is initially absorbed by an increase in pore pressure, but as pore water escapes under pressure the stress is transferred to effective stress.

This effect is most obvious in impermeable soils. In sandy soils, stress is almost immediately transferred to effective stress.

For a given increase in stress $\Delta\sigma$ the immediate increase in pore pressure is

$$\Delta u = \Delta\sigma$$

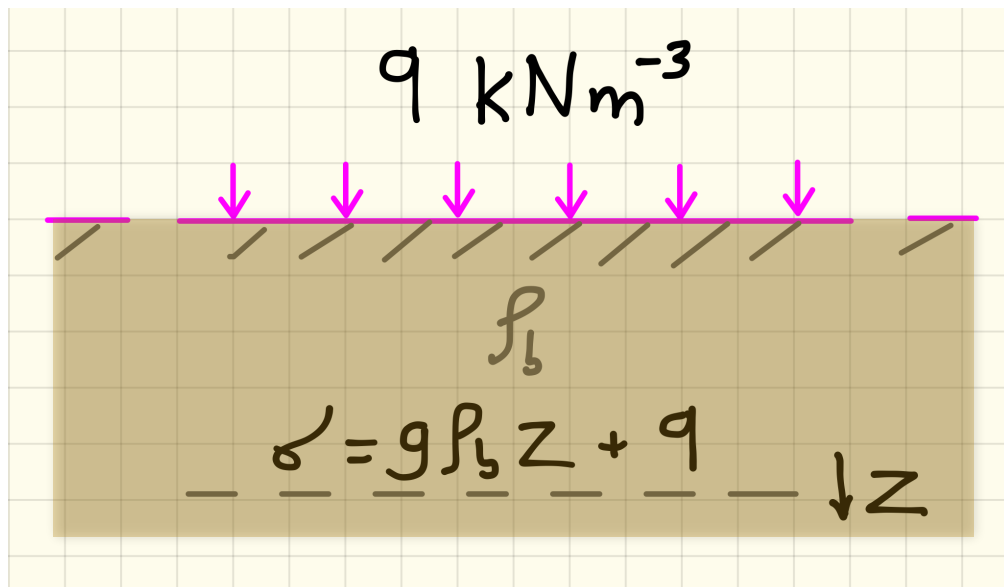
After a period of consolidation, the stress transfers to effective stress such that:

$$\Delta\sigma' = \Delta\sigma$$

$$\Delta u = 0$$

Soil Stresses from Surface Loads

1. "Infinite" Uniform Load. This idealised load is realistic close to the centre and base of any finite load, or for wide-area loads such as imported fill or extensive raft foundations over an entire site.



If the extent of area of the applied load is large relative to the depth ($Z < \text{half of the width}$ and near the centre) the total stress is modified by simply adding the applied load:

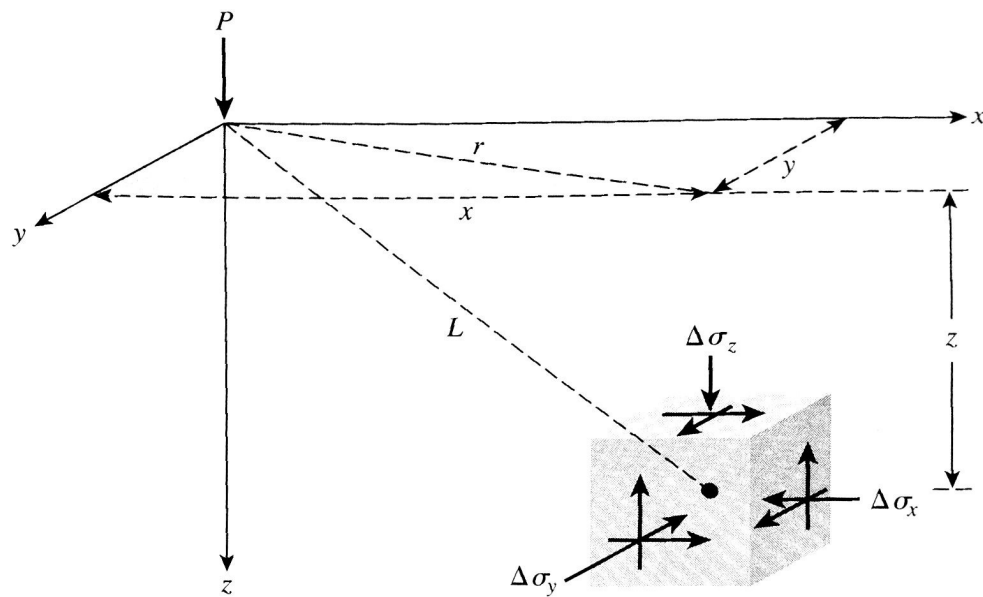
$$\sigma = \gamma_b Z + q$$

$$\sigma' = \sigma - u$$



Soil Stresses from Surface Loads

2. Point Load. Another idealised load: this is accurate at a reasonable distance from the centre and base of any finite and symmetrical load such as a square or circular footing.



Boussinesq's solution (1883) provides corrections in the three principal directions x, y (horizontal) and z (vertical).

Assuming no other horizontal stresses, the x, y corrections are the total stress in those directions.

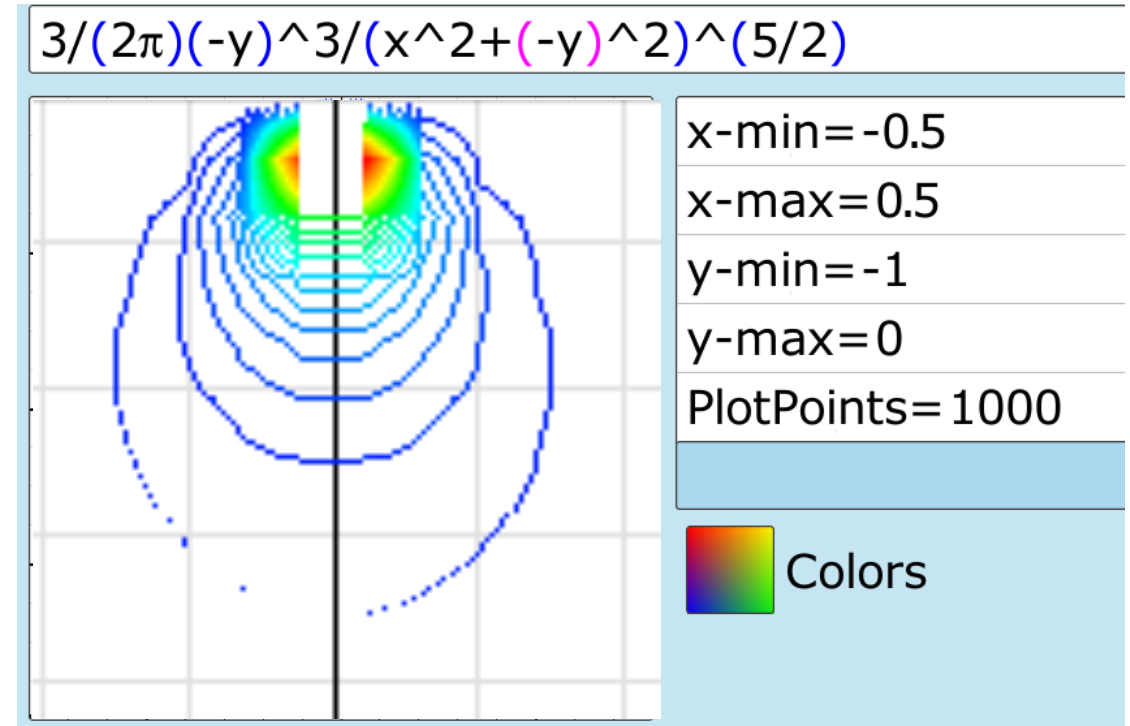
The z correction is added to vertical stresses due to self weight, seepage etc.

$$\Delta\sigma_z = \frac{3P}{2\pi} \frac{z^3}{(r^2 + z^2)^{5/2}}$$

$$\Delta\sigma_x = \frac{P}{2\pi} \left\{ \frac{3x^2z}{L^5} - (1 - 2\mu) \left[\frac{x^2 - y^2}{Lr^2(L + z)} + \frac{y^2z}{L^3r^2} \right] \right\}$$

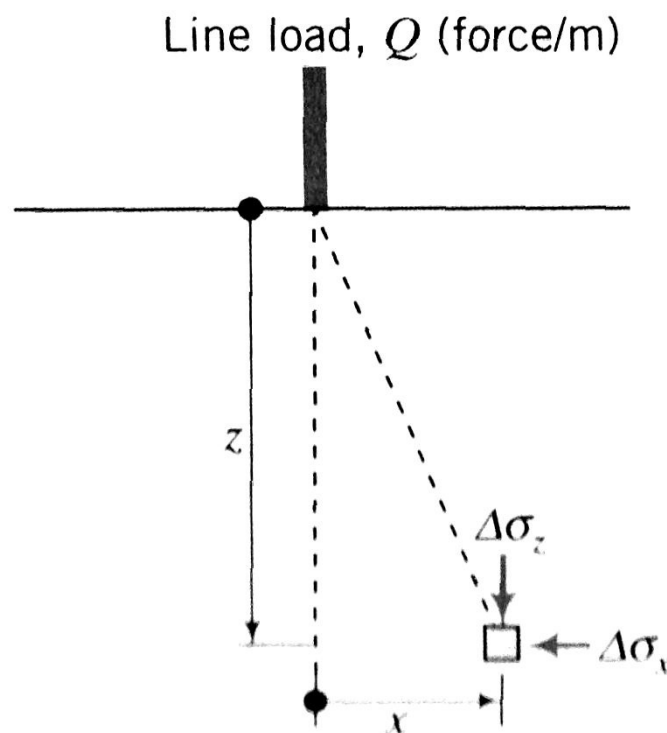
$$\Delta\sigma_y = \frac{P}{2\pi} \left\{ \frac{2y^2z}{L^5} - (1 - 2\mu) \left[\frac{y^2 - x^2}{Lr^2(L + z)} + \frac{x^2z}{L^3r^2} \right] \right\}$$

μ = Poisson's ratio = lateral strain per vertical strain



Soil Stresses from Surface Loads

3. Line Load. Similar to a point load, the idealised line load solution is realistic at a reasonable distance from the centre and base of any finite strip load such as a wall footing.



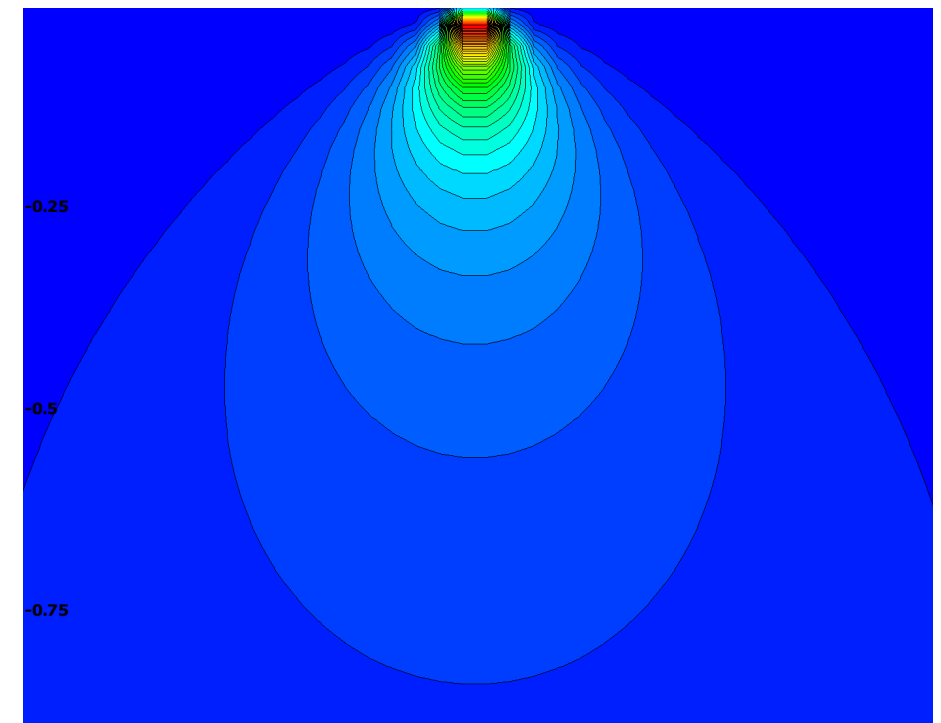
Again, the formula determines the increase in total stress due to an applied load. Other total stresses due to self-weight and seepage must be added to this:

As with a point load there are increases in horizontal and vertical stresses. Assuming the line load is parallel to the y -axis, however, there is no effect on lateral stresses in that direction.

$$\Delta\sigma_z = \frac{2Q}{\pi} \frac{z^3}{(x^2 + z^2)^2}$$

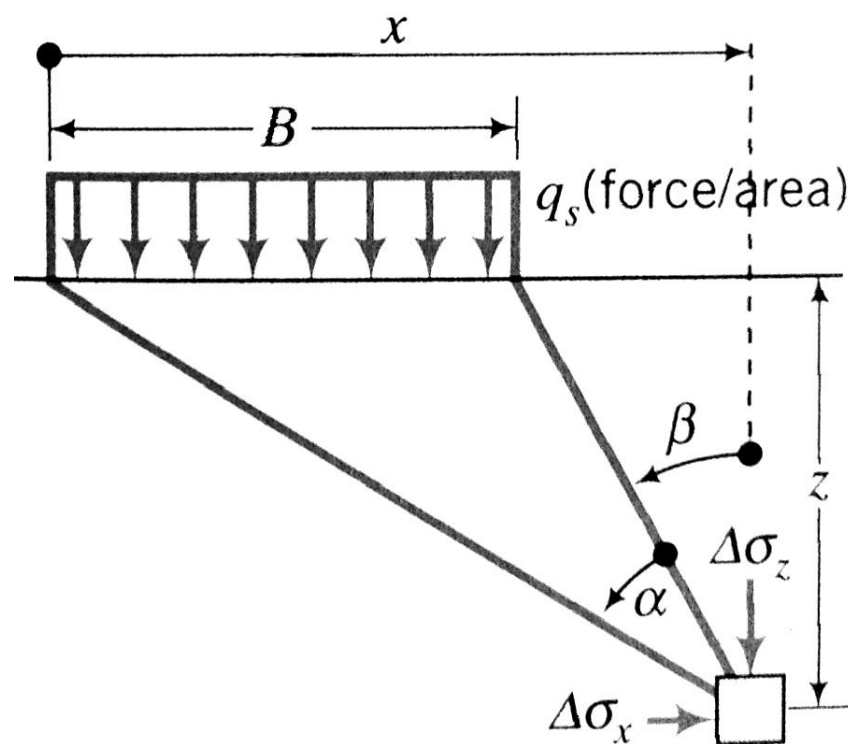
$$\Delta\sigma_x = \frac{2Q}{\pi} \frac{x^2 z}{(x^2 + z^2)^2}$$

As with a point load, the resulting stress distribution resembles a bulb with diminishing stress at increasing distance from the load



Soil Stresses from Surface Loads

4. Strip Load. More realistic than the idealised line load, the strip load solution is accurate at any distance from a finite strip load, including any point immediately below the strip.



In this case the formula depends on the geometry of angles between the two sides of the strip load.

To determine these angles correctly, note that at the base of the load, $\alpha = \pi$ and $\beta = 0$. At any other point $\alpha < \pi$. For $x < B$, $\beta < 0$ (right hand rule).

$$\Delta\sigma_z = \frac{q_s}{\pi}(\alpha + \sin(\alpha) \cos(\alpha + 2\beta))$$

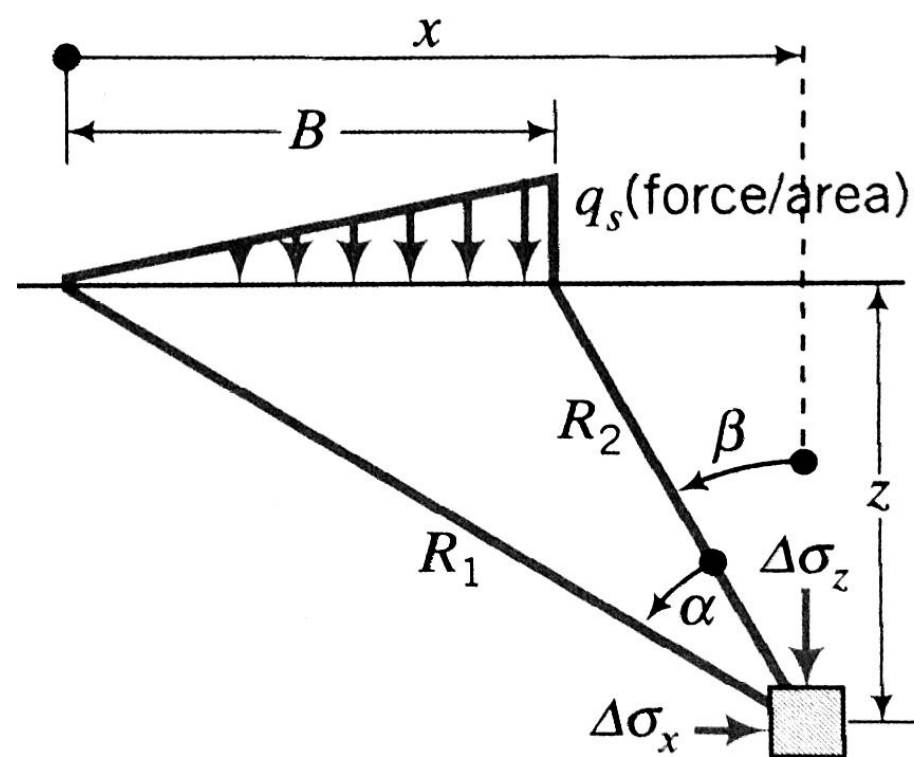
$$\Delta\sigma_x = \frac{q_s}{\pi}(\alpha - \sin(\alpha) \cos(\alpha + 2\beta))$$

$$\beta = \pm \cos^{-1} \left[\frac{z}{\sqrt{z^2 + (x - B)^2}} \right]$$

$$\alpha = \cos^{-1} \left[\frac{z}{\sqrt{z^2 + x^2}} \right] - \beta$$

Soil Stresses from Surface Loads

5. Banked Strip Load. A common problem is the stress imposed by an embankment with sloping faces. The trick is to superimpose triangular load solutions with a uniform strip load.



Again, the formula depends on the geometry of angles between the two sides of the strip load. The angles are defined in the same way as before, noting that the triangular embankment is to the left or right of the uniform central area of the embankment..

$$\Delta\sigma_z = \frac{q_s}{\pi} \left(\frac{x}{B} \alpha - \frac{1}{2} \sin(2\beta) \right)$$

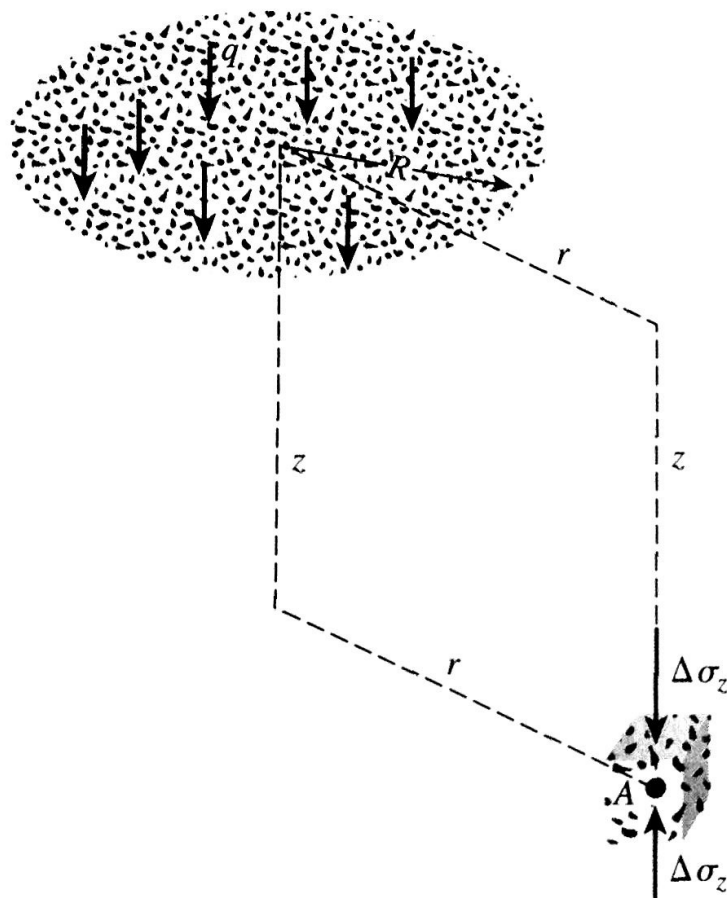
$$R_1 = \sqrt{x^2 + z^2}$$

$$\Delta\sigma_x = \frac{q_s}{\pi} \left(\frac{x}{B} \alpha - \frac{z}{B} \ln \left(\frac{R_1^2}{R_2^2} \right) + \frac{1}{2} \sin(2\beta) \right)$$

$$R_2 = \sqrt{(x - B)^2 + z^2}$$

Soil Stresses from Surface Loads

6. Uniform Circular Load. Finite load geometries more complex than a strip generally do not have a simple analytic solution. Instead, numerical analyses have been tabulated or graphed.



$$\Delta\sigma_z = q(A' + B')$$

(Vertical stress only)

Table 10.6 Variation of A' with z/R and r/R^*

z/R	0	0.2	0.4	0.6	0.8	1	1.2	1.5
0	1.0	1.0	1.0	1.0	1.0	0.5	0	0
0.1	0.90050	0.89748	0.88679	0.86126	0.78797	0.43015	0.09645	0.02787
0.2	0.80388	0.79824	0.77884	0.73483	0.63014	0.38269	0.15433	0.05251
0.3	0.71265	0.70518	0.68316	0.62690	0.52081	0.34375	0.17964	0.07199
0.4	0.62861	0.62015	0.59241	0.53767	0.44329	0.31048	0.18709	0.08593
0.5	0.55279	0.54403	0.51622	0.46448	0.38390	0.28156	0.18556	0.09499
0.6	0.48550	0.47691	0.45078	0.40427	0.33676	0.25588	0.17952	0.10010
0.7	0.42654	0.41874	0.39491	0.35428	0.29833	0.21727	0.17124	0.10228
0.8	0.37531	0.36832	0.34729	0.31243	0.26581	0.21297	0.16206	0.10236
0.9	0.33104	0.32492	0.30669	0.27707	0.23832	0.19488	0.15253	0.10094
1	0.29289	0.28763	0.27005	0.24697	0.21468	0.17868	0.14329	0.09849
1.2	0.23178	0.22795	0.21662	0.19890	0.17626	0.15101	0.12570	0.09192
1.5	0.16795	0.16552	0.15877	0.14804	0.13436	0.11892	0.10296	0.08048
2	0.10557	0.10453	0.10140	0.09647	0.09011	0.08269	0.07471	0.06275
2.5	0.07152	0.07098	0.06947	0.06698	0.06373	0.05974	0.05555	0.04880
3	0.05132	0.05101	0.05022	0.04886	0.04707	0.04487	0.04241	0.03839

Table 10.7 Variation of B' with z/R and r/R^*

z/R	0	0.2	0.4	0.6	0.8	1	1.2	1.5
0	0	0	0	0	0	0	0	0
0.1	0.09852	0.10140	0.11138	0.13424	0.18796	0.05388	-0.07899	-0.02672
0.2	0.18857	0.19306	0.20772	0.23524	0.25983	0.08513	-0.07759	-0.04448
0.3	0.26362	0.26787	0.28018	0.29483	0.27257	0.10757	-0.04316	-0.04999
0.4	0.32016	0.32259	0.32748	0.32273	0.26925	0.12404	-0.00766	-0.04535
0.5	0.35777	0.35752	0.35323	0.33106	0.26236	0.13591	0.02165	-0.03455
0.6	0.37831	0.37531	0.36308	0.32822	0.25411	0.14440	0.04457	-0.02101
0.7	0.38487	0.37962	0.36072	0.31929	0.24638	0.14986	0.06209	-0.00702
0.8	0.38091	0.37408	0.35133	0.30699	0.23779	0.15292	0.07530	0.00614
0.9	0.36962	0.36275	0.33734	0.29299	0.22891	0.15404	0.08507	0.01795
1	0.35355	0.34553	0.32075	0.27819	0.21978	0.15355	0.09210	0.02814
1.2	0.31485	0.30730	0.28481	0.24836	0.20113	0.14915	0.10002	0.04378
1.5	0.25602	0.25025	0.23338	0.20694	0.17368	0.13732	0.10193	0.05745
2	0.17889	0.18144	0.16644	0.15198	0.13375	0.11331	0.09254	0.06371
2.5	0.12807	0.12633	0.12126	0.11327	0.10298	0.09130	0.07869	0.06022
3	0.09487	0.09394	0.09099	0.08635	0.08033	0.07325	0.06551	0.05354

Soil Stresses from Surface Loads

7. Uniform Rectangular Load. In most cases the vertical stress increase below the centre of a rectangular load is most important.

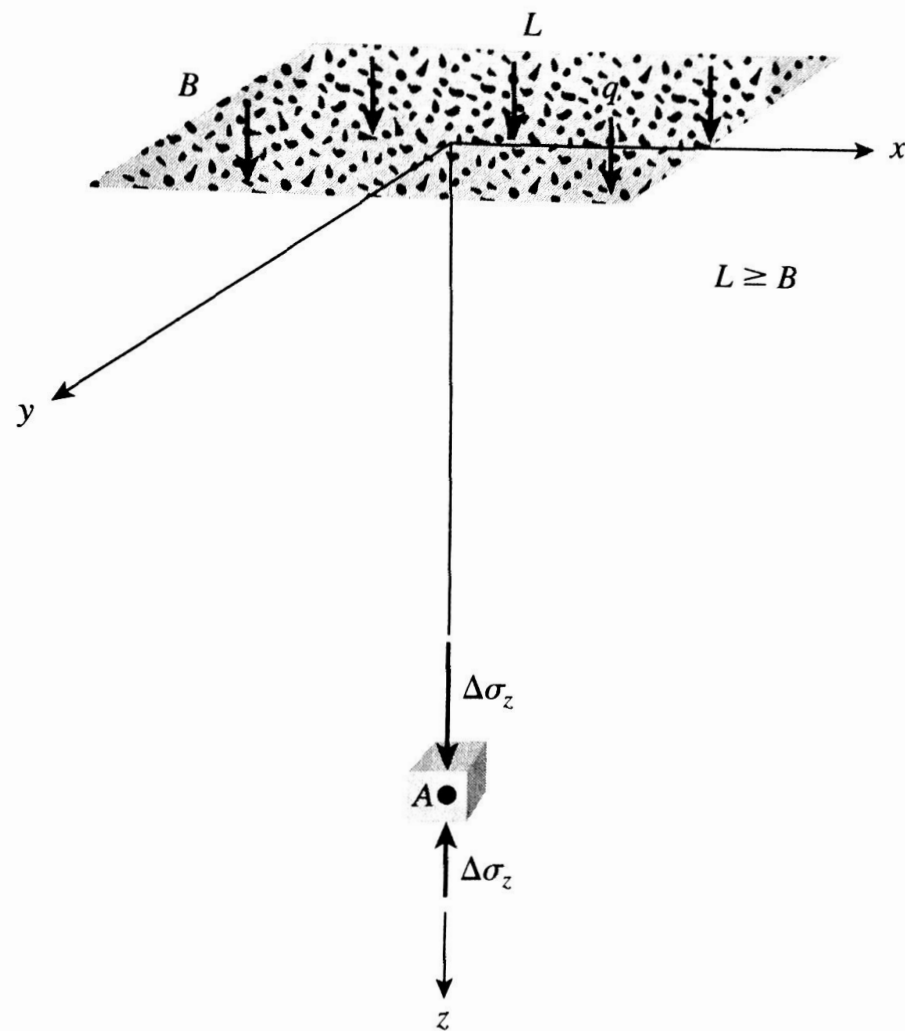


Table 10.9 Variation of I_4 with m_1 and n_1 [Eq. (10.35)]

n_1	m_1									
	1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

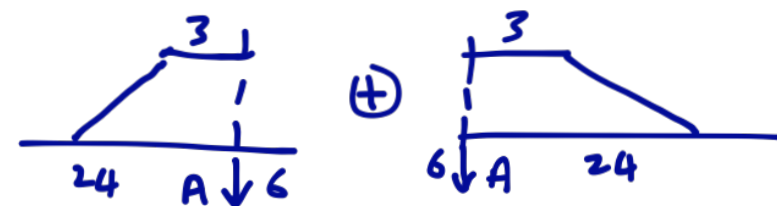
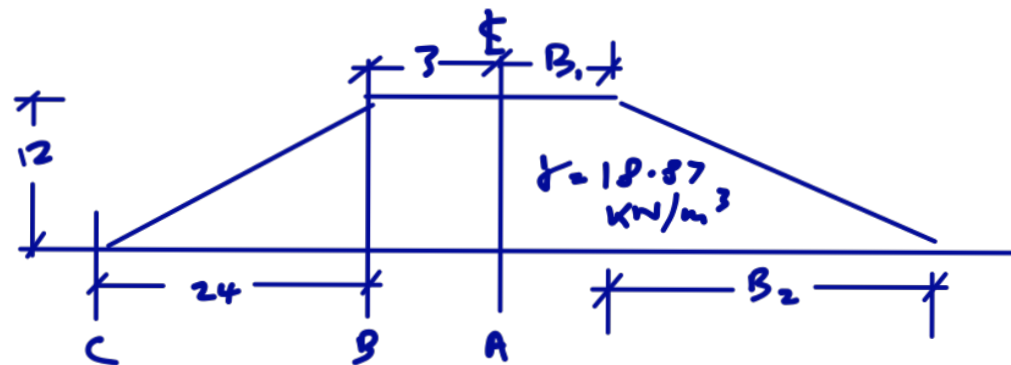
$$\Delta\sigma_z = qI_4$$

(Vertical stress only)

$$m_1 = \frac{L}{B} \quad n_1 = \frac{2z}{B}$$

Soil Stresses from Surface Loads

7. Embankments. Embankment loads can be calculated by superposition of partial sections of the embankment:



$$\text{Eq 10.20: } \Delta \sigma_z = \sum \frac{q_0}{\pi} \left[\left(\frac{B_1 + B_2}{B_2} \right) (\alpha_1 + \alpha_2) - \frac{B_1}{B_2} \alpha_2 \right]$$

$$\alpha_1 = \tan^{-1} \left(\frac{B_1 + B_2}{z} \right) - \tan^{-1} \left(\frac{B_1}{z} \right) \text{ rads}$$

$$\alpha_2 = \tan^{-1} \left(\frac{B_1}{z} \right) \text{ rads}$$

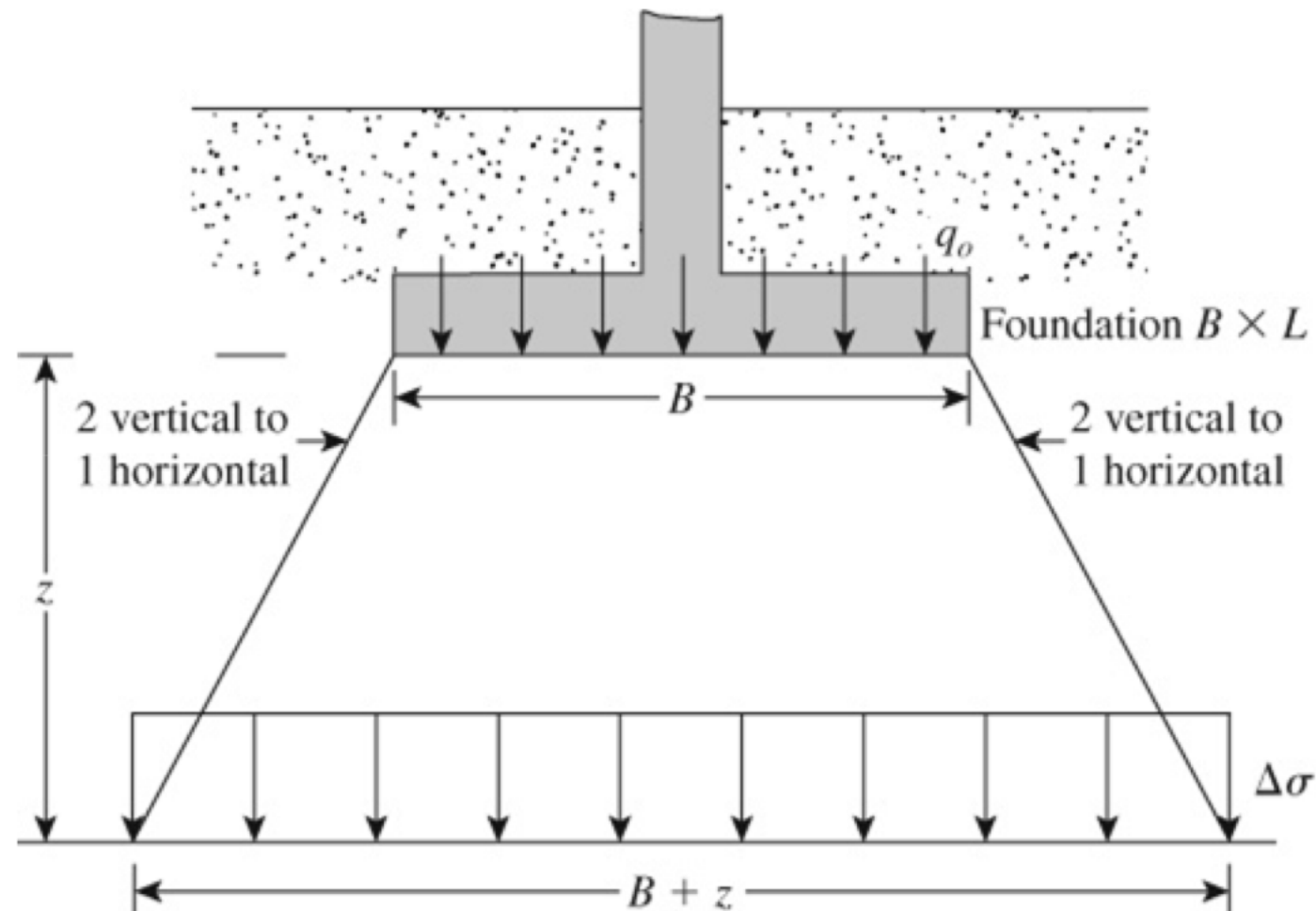
$$q_0 = \gamma H = 18.87(12) = 226 \text{ kPa}$$

Superposition: A : $B_1 = 3 \text{ m}$ $B_2 = 24 \text{ m}$ $\alpha_1 = 0.88 \text{ rad}$ $\alpha_2 = 0.46 \text{ rad}$

$$\Delta \sigma_z = (2) \times \frac{226}{\pi} \left(\frac{27}{24} (1.34) - \frac{3}{24} 0.46 \right) = \underline{209 \text{ kPa}}$$

$$2 \frac{226}{\pi} \left[\left(\frac{3+24}{24} \right) (.46 + .88) - \left(\frac{3}{24} \right) .46 \right] \quad (212)$$

Approximate 60 degree Method

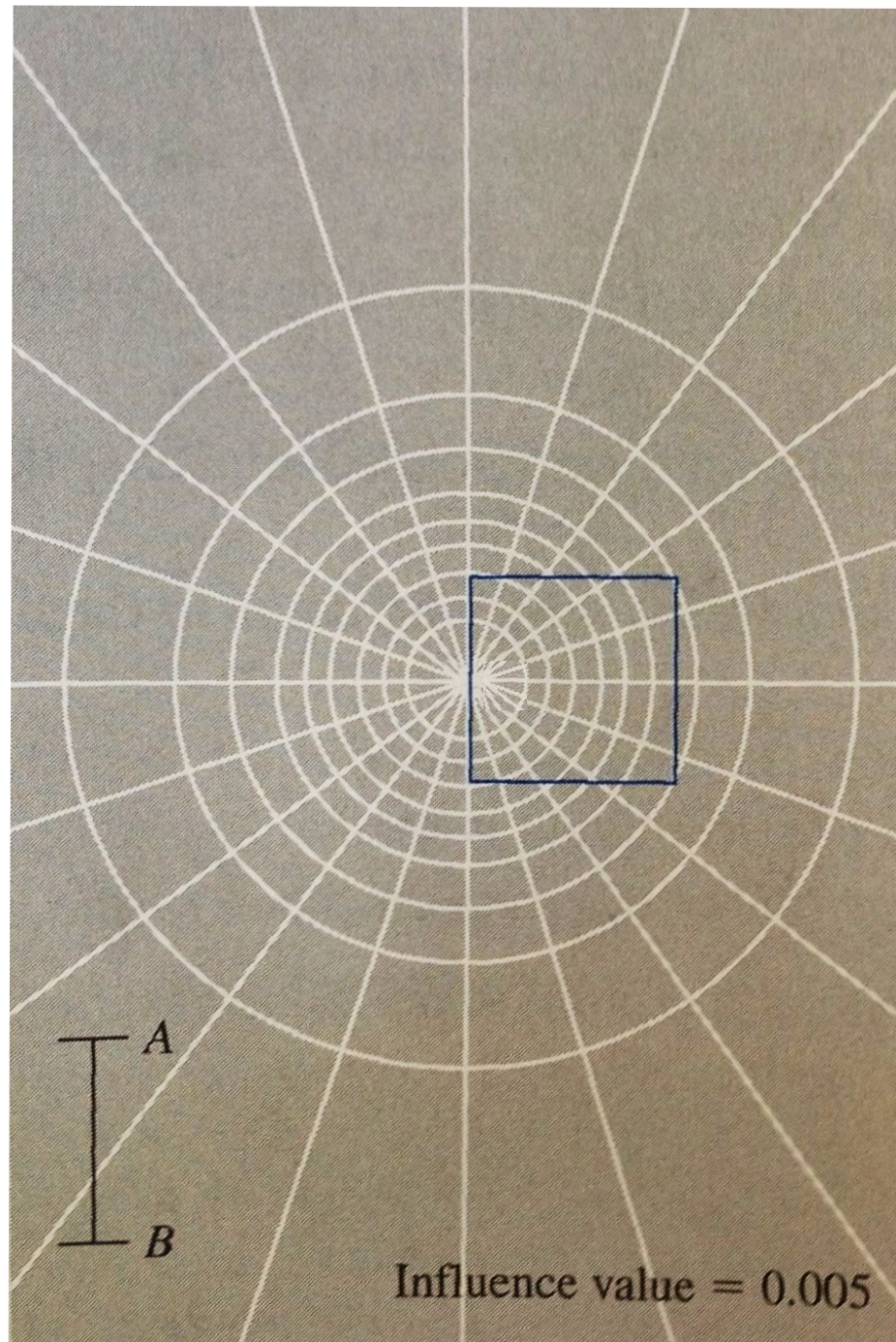


According to this method, the increase in stress at depth z is

$$\Delta\sigma = \frac{q_0 \times B \times L}{(B + z)(L + z)}$$

Soil Stresses from Surface Loads

8. Uniform Load of any shape. Boussinesq's solution can be generalised in such a way that the effect of loading by any shape can be calculated from an *influence diagram*.



$$\Delta\sigma_z = qIm$$

(Vertical stress only)

I is the influence value plotted on the chart, equal to $1/\text{the number of grid elements in the chart}$ eg $0.005 = 1/190$

m is the number of grid elements inside the plan of the loaded area, plotted to scale such that the distance AB is equal to the depth z .

This routine can be automated for computer program solution, which provides the basis for load-stress calculations in some commercial geotechnical software.

Example 10.10

The cross section and plan of a column footing are shown in the figure. Find the increase in vertical stress at point A.

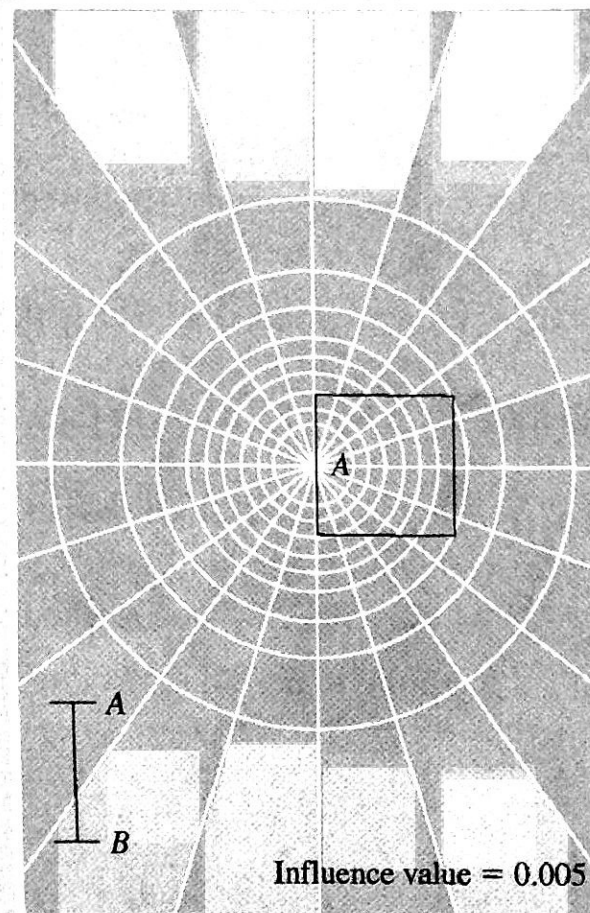
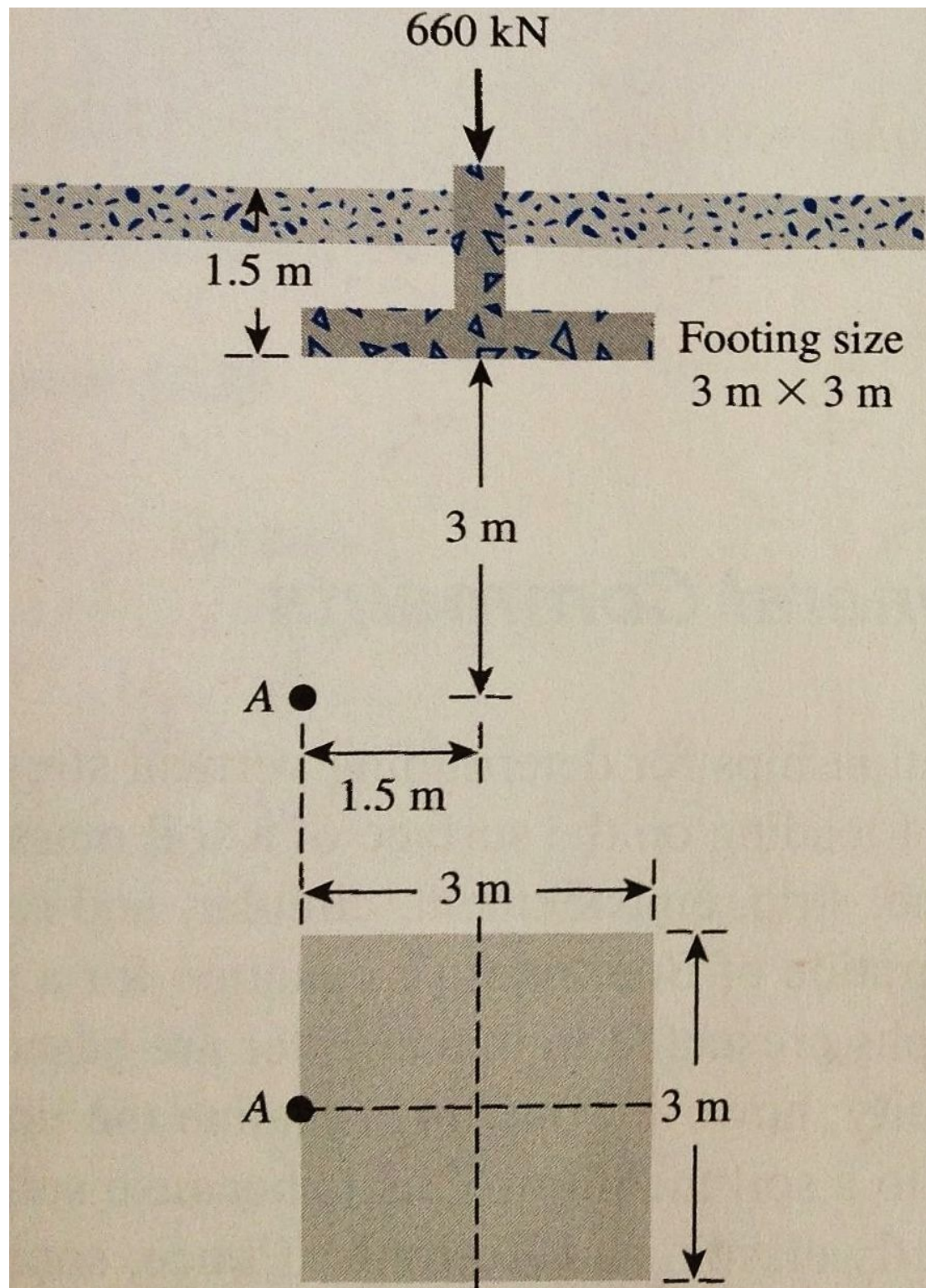


Figure 10.29 Determination of stress at a point by use of Newmark's influence chart

Solution

Point A is located at a depth 3 m below the bottom of the footing. The plan of the square footing has been replotted to a scale of $\overline{AB} = 3$ m and placed on the influence chart (Figure 10.29) in such a way that point A on the plan falls directly over the center of the chart. The number of elements inside the outline of the plan is about 48.5. Hence,

$$\Delta\sigma_z = (IV)qM = 0.005 \left(\frac{660}{3 \times 3} \right) 48.5 = 17.78 \text{ kN/m}^2$$

Readings

Chapter 9-10 Das

Decker, 1910: *Load Stress* Reading 8

